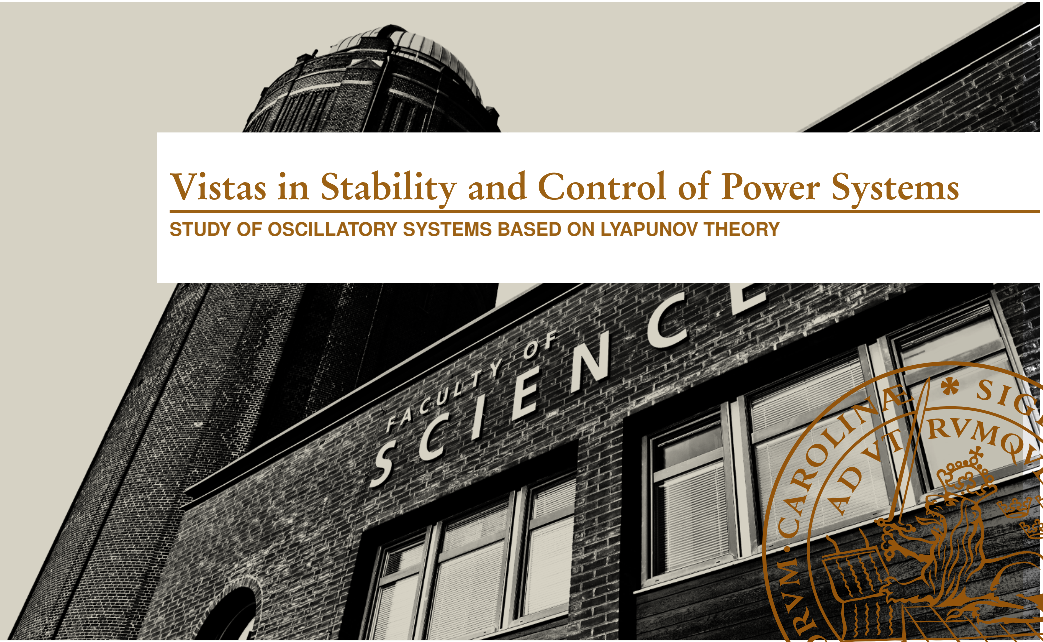
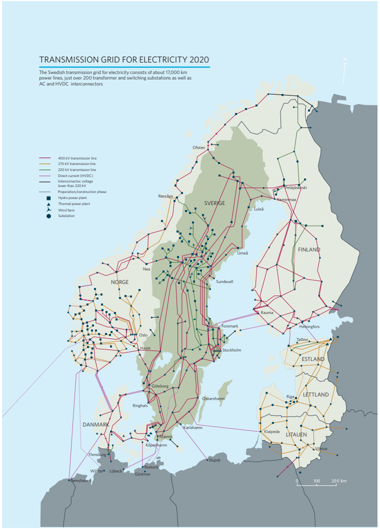
LUND UNIVERSITY

Vistas in Stability and Control of Power Systems

STUDY OF OSCILLATORY SYSTEMS BASED ON LYAPUNOV THEORY



Control of renewable power generation

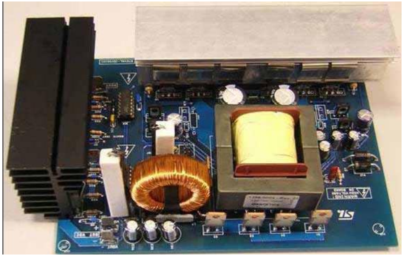


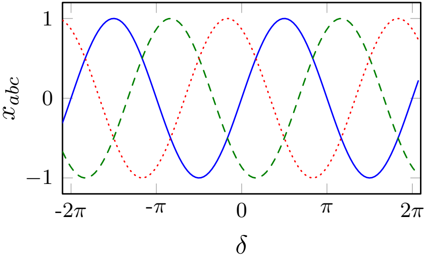
Problem setup

- power system network $\mathscr{G} = (\mathscr{V}, \mathscr{E})$
  - $\mathscr{V}$: **identical** (averaged and balanced) DC/AC converters
  - $\mathscr{E}$: **inductive** and **resistive** power transmission lines
- power system dynamics

