LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control Basic Course

Exam March 16 2023, 14-19

Points and grading

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminary grades:

Betyg 3: 12 p
4: 17 p
5: 22 p

Aids

Mathematical collections of formulae (e.g. TEFYMA), Collections of Formulae in Automatic Control, and calculators that are not programmed in advance.

Tentamensresultat

The results are announced through LADOK. Date and time for exam viewing will be announced on the course web (Canvas).

Instructions (N.B.!)

Only write on one side of each sheet of paper.

Treat only one problem (1-6) on each page. Subproblems (a,b,c,...) can be treated on the same page.

Good luck!

1. A system is described by the following differential equation

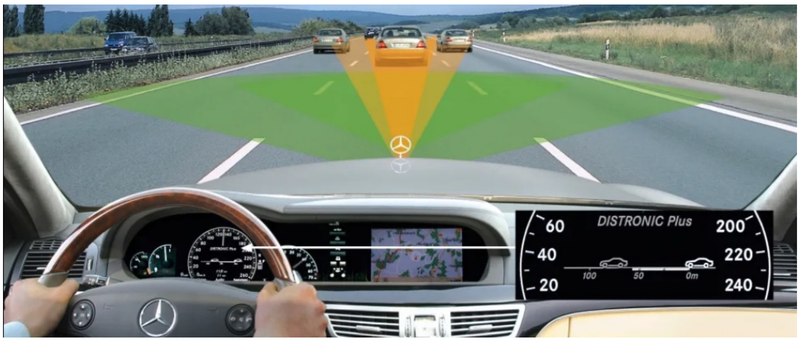
$$\dddot{y} + 4\ddot{y} + 5\dot{y} = \dot{u} + 3u$$

a. Determine the transfer function of the system $G_P(s)$. (1 p)

b. Determine the system's poles and zeros. Is the system asymptotically stable? (1.5 p)

c. Write the system on controllable canonical form. (1 p)

2. Modern cars are often equipped with an adaptive cruise controller (ACC). It adapts the speed of the car in order to keep a constant safety distance to the car ahead. The distance is measured through onboard sensors such as radar or laser. The desired safety distance is chosen by the driver, for example based on the "three second rule."



Figur 1: Illustration of the adaptive cruise controller (ACC) in Problem 2.

a. Regard the adaptive cruise controller, as it is described above, as a control problem and suggest its

i. measurement (or output) signal y

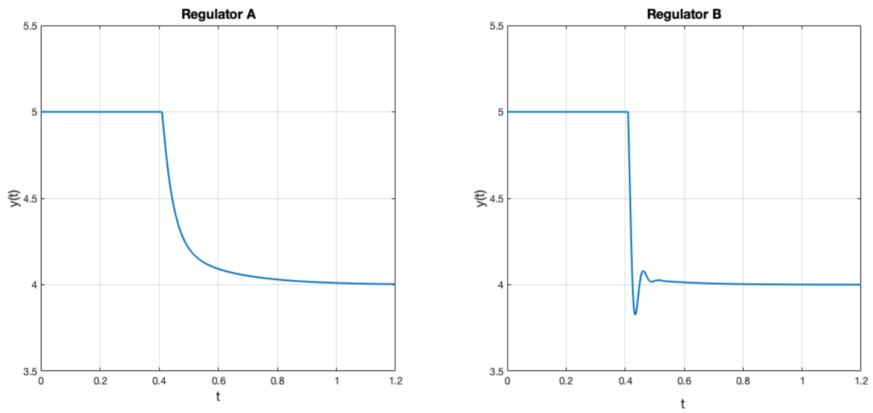
ii. control (or input) signal u

iii. reference signal r

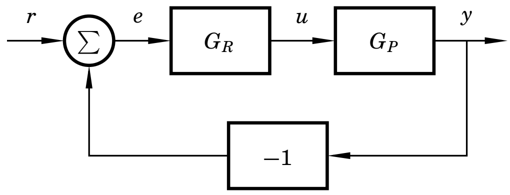
Motivate your answer. (1 p)

b. One car company is not satisfied with their adaptive cruise controller. The engineers propose two new controllers and simulate the system's response to a step change in the reference. The result is shown in Figure 2.

Which of the two controllers would you recommend they use (and which should they not use?). Motivate your answer based on the control problem in question. (1 p)



Figur 2: Simulated responses to a step change in the reference signal of the ACC with controller A and B, respectively. (The time t and output y are plotted with arbitrary units).



