Problem 1

Consider the ARMAX model

$$y(k) + ay(k-1) = bu(k-1) + w(k) + cw(k-2), \tag{1}$$

where $\{u(k)\}_{k=1}^{N}$ and $\{w(k)\}_{k=1}^{N}$ are zero mean white noise processes with data length $N = 10^4$.

The data $\{u(k)\}_{k=1}^{N}$ and $\{y(k)\}_{k=1}^{N}$ are provided in the downloadable files below.

1. Use MATLAB or Julia to compute $\theta^{LS} = [a^{LS}, b^{LS}]^\top$ the least square estimate and $\Sigma(\theta^{LS})$ the covariance matrix of the estimate.

2. Let $z(k) = a^{LS}z(k-1) + a^{LS}u(k-1)$, where a^{LS} and b^{LS} are the least square estimates from above. Consider the regressor matrix

$$Z = \begin{bmatrix} z_2 & u_2 \\ \vdots & \vdots \\ z_N & u_N \end{bmatrix},$$

   and use MATLAB or Julia to find $\theta^{IV} = [a^{IV}, b^{IV}]^\top$ the instrumental variable estimate. Compute $\Sigma(\theta^{IV})$ the covariance matrix of the estimate.

   Assuming that θ^{LS} is deterministic, are the obtained estimates consistent?

3. Use pseudolinear regression method to estimate the system given in (1) .

   Hint Every time you compute the covariances of the estimates, assume that w is uncorrelated $E\{ww^T\} = \sigma^2 I$, where $\sigma^2 = E\{w^2\}$.

Return your implemented MATLAB functions (.m-files) as well as a self-explanatory answer including your numerical results and computations (.pdf file). Explain and motivate your reasoning clearly.

Note that you are supposed to implement the algorithms yourself, i.e., do not use the built-in MATLAB commands in the System Identification Toolbox.