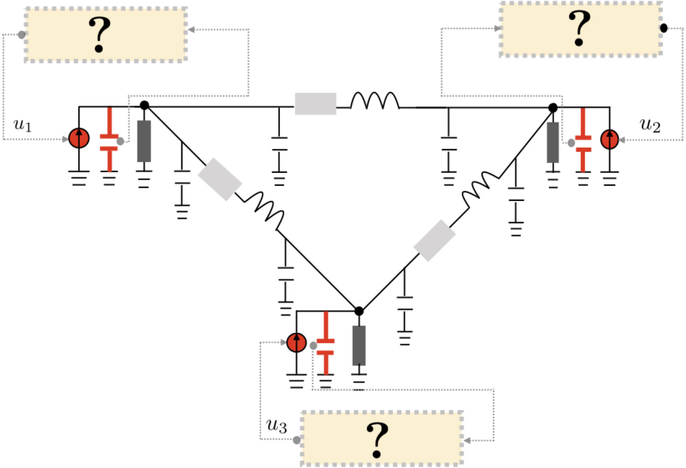
$$
\begin{aligned}
C_{dc}\dot{v}_{dc} &= -G_{dc}\, v_{dc} - \frac{1}{2}\,\mathrm{diag}(m^\top)\, i_{dq} + i_{dc}\\
L\dot{i}_{dq} &= -R\, i_{dq} + \frac{1}{2}\,\mathrm{diag}(m)\, v_{dc} - v_{dq}\\
C\dot{v}_{dq} &= -G\, v_{dq} + i_{dq} - \mathbf{B}\, i_{net,dq}\\
L_{net}\dot{i}_{net,dq} &= -R_{net}\, i_{net,dq} + \mathbf{B}^\top v_{dq}
\end{aligned}
$$

B: incidence matrix.





Control design

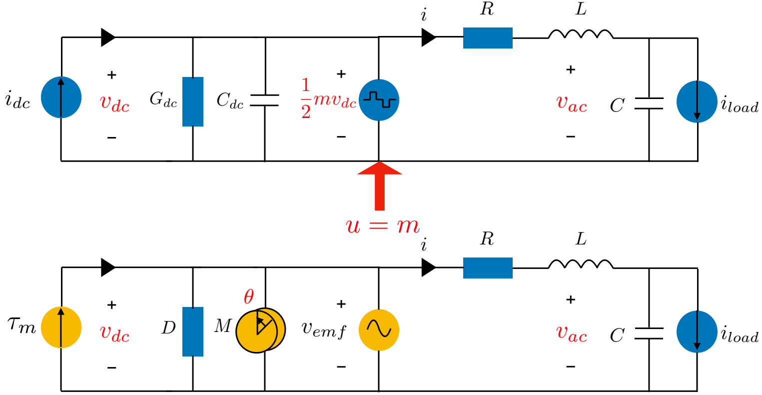


Solution 1: synchronous machines matching control

$$\dot{\gamma} = \eta(v_{dc} - v_{dc}^{*}),$$

$$u = \mu \begin{bmatrix} \sin(\gamma) \\ \cos(\gamma) \end{bmatrix},$$

$$\eta > 0, 1 > \mu > 0$$



Solution 1: structural properties of the vector field

- vector field has a **rotational symmetry**,

$$\dot{z} = F(s\theta + S(\theta)z) = F(z), \theta \in \mathbb{S}^1$$

  - invariance under group action: angle **shift** by $s = \begin{bmatrix} \mathbb{1}_n^\top & 0^\top & 0^\top \end{bmatrix}^\top$ and AC states **rotation** by $S(\theta)$
  - induces a **continuum**

$$[z] = \left\{ [\gamma^\top + \theta \mathbb{1}_n^\top, (v_{dc} - v_{dc}^* \mathbb{1}_n)^\top, (\mathbb{R}(\theta)x_{ac})^\top]^\top \,|\, \theta \in \mathbb{S}^1 \right\}$$

- **steady state** manifold

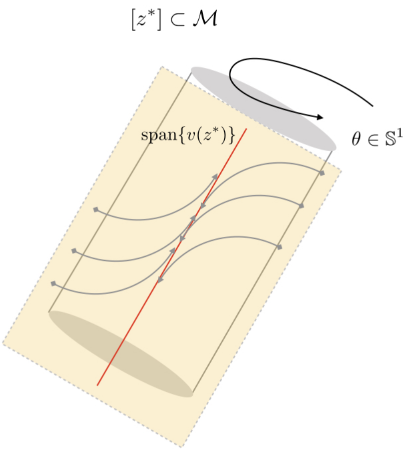
$$\mathscr{M} = \{z^* \in \mathbb{R}^N | F(z^*) = 0\}$$

  - **frequency** synchronous at $\omega^* = 2\pi \cdot 50$
  - **inherits** the rotational symmetry
  - 2π- **periodic**

Solution 1: system linearization

consider the linearized system with $\delta z = z - z^*$

$$\delta \dot{z} = \frac{\partial F}{\partial z}\Big|_{z=z^*} \delta z$$



- study of trajectories living on the tangent space
  - Jacobian with **zero-subspace** $\mathrm{span}\{v(z^*)\} \in \ker\left(\frac{\partial F}{\partial z}\Big|_{z^*}\right)$
  - **asymptotic stability** of the tangent vector of $[z^*]$
  - **sufficient** stability conditions

$$Q^* > \frac{\mu^2 v_{dc}^{*2}}{16R}, \; \|G\|_\infty < 1.$$

finite-gain $\mathscr{L}_2$- stability (from DC to AC circuit)

