



LUNDS TEKNISKA  
HÖGSKOLA  
Lunds universitet

Institutionen för  
**REGLERTEKNIK**

## **Automatic Control, Basic Course FRT 010**

**Exam March 6, 2012**

### **Points and grades**

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Grades:

Betyg 3: least 12 points

4: least 17 points

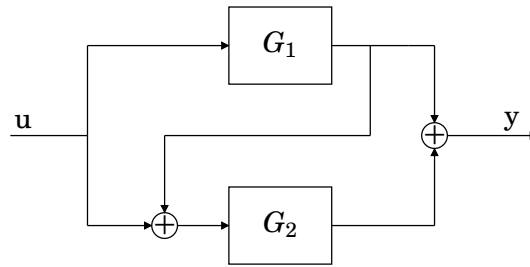
5: least 22 points

### **Aids**

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

### **Results**

The results are presented on March 16 at the latest, at the information board of the the department at the first floor in the M building and at the web page for the course.



**Figur 1** Block diagram of the system in Problem 1.

1. Two systems,  $G_1$  och  $G_2$  are coupled according to Figure 1.
  - a. Determine the transfer function between  $u$  and  $y$ . (1 p)
  - b. Determine the poles and the zeros of the system when the transfer functions are

$$G_1(s) = \frac{1}{s+4}, \quad G_2(s) = \frac{2}{s+5}.$$

(1 p)

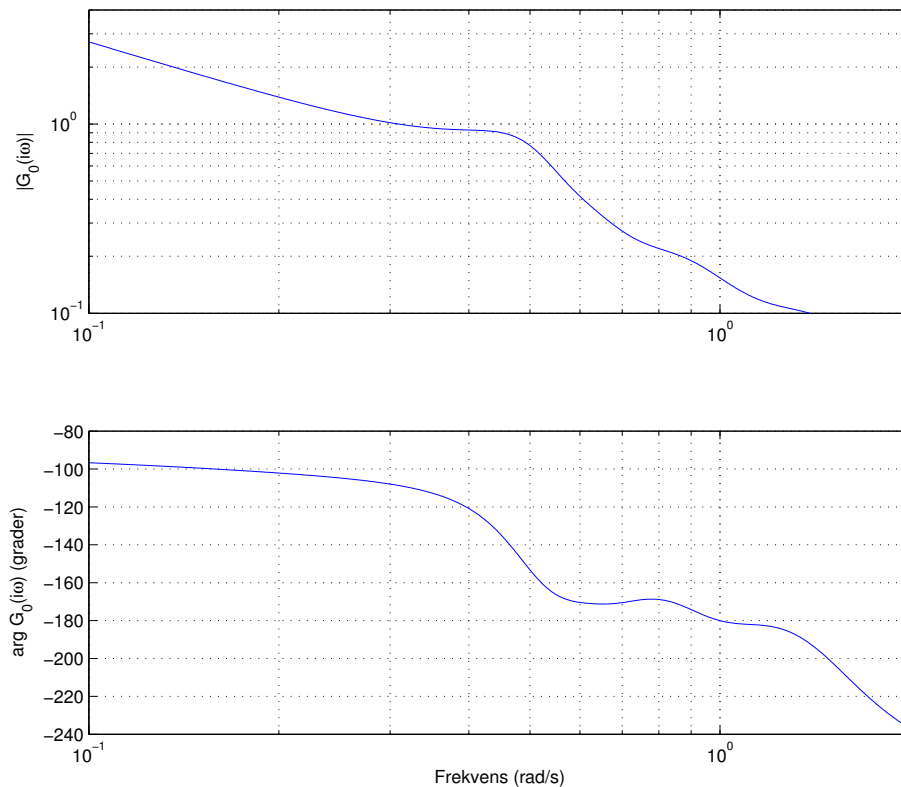
- c. Determine and draw the step response of the system. (1 p)
2. The dynamics of a system is given by the following nonlinear differential equation

$$\ddot{z} + \frac{\dot{z}^4}{z^2} - z = \sqrt{u}$$

where  $u$  is the input signal and the output is given by  $y = z^2 + u^2$ .

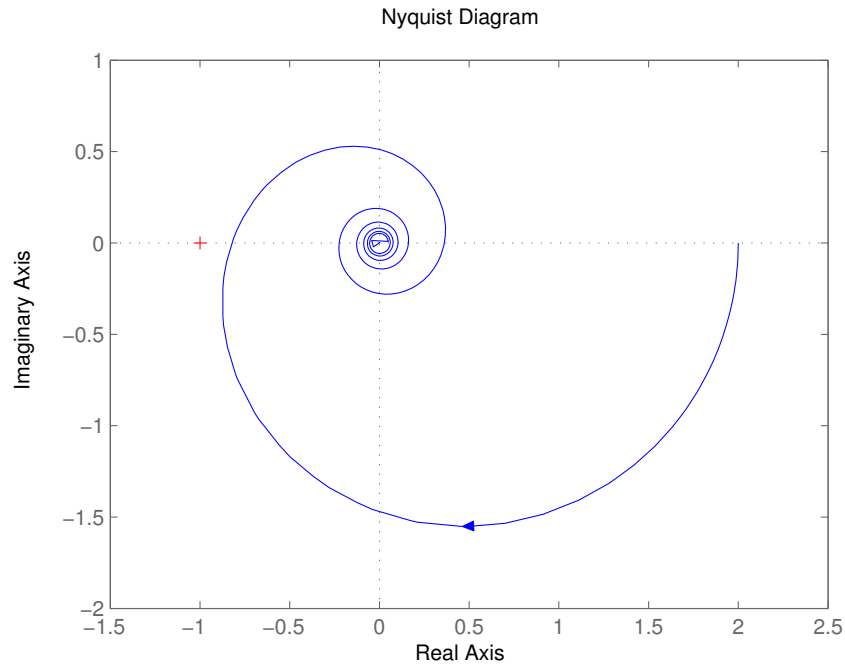
- a. Introduce the states  $x_1 = z$  and  $x_2 = \dot{z}$ , and write the system on state-space form. (0.5 p)
  - b. Determine the stationary points. (1 p)
  - c. Linearize the system around the stationary point corresponding to  $u = 9$ . (1.5 p)

