Lund University

Department of
AUTOMATIC CONTROL

Automatic Control, Basic Course

Exam 28 October 2019, 8:00-13:00

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem.

Preliminary grades:

Grade 3: at least 12 points,
4: at least 17 points,
5: at least 22 points.

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!

Solutions to exam in Basic Control 2019-10-28

1. A process can be described by the following differential equation:

$$\ddot{y} + 9\dot{y} + 8y = \dot{u} - 4u$$

a. What is the transfer function of the process from U to Y? (1 p)

b. What are the poles and zeros of the process? Is it asymptotically stable? Motivate your answer. (1 p)

c. The system is controlled with a P-controller with gain K, see Figure 1. For what values of K is the closed loop system asymptotically stable? (answer for both positive and negative values of K). (2 p)

d. Suppose that we make a unit step change in setpoint R. Determine the stationary control error E. How small can we make the error using the P controller? (2 p)

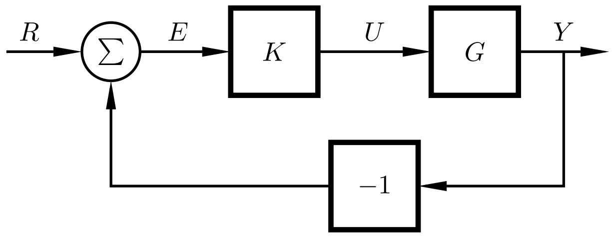

Figure 1: The closed loop system for problem 1.c.

2. The motion of a frictionless pendulum may be described by the following differential equation

$$\ddot{\theta} + \sin(\theta) = u$$

Here, $\theta(t)$ denotes the angle between its current position and its downward position at rest, and $u(t)$ denotes the control signal with which we influence the pendulum.

a. Introduce the states $x_1 = \theta$ and $x_2 = \dot{\theta}$ and express the system on state-space form. Assume that we can only measure the angle, i.e. $y = x_1$. (1 p)

b. Find all stationary points when we don't influence the system, i.e. when $u(t) = 0$. (1 p)

c. We are especially interested in the behavior of the pendulum when it hangs down. Linearize the system around a stationary point corresponding to such a position. (1 p)

d. Show that the linearized system is stable but not asymptotically stable.

Suppose that we allow the pendulum to start from a vertical upright position with angular velocity 1 rad/s. Then, because there is no friction, the pendulum will rotate about its joint forever. Why is this behavior impossible for a stable system? How did we manage to prove stability despite this?

(1 p)

3. State which step response (I-VI, see Figure 2) and Bode plot (A-F, see Figure 3) match each of the four transfer functions below. Each correctly justified match is given 0.5 points. (4 p)

$$G_1(s) = \frac{s + 0.1}{s^2} \qquad G_2(s) = \frac{1}{s^2 + 0.4s + 1}$$

$$G_3(s) = \frac{1.3}{s^2 + s + 1} \qquad G_4(s) = \frac{2}{s + 2}$$

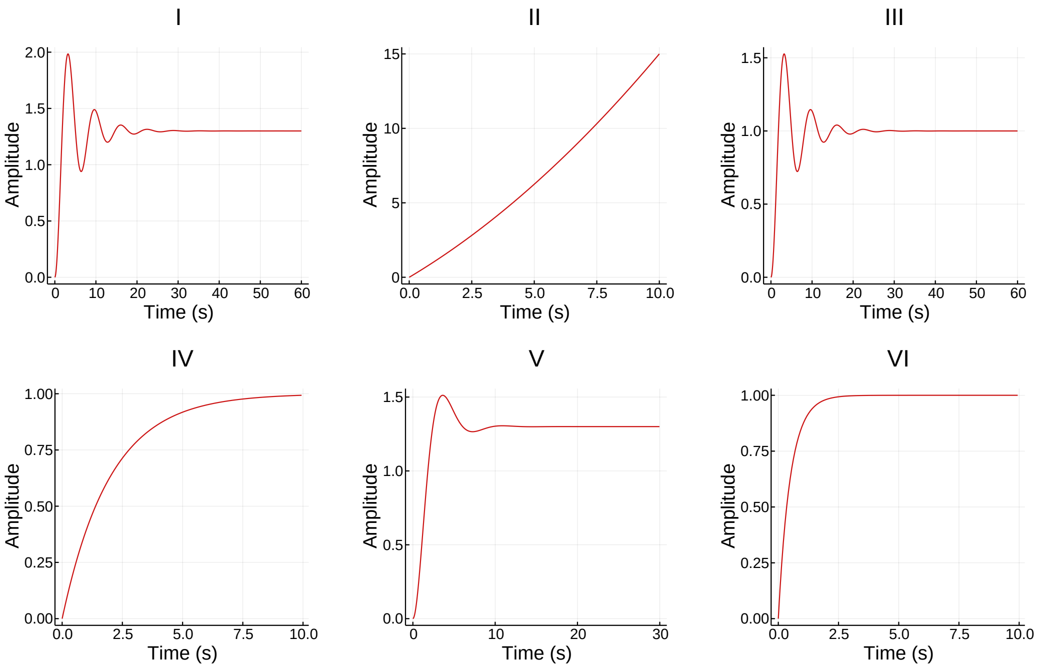

Figure 2: Step responses for problem 3.

