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Scheduling Theory Real-Time Systems, Lecture 12 Martina Maggio 25 February 2020 Lund University, Department of Automatic Control www.control.lth.se/course/FRTN01	 [Real-Time Control System: Chapter 8] 1. Introduction 2. Execution Time Estimation 3. Scheduling Approaches 3.1 Static Cyclic Scheduling 3.2 Fixed Priority Scheduling 3.3 Earliest Deadline Scheduling 3.4 Reservation Based Scheduling
	Scheduling Theory
Introduction	Goal: guarantee that a set of tasks sharing resources (CPU) meet their deadlines. Notation: events occur (they require computations, like interrupts) which can be separated into aperiodic (sporadic) and periodic events; a task executes a piece of code in response to an event, the worst-case execution time (WCET) is an upper bound to the amount of CPU time the task execution requires (when the task is alone in the system); a deadline is the maximum allowed time for the task completion. Scheduling is the act of choosing which event to process at a given time (which task to execute at a given time).
 For hard real-time systems the deadlines must always be met. An offline test (performed before the system is started) is required, to check that there are no circumstances when deadlines are missed. A system is unschedulable if the scheduler does not find a way to switch between the tasks and meet all the deadlines. A test can be: sufficient if when the answer is yes all the deadlines are met; necessary if when the answer is no there really is a situation in which deadlines are missed; exact if it is both sufficient and necessary. We require a sufficient test and we would like it to be as close as possible to necessary. 	Execution Time Estimation

Execution Time Estimation	Measuring Execution Time
Basic question: How much CPU time does this piece of code need? Two major approaches: • measuring execution time; • analyzing execution time.	The code is compiled and run with measuring devices (for example a logical analyzer) connected or the operating system provides execution time measurements. If the piece of code has input data, a large set of test input data must be used and the longest time required is the longest time masured plus some safety margin.

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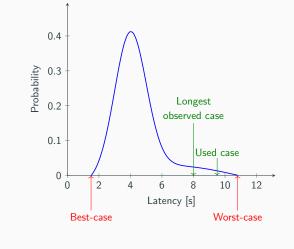
Measuring Execution Time

Measuring Execution Time

Problems:

- execution times are data dependent (for example sensor readings);
- caching (memories have different speed, a memory reference causing a cache miss takes much longer time that one that finds the data in the cache);
- pipelining and speculative execution;
- garbage collectors, etc.

The general problem of this approach is that we are **not guaranteed** that we have experienced the longest execution time.



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Analyzing Execution Time

Analyzing Execution Time

Problem: branching statements (if, case) we don't know which code is executed before runtime.

MOV(_X) DO CMP D0 5 Β1 if (x>5) BGT L1 if B1 x=x+1;MUL DO 3 B3 else MOVE DO B2 else x=x*3; JMP L2 B2 L1: ADD DO 1 B3 L2: ... Longest execution time: time(B1) + max(time(B2), time(B3))

Aim: a tool that takes the source code and automatically decides the longest execution time with formal correctness (research area the last 20 years).

 $\label{eq:problems: compiler dependent (different compilers generate different code, the remedy is working with the machine code).$

 $\mbox{Approach}:$ use the instruction tables from the CPU manufacturer and add up the instruction times of the individual statements.

Notation	Notation
NotationDescription C_i Worst case execution time of task i T_i Period of task i D_i Relative deadline of task i CPU Utilization: $U = \sum_{i=1}^{N} \frac{C_i}{T_i}$	$\tau \xrightarrow{\text{Period}} \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$
¹⁵ The critical instant	16 Scheduling Approaches
It can be shown that, in the uni-processor case, the worst situation from a schedulability perspective occurs when all the tasks want to start their execution at the same time instant. This is known as the <i>critical instant</i> . If the task set is schedulable in this situation it will also be schedulable in all other situations. If we can show that the task set is schedulable for the worst-case execution times, the task set will be schedulable if execution times are shorter. In uni-processor analysis, we just need to check for the worst case execution time and the critical instant.	 Static Cyclic Scheduling Fixed Priority Scheduling Earliest Deadline Scheduling Reservation Based Scheduling
17 Static Cyclic Scheduling	18 Static Cyclic Scheduling: Analysis
 Offline approach. Configuration algorithm generates an execution table (or calendar) using many different algorithms (optimization). The table repeats cyclically. The runtime dispatcher simply follows the table (sets up an hardware interrupt when a context switch should be performed, starts the first task in calendar, when the hardware interrupt arrives the first task is preempted and the second is run). Both preemptive and non-preemptive scheduling. 	 Analysis: trivial, run through the table and check timing requirements. Limitations: can only handle periodic tasks, aperiodic are made periodic using polling; the task calendar cannot be too large (shortest repeating cycle is the hyperperiod – the least common multiple, LCM, of the task periods – example: periods 5, 10 and 20 gives a cycle of 20, periods 7, 13 and 23 gives a cycle of 2093 – to reduce the calendar the periods can be made shorter).
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Static Cyclic Scheduling: Analysis