Figur 3: Simple feedback loop.

3. The process $G_P(s)$ is connected in feedback with the P controller $G_R(s) = K$ as depicted in Figure 3. One is interested in how the closed-loop system is affected by the choice of K. Therefore, one has drawn a root locus, shown in Figure 4. For the particular choices $K = 0.5$ och $K = 7$ one has also drawn the amplitude curves of the Bode plots for the *closed-loop* system, see Figure 5. Answer the following questions and motivate your answers.

a. Will the closed-loop system be stable for all $K > 0$? (0.5 p)

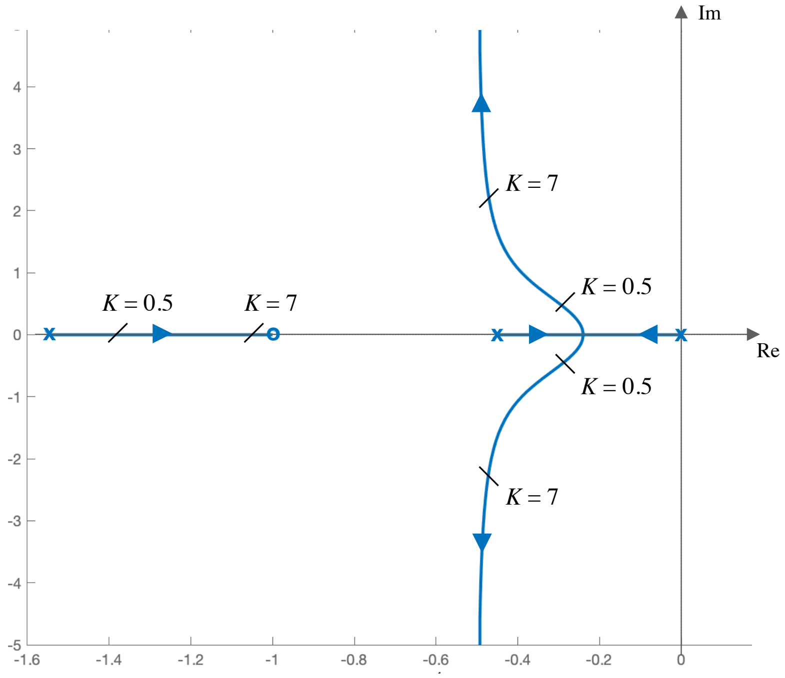
b. Pair the respective Bode plot in Figure 5 with the right value of K, that is, $K = 0.5$ or $K = 7$. (1 p)

For full points you need to argue based on the root locus.

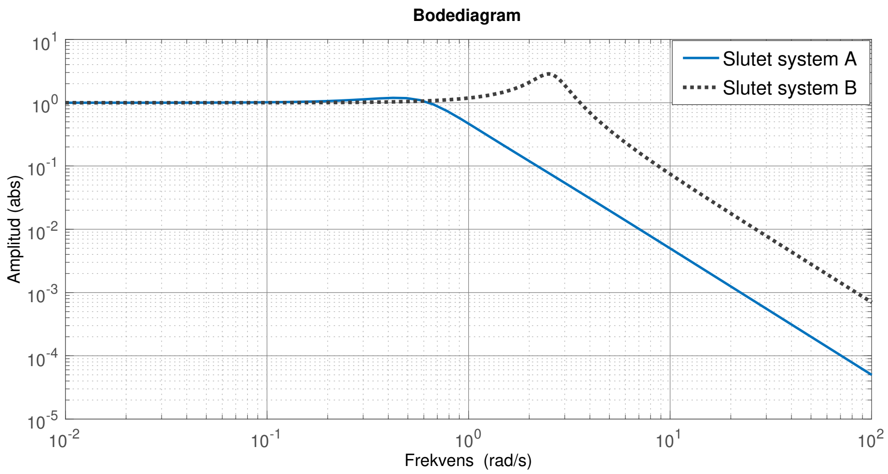
c. For the application in question, it is important that the system can track the reference

$$r(t) = A\sin(t)$$

(with arbitrary A) well. What is the amplitude of the signal $y(t)$ (expressed in terms of A) for each of the two choices of K, when this $r(t)$ has been applied for a long time? (1 p)



Figur 4: Root locus for problem 3. Two particular values of K are indicated on the root locus.



