Real-Time Systems
Model for the Ball and Beam Process

1. The Process

The ball and beam process, see Figure 1, consists of a horizontal beam and a motor. A rolling ball should be balanced on the beam. The beam angle is controlled by the motor.

Figure 1 The ball and beam process.

The output signals from the process are voltages in the interval ±10 V, representing the beam angle relative the horizontal plane (ϕ) and the position of the ball (x). The input signal to the process is a voltage in the interval ±10 V that causes the beam to rotate with a rotation speed proportional to the voltage.

Process Model

The process model can be decomposed into the model of the angle process, $G_\phi(s)$, and a model of how the ball position is influenced by the beam angle, $G_x(s)$. The total transfer function from the input voltage to the voltage that indicates the ball position is then $G_\phi(s)G_x(s)$.

The angle process Neglecting the dynamics in the motor, the angular velocity of the beam is proportional to the motor voltage, i.e.

$$\dot{\phi}(t) = k_\phi u(t)$$

Hence

$$G_\phi(s) = \frac{k_\phi}{s}$$

Through experiments it has been determined that $k_\phi \approx 4.4$.

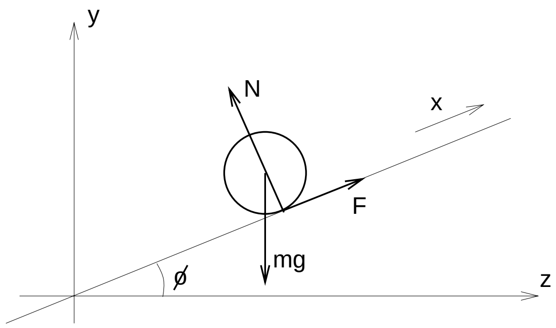

Figure 2 Forces acting on the ball on the beam.

The ball position process The dynamics of the ball position can be determined by classical mechanics. Assume that the ball position is x, the beam angle is ϕ and that the beam lies in the center of rotation ϕ. Coordinates and forces are defined according to Figure 2. The force equations are

$$\begin{aligned} m\ddot{y} &= -mg + N\cos\phi + F\sin\phi \\ m\ddot{z} &= -N\sin\phi + F\cos\phi \end{aligned}$$

Multiply with $\sin\phi$ and $\cos\phi$ and sum:

$$m(\ddot{y}\sin\phi + \ddot{z}\cos\phi) = -mg\sin\phi + F \tag{1}$$

The condition for a rolling ball without friction is $Fr = J\dot{\omega} = -J\ddot{x}/r$ For a solid ball we have $J = 2mr^2/5$, hence $F = -2m\ddot{x}/5$. Further, we have $z = x\cos\phi$ and $y = x\sin\phi$. Derive these expressions twice, multiply with $\cos\phi$ and $\sin\phi$, sum. This gives

$$\ddot{y}\sin\phi + \ddot{z}\cos\phi = \ddot{x} - x\dot{\phi}^2$$

Insert this expression and the expression for F in (1). We then get

$$m(\ddot{x} - x\dot{\phi}^2) = -mg\sin\phi - \frac{2}{5}m\ddot{x}.$$

If we finally assume that ϕ and $\dot{\phi}$ are small we obtain

$$G_x(s) = -\frac{5g}{7s^2} = -\frac{k_x}{s^2}$$

where $k_x \approx 7.0$. Notice that the gain of this process is negative.