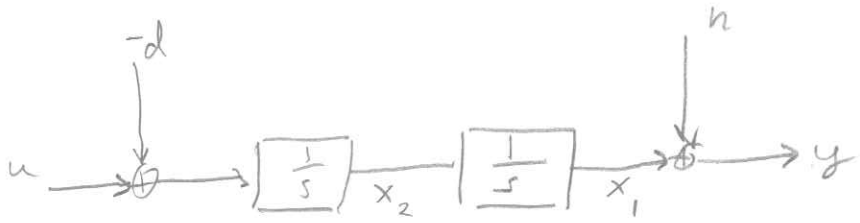
Car speed estimation

(background to car.m
carModel.stx)



u: throttle
Acceleration: $u-d$, offset d caused by sloping road

y: position measurement (GPS)
Position: $y-n$, noise n introduced by GPS

x_1: position
x_2: speed

Strategy 1 $\hat{x}_2 = \int u\,dt = x_2 + dt$ — growing error

Strategy 2 $\hat{x}_2 = \frac{d}{dt} y = x_2 + \frac{d}{dt} n$ — very noisy

Kalman

undisturbed system ($n=d=0$):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \\ y = x_1 \end{cases} \Rightarrow \dot{x} = \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{A} x + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B} u$$

$$y = \underbrace{(1\ 0)}_{C} x$$

observer:

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y-\hat{y})$$
$$= (A-KC)\hat{x} + Bu + Ky$$

choise of K:

$$\det(sI - A + KC) = \det\left(\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}(1\ 0)\right) = \det\begin{pmatrix} s+k_1 & -1 \\ k_2 & s \end{pmatrix} = s^2 + k_1 s + k_2$$
$$= s^2 + 2\xi\omega_0 s + \omega_0^2$$

$k_1 = 2\xi\omega_0$ $\quad \xi = 0.7$

$k_2 = \omega_0^2$ $\quad \omega_0$ - design parameter (observer speed)