Figur 5: Bode plot (amplitude plot) for the *closed-loop* system in Problem 3. One of these corresponds to $K = 0.5$ and the other $K = 7$.

4. A linear system can be described through the following state-space description:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -2 & a+2 \\ 0 & -a \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned} \quad . \tag{1}$$

a. For which values of a is the system asymptotically stable? (1 p)

b. For which values of a is the system observable? (1 p)

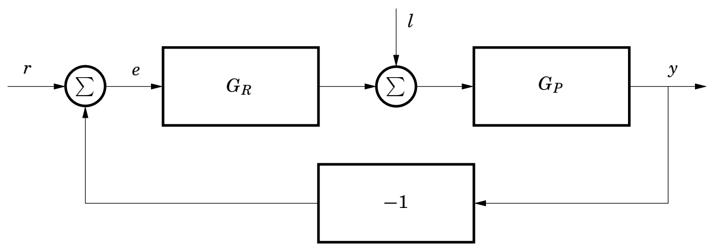
c. Let $a = 1$ and design a state feedback on the form $u = k_r r - Kx$ that places both poles in $s = -5$. Also make sure that the static gain from reference r to output y is 1. (2 p)

d. If we cannot measure both states but only $y = x_1$, we need a Kalman filter (an observer). Let $a = 1$ and design a Kalman filter to be used together with the state feedback above. Choose the pole placement for the Kalman filter yourself and motivate your choice briefly. (2 p)

e. The system (1) above is the result of the linearization of a nonlinear system around the stationary point $(x_0, u_0) = (0, 0, 0)$. Could the nonlinear system have been the following:

$$\begin{aligned}\dot{x}_1 &= -4x_1 + a\sin x_2 + 2x_1x_2\\ \dot{x}_2 &= -a\sin x_2 + u\\ y &= x_1.\end{aligned}$$

Answer yes or no and motivate your answer. (1.5 p)



Figur 6: Block diagram for problem 5

5. Consider the feedback system in Figure 6, which is subjected to the load disturbance l.

a. State the transfer functions from r to y and l to y. (1 p)

b. Next, assume

$$G_P = \frac{10}{s(s+5)^2}.$$

We wish to control the system with a controller with the transfer function

$$G_R = K(Ts + 1)$$

i. What type of controller is this?

ii. What is the stationary error $\lim_{t\to\infty} e(t)$ due to a unit step in the reference signal?

iii. What is the stationary error $\lim_{t\to\infty} e(t)$ due to a unit step in the load disturbance?

(2 p)

Hint: You may set $l = 0$ when evaluating the error due to a step in r, and vice versa.

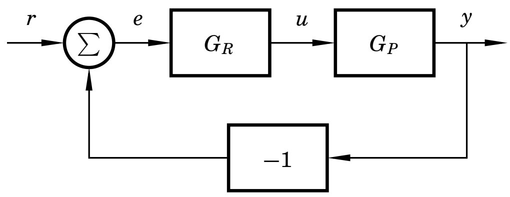
c. One discovers that, for this application, it is simple and cheap to measure the load disturbance directly. One therefore chooses to implement another controller tasked with compensating for the load disturbance before it affects the measurement signal via the process G_P.

What is this technique called? Draw the block diagram for the new controller structure. (1 p)

6. At the International Space Station ISS there is a long robotic arm that is used for service of the solar panels. To make it light, it is very thin. Therefore, it has a tendency to oscillate when it is moved rapidly. The transfer function from the motor controlling the arm to the position of the tip of the arm is given by:

$$G_P(s) = \frac{32 - s}{s^2 + 2s + 4}.$$

The Bode plot for $G_P(s)$ is shown in Figure 9. One wishes to control the arm through feedback from the tip position by designing an appropriate controller. The feedback loop is shown in Figure 7.



Figur 7: Feedback loop for Problem 6.

First, one tries a P controller $G_R(s) = K$ with $K = 1$. The closed-loop system is stable, but with unsatisfactory performance. Its step response is shown in Figure 10.