  - **resistive** damping/ reactive power: inter-area oscillations
  - extendable to **heterogeneous** converters and lines

Solution 1: local convergence

- study **variational** system with $\delta z \in T_z\mathscr{D}$, $[z^*] \subset \mathscr{D}$

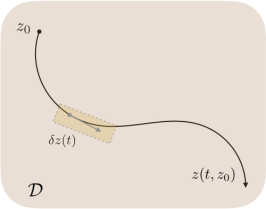
$$\delta\dot{z} = \frac{\partial F(z)}{\partial z}\delta z, \quad \left.\frac{\partial F(z)}{\partial z}\right|_{z=z^*} v(z^*) = 0$$

- **key** observation: $[z^*] \Leftrightarrow \text{span}\{v(z^*)\}$

$$[z^*] = z^* + \int_0^\theta \text{span}\{v(z^*)\}\, ds$$

- study **tangent vector**, then **curve integration**

$$[z] = z_0 + \int_0^\theta \delta z(s) ds, \; z_0 \in \mathscr{D}$$



Solution 1: distance to $[z^*]$

- define $\mathscr{F}$ Finsler structure. Then **Finsler** distance: $d : \mathbb{R}^N \times \mathbb{R}^N \longrightarrow \mathbb{R}_{\geq 0}$

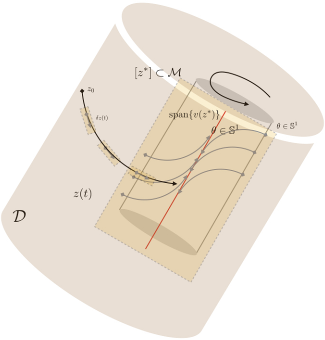
$$d(z_1, z_2) := \inf_{\Gamma(z_1, z_2)} \int_\nu \mathscr{F}\left(\nu(s), \frac{\partial \nu}{\partial s}, t\right) ds$$

- our Finsler structure: **differential** Lyapunov function

$$V(\delta z) = \delta z \left(P - \frac{P v(z^*) v^\top(z^*) P}{v^\top(z^*) P v(z^*)}\right) \delta z,\ P > 0$$

squared pseudo-distance of δz to $\mathrm{span}\{v(z^*)\}$

- **quotient space**: pseudo-distance by $\sqrt{V}$ on $\mathscr{D}$ is a distance on $\mathscr{D}/\sim$



Solution 1: summary

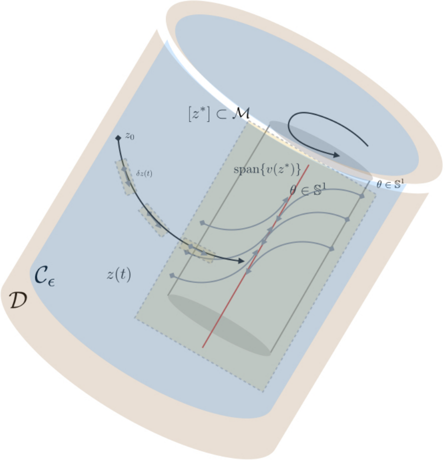
- **local contraction** for negative-definite Lie derivative with respect to span$\{v(z^*)\}$
- parametric **region of contraction** $\mathscr{C}_\epsilon(z)$

$$\mathscr{C}_\epsilon = \{(z, \delta z) \in \mathscr{D} \mid V(z, \delta z) \leq \epsilon\},$$

ϵ positive and sufficiently small

- **reminder**: steady state of interest

$$Q^* > \frac{\mu^2 v_{dc}^{*2}}{16R}, ||G(s)||_\infty < 1$$



Relevant papers/drafts

1. Steady state characterization and frequency synchronization of a multi-converter power system on high-order manifolds, T. Jouini and Z. Sun (ArXiv'20, submitted)
2. Fully decentralized conditions for local convergence of DC/AC converter network based on matching control, T. Jouini and Z. Sun (CDC'20)