B();

A();

END;

IncTime(t, 4);

WaitUntil(t);

IncTime(t, 4);
WaitUntil(t);

Static Cyclic Scheduling: Example

• Advantages: a number of different constraints can be handled – for example exclusion constraints, precedence constraints. It is possible to use constraint programming to find a schedule.

• **Disadvantages**: inflexible (static design), building a schedule is NP-hard (potentially even if a schedule exists it can be difficult to find, good heuristic algorithms).

Task Name	Т	D	С
А	5	5	2
В	10	10	4

CPU Utilization: $U = \sum_{i=1}^{n} \frac{C_i}{T_i} = \frac{2}{5} + \frac{4}{10} = 0.8$ Schedule lenght: LCM(5, 10) = 10

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Static Cyclic Scheduling: Example	Static Cyclic Scheduling: implementation
Schedule (repeat): • from 0 to 2, task A; • from 2 to 6, task B; • from 6 to 8, task A; • from 8 to 10, no task. Worst case response time for task A: $3 \le D_A$ Worst case response time for task B: $6 \le D_B$	<pre>CurrentTime(t); LOOP A(); B(); A(); IncTime(t, 10); WaitUntil(t); END; Problem: what if it only takes 2 time units to execute task B? Then A would start before it should.</pre>
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Static Cyclic Scheduling: Implementation	Fixed Priority Scheduling
CurrentTime(t); LOOP A(); IncTime(t, 2); WaitUntil(t);	• Each task has a fixed priority.

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- The dispatcher selects the task with the highest priority.
- Preemptive.
- Used in most real-time kernels and real-time operating systems.

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Fixed Priority Scheduling: Rate Monotonic	Fixed Priority Scheduling: Rate Monotonic analysis
 Rate Monotonic is a scheme for assigning priorities to tasks. Priorities are set monotonically with rate. A task with a shorter period is assigned a higher priority. Introduced in C.L. Liu and J.W. Layland, <i>Scheduling Algorithms for Multiprogramming in a Hard Real-Time Environment</i>, JACM, Vol. 20, Number 1, 1973. 	Model: • Periodic tasks, • $D_i = T_i$, • Tasks are not allowed to be blocked or to suspend themselves, • Priorites are unique, • Task execution times bounded by C_i , • Task utilization $U_i = \frac{C_i}{T_i}$, • Interrupts and context switches take zero time.

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Fixed Priority Scheduling: Rate Monotonic analysis	Fixed Priority Scheduling: Rate Monotonic analysis
Result: • If the task set has a utilization below a utilization bound then all deadlines will be met. $U = \sum_{i=1}^{n} \frac{C_i}{T_i} \le n(2^{1/n} - 1)$ Sufficient condition (if the utilization is larger than the bound, the task set may still be schedulable). As $n \to \infty$ the utilization bound tends to	Since 1973 the models have become more flexible and the analysis better; M. Joseph and P. Pandaya, <i>Finding Response Times in a Real-Time</i> <i>System</i> , The Computer Journal, Vol. 29, No. 5, 1986. Notation: Notation Description
0.692 (log 2). If the CPU utilization is less than 69% all the deadlines are met. Alternative (tighter) test – Hyperbolic Bound: $\prod_{i=1}^{n} (\frac{C_i}{T_i} + 1) \le 2$	C_i Worst-case execution time of task i T_i Period of task i D_i Relative deadline of task i R_i Worst-case response time of task i