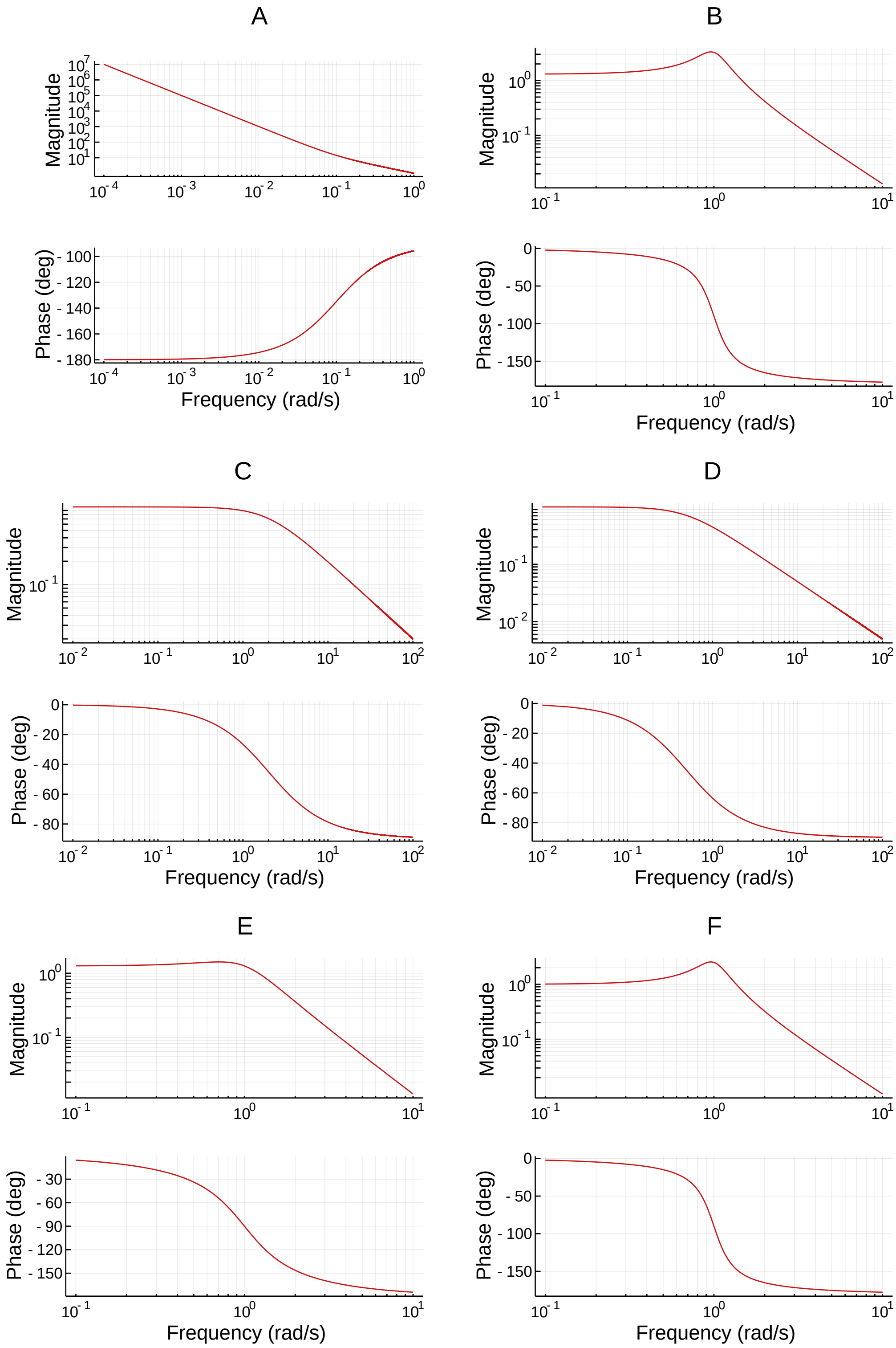

Figure 3: Bode plots for problem 3.

4. In the second laboratory exercise we considered a process consisting of two tanks, one on top of the other. The control signal was the inflow to the upper tank, and its outflow was the inflow to the lower tank. Recall now that part of the preparatory assignments consisted of deriving a state-space description for the lower tank system. The linearization around a certain stationary point was then found to be

$$\dot{x} = \begin{pmatrix} -\gamma_1 & 0 \\ \gamma_1 & \gamma_2 \end{pmatrix} x + \begin{pmatrix} \delta \\ 0 \end{pmatrix} u$$
$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} x$$

Here, x_1 and x_2 denote the deviations from the stationary values of the upper and lower water levels respectively, given a constant inflow. The deviation of the control signal from the latter is denoted by u. The constants $\gamma_1, \gamma_2, \delta > 0$ are material parameters.

