Predictive Control – Exercise Session 2
Real-Time Parameter Estimation

1. Consider the FIR model

$$y(t) = b_0 u(t) + b_1 u(t-1) + e(t) \quad t = 1, 2, \ldots$$

where $\{e(t)\}$ is a sequence of independent normal random variables with zero mean and standard deviation σ.

a. Determine the regressor vector and parameter vector of the linear regression model.

b. Write up the normal equation for the least-square estimation of the parameters b_0 and b_1.

2. Consider the FIR model in Problem 1. Assume that the input signal is a step, that is

$$u(t) = \begin{cases} 0, & t = 1 \\ 1, & t > 1. \end{cases}$$

a. Determine the least-square estimate of the parameters b_0 and b_1.

b. Analyze the covariance of the estimate when the number of estimates goes to infinity.

c. Analyze the covariance for the estimate of $b_0 + b_1$, when the number of estimates goes to infinity.

d. Relate the results to the notion of persistent excitation.

3. Consider the FIR model in Problem 1. Assume that the input signal is white noise with unit variance.

a. Determine the least-square estimate of the parameters b_0 and b_1.

b. Analyze the covariance of the estimate when the number of estimates goes to infinity.

c. Relate the results to the notion of persistent excitation.

4. The above questions relate to the standard least squares (LS) algorithm for estimating parameters given a set of data.

a. What problems would arise if this algorithm were to be applied in a real time setting, with the aim of continuously providing new parameter estimates?

b. How can the algorithm be modified in order to operate in such a way?

c. What further problems do you anticipate in the case where the parameters are time varying, $\theta = \theta(t)$?