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Fixed Priority Scheduling: Rate Monotonic analysis

Scheduling test: $R_i \leq D_i$ (necessary and sufficient)

Model:

$$D_i \leq T_i, \qquad R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

where hp(i) is the set of tasks of higher priority than task *i*. The function $\lceil x \rceil$ is the *ceiling function* that returns the smallest integer $\ge x$. Recurrence relation, solved by iteration. The smallest solution is searched for.

$$R_i^{n+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

Start with $R_i^0 = 0$.

Limitations of the exact analysis: if the response time is larger than the period then the quantitative value cannot be trusted

Fixed Priority Scheduling: Rate Monotonic analysis

- Reason: The analysis does not take interference from previous jobs of the same task into account.
- More advanced analysis exists.

However, one still knows that the deadline will not be met, which is normally what one is interested in.

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Fixed Priority Scheduling: Rate Monotonic analysis

Best-case response time: Under rate-monotonic priority assignment one can also calculate the best-case response time R_i^b of a task *i*.

$$R_i^b = C_i^{\min} + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^b - T_j}{T_j} \right\rceil_0 C_j^{\min}$$

where C_i^{\min} is the best-case execution time of the task and $\lceil x \rceil_0 = \max(0, \lceil x \rceil)$.

Can be used to calculate the worst-case input-output latency of a control task.

Fixed Priority Scheduling: Rate Monotonic example

Task name	Т	D	С	Priority
А	52	52	12	low
В	40	40	10	medium
С	30	30	10	hight

Original (approximative) analysis:

$$\sum_{i=1}^{3} \frac{c_i}{\tau} = 0.814.$$

 $\sum_{i=1}^{n} T_i$ of T_i $3(2^{1/3} - 1) = 0.7798$. Using this analysis we cannot say if the task set is schedulable or not.

Hyperbolic bound:

 $\prod_{i=1}^{3} \left(\frac{C_i}{T_i} + 1\right) = 2.0508 \text{ (not } \leq 2\text{)}.$ Using this analysis we cannot say if the task set is schedulable or not.

Priority Scheduling: Rate Monotonic example sk name T D C Priority A 52 52 12 Jow	Fixed Priority Scheduling: Rate Monotonic example
$\frac{B}{C} = \frac{40}{40} \frac{40}{40} \frac{10}{10} \frac{\text{medium}}{\text{medium}}$ $\frac{B}{C} = \frac{40}{30} \frac{40}{30} \frac{10}{10} \frac{\text{medium}}{\text{hight}}$ $\frac{B}{C} = \frac{10}{30} \frac{10}{30} \frac{10}{10} \frac{10}{10} \frac{\text{medium}}{\text{hight}}$ $\frac{R_B^0 = 0, R_C^1 = C_C = 10, R_C^2 = C_C = 10}{R_B^0 = 0, R_B^1 = C_B = 10, R_B^0 = 0, R_B^0 = $	Task C has highest priority \rightarrow will not be interrupted and hence $R_C = C_C = 10$ (R_C^1) Task B has medium priority. The response time will be at least equal to $C_B = 10$ (R_B^1). During that time B will be interrupted once by C. Hence, the response time will be extended by the execution time of C, i.e. $R_B^2 = 10 + 10 = 20$. During this time B will only be interrupted once by C and that has already been accounted for, i.e. $R_B^3 = 20$. Task A has lowest priority. The response will be at least equal to $C_A = 12$ (R_A^1). During that time A will be interrupted once by C and once by B, i.e., $R_A^2 = 12 + 10 + 10 = 32$. During this time A will be interrupted twice by C and once by B, i.e., $R_A^3 = 32 + 10 = 42$. During this time A will be interrupted twice by C and twice by B, i.e., $R_A^4 = 42 + 10 = 52$. During this time no more unaccounted for interrupts

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• The rate monotonic policy is not very good when $D \leq T$.

- An infrequent but urgent task would still be given a low priority.
- The *deadline monotonic* ordering policy works better.