During the lab session we only considered the control error for our feedback law; now we would like to exploit the states. Unfortunately, we only measure the water level of the lower tank and therefore do not know x_1, which we must estimate.

a. Design a Kalman filter such that the poles are placed at $s = -2 \pm 2i$. (2 p)

b. Suppose now that we only measure the water level of the upper tank. Show that the system is not observable. From a physical point of view, can you tell which states are non-observable? Motivate using the definition of observability. (1 p)

5. Consider the Bode plot of the open-loop transfer function G_0 in Figure 4.

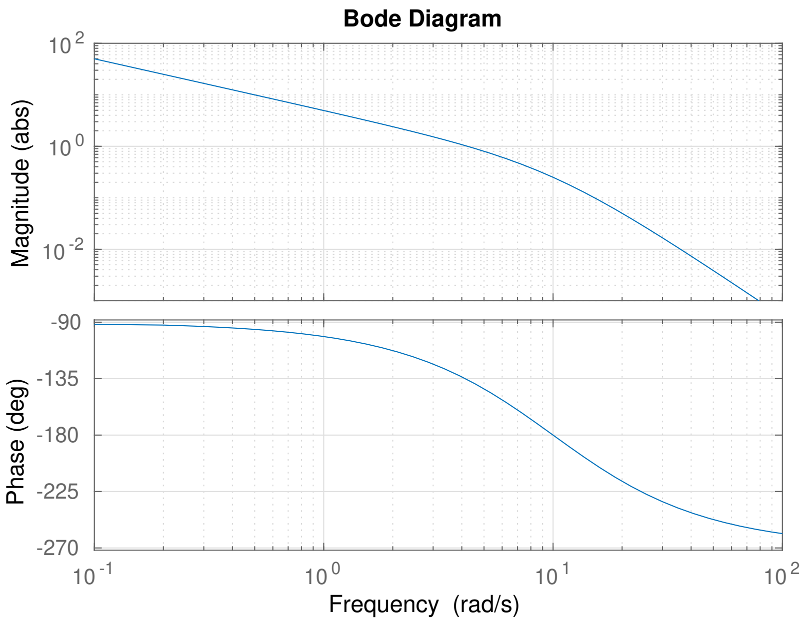

Figure 4: The open-loop transfer function G_0 in problem 5.

a. Give a short description of the purpose of the phase and amplitude margins. Use the figure to determine these values. (1 p)

b. As it is now, the closed-loop system can't track ramp references without stationary errors. Design a compensation link that reduces the stationary error by a factor 5. (2 p)

c. Consider the compensation link below

$$G_K(s) = \frac{s + 10}{s + 1}$$

Why would such an addition to G_0 not be a good idea?

Hint: Sketch the Bode plot of the compensation link! (1 p)

6. The electonic circuit in Figure 5 is a bandstop filter. The relationship between the input and output voltage is given by

$$C\ddot{V}_{\text{out}} + \frac{1}{R}\dot{V}_{\text{out}} + \frac{1}{L}V_{\text{out}} = C\ddot{V}_{\text{in}} + \frac{1}{L}V_{\text{in}}.$$

a. Find the transfer function $G(s)$ from V_{in} to V_{out}. (1 p)

b. What is the gain of the circuit for an arbitrary frequency ω? (0.5 p)

c. At which frequency is the circuit gain 0 (the frequency that the circuit "stops")? (0.5 p)

d. A disturbance V_D enters the system as illustrated in Figure 6. However, you're able to measure the disturbance and decide to add a feed-forward link G_{FF} to try to eliminate the effect of the disturbance. What is the new transfer function from V_D to V_{out}? Choose G_{FF} to eliminate the effect of the disturbance V_D. (2 p)

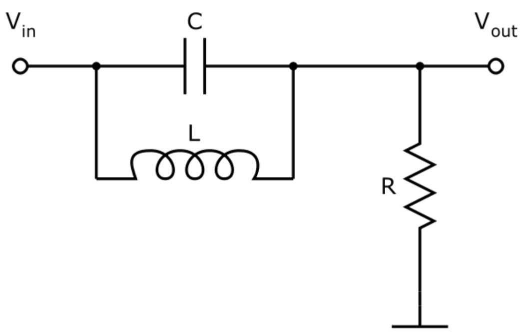

Figure 5: Passive electric bandstop filter in problem 6.

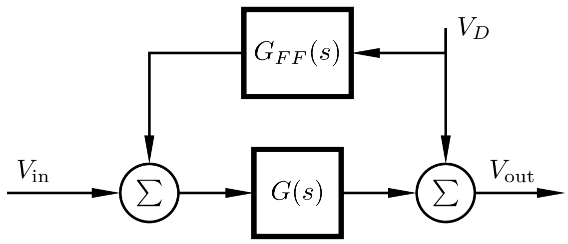

Figure 6: A block diagram illustrating the bandstop filter with disturbance voltage V_D and feed forward block G_{FF} from problem 6.