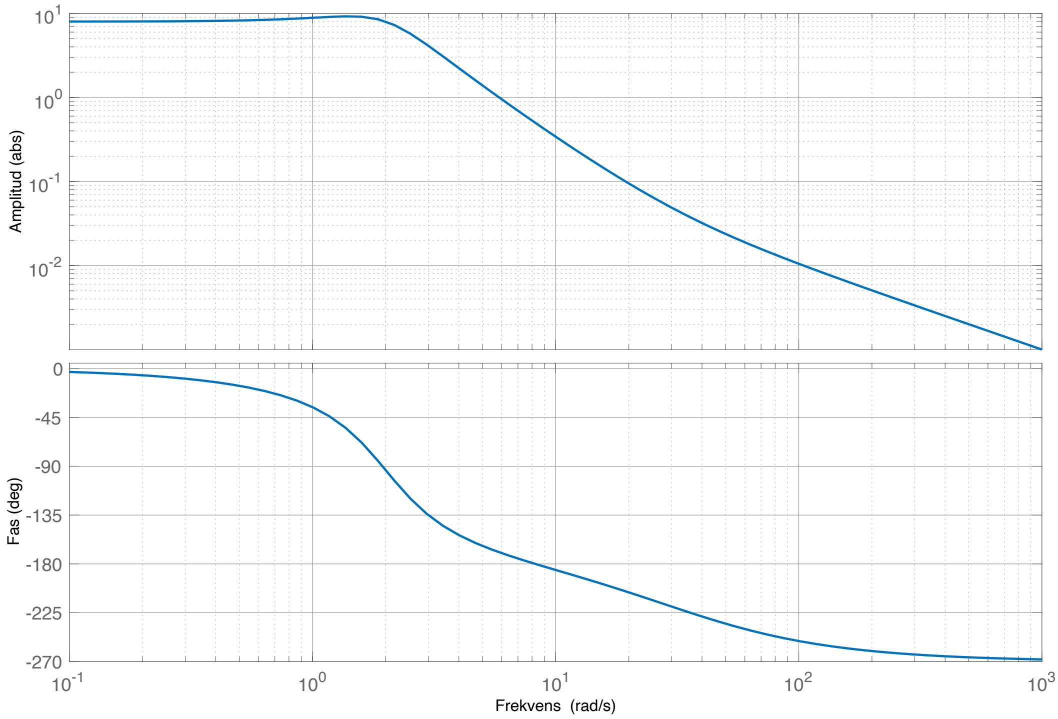
a. It is very important to be able to track changes in the reference position with high accuracy. According to the specifications, the stationary error due to a step change in the reference position should be less than 1% of the step size. This is impossible to achieve with a P controller $G_R(s) = K$. Why? (1.5 p)

b. Take the P controller with $K = 1$ as a starting point. Design a compensator that ensures the stationary error due to a step change in the reference is less than 1% of the step size. The phase margin is not allowed to decrease by more than 6°. (2 p)

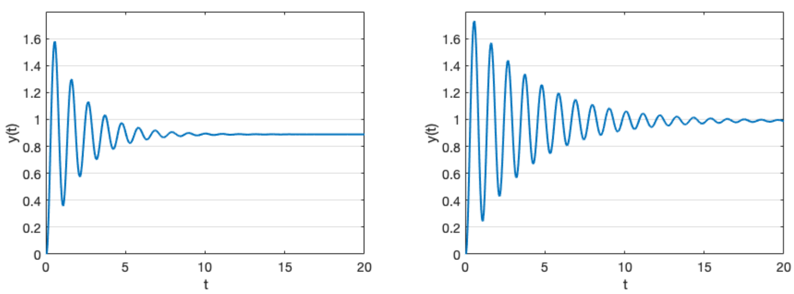
c. As previously mentioned, the arm has a tendency to oscillate when it is moved, see Figure 10. It is a consequence of a too small phase margin. To reduce the oscillations, add a compensator that gives the open loop system a phase margin of at least 30°. The speed of the system can be left unchanged. (2 p)

Account for the effect of the compensator in b when you design your new compensator. This is possible even if you have not solved subproblem b.

Figur 8: European Robotic Arm, which is at ISS since 2021. In this exam it is modeled by $G(s)$ in Problem 6. Photo from the European Space Agency.



Figur 9: Bode plot of the process $G_P(s) = \frac{32-s}{s^2+2s+4}$ in Problem 6.



Figur 10: Step response of the robotic arm in Problem 6 with the controller $G_R(s) = 1$ (left) and the compensator from 6-b (right).