Fixed Priority Scheduling: Deadline Monotonic

- A task with a short relative deadline D gets a high priority.
- This policy has been proved optimal when D ≤ T (if the system is unschedulable with the deadline monotonic ordering then it is unschedulable with *all* other orderings).
- With $D \leq T$ we can control the jitter in control delay.

For a system with n tasks, all tasks will meet their deadlines if the sum over all tasks of the ratio between the worst-case execution time of the task and the deadline of the task is below a certain bound.

Fixed Priority Scheduling: Deadline Monotonic analysis

$$\sum_{i=1}^{n} \frac{C_i}{D_i} \le n(2^{1/n} - 1)$$

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Fixed Priority Scheduling: Deadline Monotonic analysis	Fixed Priority Scheduling: Extension – the blocking problem
Exact Analysis : The response time calculations from the rate monotonic theory is also applicable to deadline monotonic scheduling. Response time calculation does not make any assumptions on the priority assignment rule.	 How should interprocess communication be handled. The analysis up to now does not allow tasks to share data under mutual exclusion constraints (e.g. no semaphores or monitors) Main problem: a task <i>i</i> might want to lock a semaphore, but the semaphore might be held by a lower priority task; task <i>i</i> is blocked.
³⁹ Fixed Priority Scheduling: Extension – the blocking problem	4 Fixed Priority Scheduling: Further extensions
The blocking factor, B_i is the longest time a task <i>i</i> can be delayed by the execution of lower priority tasks $R_i = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$ Priority inversion may cause unbounded blocking time if ordinary locks are used. Different locking schemes have different blocking times: • ordinary priority inheritance; • priority ceiling protocol; • immediate inheritance protocol.	 Release Jitter: the difference between the earliest and latest release of a task relative to the invocation of the task; Context Switch Overheads; Clock Interrupt Overheads; Distributed systems using CAN.
41 Fixed Priority Scheduling: Overrun behavior	42 Earliest Deadline First (EDF) Scheduling
 Overrun means exceeding the worst-case execution time. With fixed priority schemes, this will only affect the current task and lower priority ones. These tasks can miss deadlines and/or not get any execution time at all. Higher priority tasks are unaffected. 	 Dynamic Approach: all scheduling decisions are made online. The task with the smallest absolute deadline runs. Preemptive. Ready-queue sorted in deadline order. Dynamic priorities. It is more intuitive to assign deadlines to tasks than priorities. Requires only local knowledge.
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Earliest Deadline First (EDF) Scheduling: analysis	Earliest Deadline First (EDF) Scheduling: analysis
 Simplest Model: Periodic Tasks, Each task <i>i</i> has a period <i>T_i</i>, a worst-case computation time requirement <i>C_i</i> and a relative deadline <i>D_i</i>, Tasks are independent, The kernel is ideal, <i>D_i</i> = <i>T_i</i>. 	Result: • If the utilization U of the system is not more than 100% then all the deadlines are met. $U = \sum_{i=1}^{n} \frac{C_i}{T_i} \le 1$ Necessary and sufficient condition. Advantage: the processor can be fully used. Less restrictive assumptions make the analysis harder (see RTCS for the analysis in the case $D_i \le T_i$).
45 Earliest Deadline First (EDF) Scheduling: example	46 Earliest Deadline First (EDF) Scheduling: example
Task name T D C A 8 8 1 B 5 5 2 C 10 10 4	Task name T D C A 8 8 1 B 5 5 2 C 10 10 4
B Image: Constraint of the second	B 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 47
Earliest Deadline First (EDF) Scheduling: example	Earliest Deadline First (EDF) Scheduling: example
Task nameTDCA881B552C10104	Task nameTDCA881B552C10104
A B C C C C C C C C C C C C C	A B C C C C C C C C C C C C C
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Earliest Deadline First (EDF) Scheduling: example	Earliest Deadline First (EDF) Scheduling: example
Task name T D C A 8 8 1 B 5 5 2 C 10 10 4	Task name T D C A 8 8 1 B 5 5 2 C 10 10 4
A + + + + + + + + + + + + + + + + + + +	A + + + + + + + + + + + + + + + + + + +
47	47
Earliest Deadline First (EDF) Scheduling: example	Earliest Deadline First (EDF) Scheduling: example
Task name T D C A 8 8 1 B 5 5 2 C 10 10 4	Task name T D C A 8 8 1 B 5 5 2 C 10 10 4
A B C C C C C C C C C C C C C	A B C O 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 C C C C C C C C C C C C C
47	47
Earliest Deadline First (EDF) Scheduling: example	Earliest Deadline First (EDF) Scheduling: example
Task name T D C A 8 8 1 B 5 5 2 C 10 10 4	Task name T D C A 8 8 1 B 5 5 2 C 10 10 4
A B C C C C C C C C C C C C C	A + + + + + + + + + + + + +
47	47