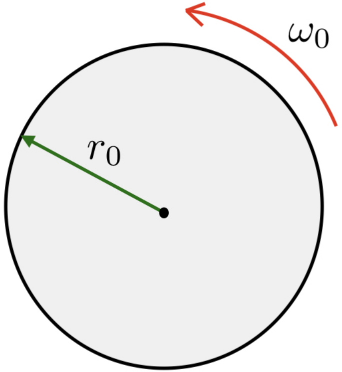
Solution 2: Harmonic oscillator

- rectangular coordinates

$$\dot{u} = \begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} u$$

- polar coordinates

$$\dot{r} = 0$$
$$\dot{\gamma} = \omega_0$$



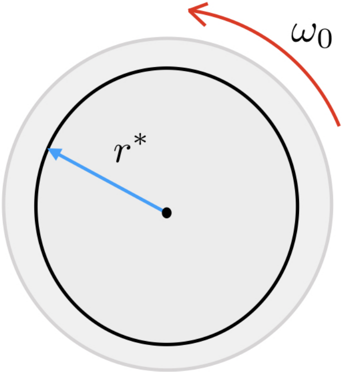
Solution 2: nonlinear oscillator

- rectangular coordinates

$$\dot{u} = \begin{bmatrix} -\zeta(r-r^*) & -\omega_0 \\ \omega_0 & -\zeta(r-r^*) \end{bmatrix} u$$

- polar coordinates

$$\dot{r} = -r\zeta(r-r^*), \zeta > 0$$
$$\dot{\gamma} = \omega_0$$



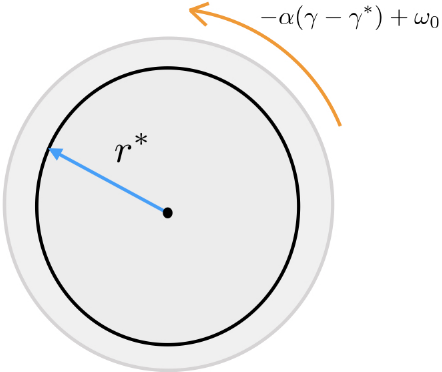
Solution 2: $\lambda-\omega$ oscillator

- rectangular coordinates

$$\dot{u} = \begin{bmatrix} \lambda(r,r^*) & -\omega(\gamma,\gamma^*) \\ \omega(\gamma,\gamma^*) & \lambda(r,r^*) \end{bmatrix} u$$

- polar coordinates

$$\dot{r} = r\lambda(r,r^*),$$
$$\dot{\gamma} = \omega(\gamma,\gamma^*)$$
$$\omega(\gamma,\gamma^*) = -\alpha(\gamma-\gamma^*) + \omega_0,\ \alpha > 0,$$
$$\gamma^* = \omega_0 t + \gamma_0^*,\ t \geq 0.$$



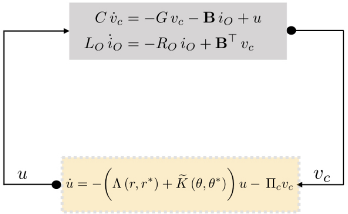
Solution 2: $\lambda-\omega$ oscillator

- prototypical models for **biological** oscillations
- **global** stability analysis

$$\max\{\alpha_1,\ldots,\alpha_n\} < \alpha^*,$$

$$\Pi(\gamma_{\max} - \Pi_{dq}L^{-1}) + (\gamma_{\max} - \Pi_{dq}L^{-1})^\top \Pi < -\xi_1 \Pi$$

$$\frac{C}{\sqrt{C^2\omega^{*2} + G^2}} < \frac{\xi_1\xi_2}{\xi_1\nu + \beta_1\beta_2},$$



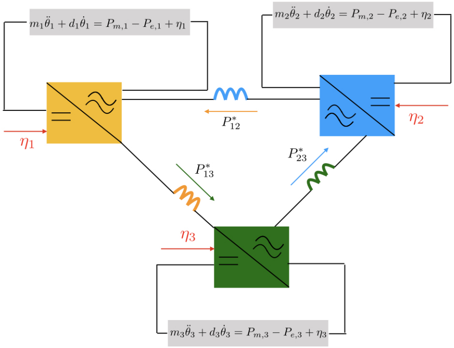
- **grid-forming** control with **droop** properties
- link to (dispatchable) virtual oscillator control
- validation on three-DC/AC converter setup

Relevant papers/drafts

1. Grid-forming ($\lambda - \omega$) virtual oscillator control in converter-based power systems, T. Jouini, E. Tegling and Z. Sun (soon on ArXiv'20)

We can do more . . .

- use $\mathscr{H}_2$ **norm optimization** for optimal control gain tuning
- use **learning algorithms** (log-linear) for optimal allocation
- use **Hamilton-Jacobi-Bellman-Equation** (HJB) for control design (future work)



Further read

1. **Gaussian processes**: Performance analysis and optimization of power systems with spatially correlated noise, T. Jouini and Z. Sun (L-CSS'20)
2. **Markov processes**: Distributed learning for optimal allocation in radial power systems, T. Jouini and Z. Sun (under review)