Earliest Deadline First (EDF) Scheduling: example	Earliest Deadline First (EDF) Scheduling: example
Task name T D C A 8 8 1 B 5 5 2 C 10 10 4 A +	Task name T D C A 8 1 B 5 5 2 C 10 10 4 A 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
47	47
Earliest Deadline First (EDF) Scheduling: example	Earliest Deadline First (EDF) Scheduling: example
Task name T D C A 8 8 1 B 5 5 2 C 10 10 4	Task name T D C A 8 8 1 B 5 5 2 C 10 10 4
A B C C C C C C C C C C C C C	A B C C C C C C C C C C C C C
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Earliest Deadline First (EDF) Scheduling: overrun behavior In the case of overrun all tasks will be affected, so they may all miss deadlines (there is a "domino" effect). However, in general EDF is more fair than priority-based scheduling – the available resources are distributed among all the tasks.	Earliest Deadline First (EDF) Scheduling: summary Also for EDF there exists a very well-developed schedulability theory. Resource access protocols similar to priority inheritance and ceiling.
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Reservation Based Scheduling	Reservation Based Scheduling: static and dynamic
 If a tasks overruns (executes longer than anticipated) this will effect other tasks negatively. In priority-based systems the priority decides which tasks that will be effected. In deadline-based systems all tasks will be effected. We want to provide temporal protection between tasks that guarantees that a certain task or group of tasks receives a certain amount of the CPU time. 	Use static cyclic scheduling for some tasks and let the other tasks be priority-based (event-based) which only may execute during the idle periods of the static schedule. Used in Rubus from Arcticus: • swedish RTOS used by Volvo, • red threads - statically scheduled, • blue threads - dynamically scheduled.
50 Reservation Based Scheduling: priority-based systems	Reservation Based Scheduling: priority-based systems
 How can, conceptually, a reservation-based scheduling system be implemented on top of ordinary priority-based scheduling. Each task or task set receives a certain percentage of the CPU. (50% + 30% + 20%). Can be viewed as if the tasks are executed on a correspondingly much slower CPU. Two variants: Each task set gets exactly its share of the CPU; Each task set gets at least its share of the CPU. Question: Over which time horizon does the CPU reservation hold? 	Each task set gets exactly its share of the CPU. The scheduler can be viewed as consisting of as many ready-queues as there are reservation sets. An external timer is set up to generate interrupts when it is time to switch which ready-queue that is active. One idle process in each ready-queue.
52 Reservation Based Scheduling: priority-based systems	Reservation Based Scheduling: SCHED_DEADLINE
Each task set gets at least its share of the CPU.One ready-queue.Make sure that the tasks belonging to the currently serviced task set all have higher priority than the tasks in the tasks sets which are not serviced.An external timer is set up to generate interrupts when it is time to switch between the tasks sets.Lower the priorities of the the tasks that have been serviced and raise the priorities of the tasks that should be serviced.A single idle task.	In Mainline Linux Kernel since 2 Feb 2014 17:12:22 (Linux 3.14.2).

Reservation Based Scheduling: Industrial practice	More Information
Beginning to emerge in commercial RTOS. Integrity from Green Hills Software Inc. 56	A nice introduction and overview of the state-of-the-art in uni-processor scheduling of real-time systems can be found in: "Real Time Systems by Fixed Priority Scheduling" by Ken Tindell and Hans Hansson, Dept. of Computer Systems, Uppsala University http://www.docs.uu.se/~hansh/fpsnotes-9710.ps