3. The Bode plot of an open-loop system is given in Figure 2.
  - a. Determine the phase and amplitude margins of the system. (1 p)
  - b. Draw the Nyquist plot of the system. Mark the phase and amplitude margins, and the point corresponding to  $\omega = 0.5$  rad/s. (1 p)
  - c. Are the amplitude and phase margins good robustness measures for this system? Motivate your answer! (1 p)



**Figure 2** Bode plot of the system in Problem 3.

4. The Nyquist plot of a process is presented in Figure 3. Determine, with the use of this plot, whether the following statements are true, false, or if you don't have enough information about the system. The system is assumed to be minimum, i.e. there is no pole-zero cancellation. All answers must be motivated. Each correct answer gives 0.5 p. (3 p)
  - a. The system will be unstable if the process is controlled using a P controller with gain  $K = 2$ .
  - b. The phase margin using unit feedback is less than  $60^\circ$ .
  - c. The delay margin using unit feedback is greater than 0.1 s.
  - d. The static gain of the process is 2.
  - e. The process contains an integrator.
  - f. The process is of second order.
  
5. An industrial robot, see Figure 4, is composed of a chain of links and joints driven by motors. The goal is to control the joints so that the robot tool performs a desired motion. The robot has normally 6 joints, and these are controlled individually. The relation between the torque from a motor and



**Figur 3** Nyquist plot of the system in Problem 4.

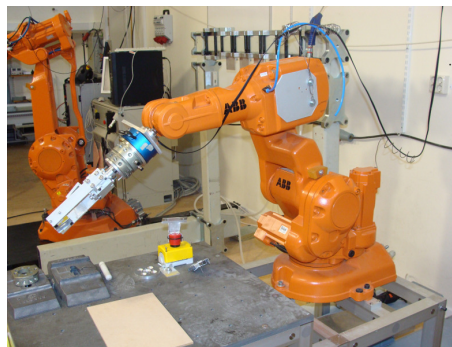
the joint position is nonlinear, but it is possible to linearize the model using so called *computed torque control*, resulting in the linear transfer function

$$G(s) = \frac{1}{s^2}$$

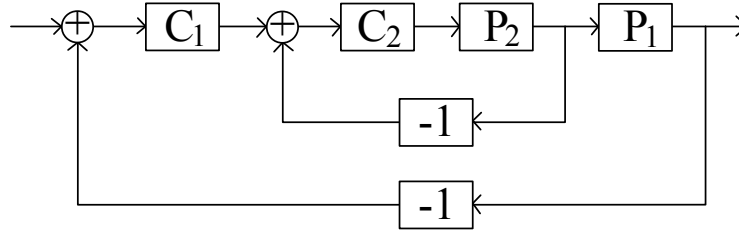
The block diagram of the control system is presented in Figure 5, where the output from  $P_2$  is the speed of the joint and the output from  $P_1$  is the position of the joint.

$$P_1 = \frac{1}{s}, \quad P_2 = \frac{1}{s}$$

- a. What kind of control structure is this? (1 p)



**Figur 4** An ABB IRB140 at the robot lab of the department



**Figur 5** Block diagram of the control system in Problem 5.

- b.** In robot control,  $C_2$  is normally a PI controller, and  $C_1$  is a P controller. Design  $C_2$  so that the poles of the inner loop gives a speed corresponding to  $\omega = 10$  rad/s, and a relative damping  $\zeta = 0.8$ . (2 p)
- c.** Explain how the outer loop should be designed so that the inner loop can be approximated by just a gain  $K$ . How should  $K$  be chosen? Why is it of interest to make such an approximation? (1 p)
- 6.** The dynamics of a chemical process can be approximated by the transfer function

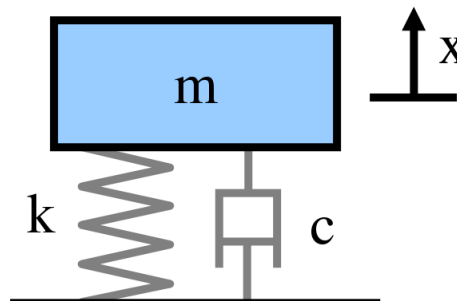
$$G_p(s) = \frac{1}{s(s/2 + 1)} e^{-0.5s}.$$

Design a compensator so that the compensated system becomes twice as fast as the uncompensated system. The phase margin must not decrease more than  $6^\circ$ . (3 p)

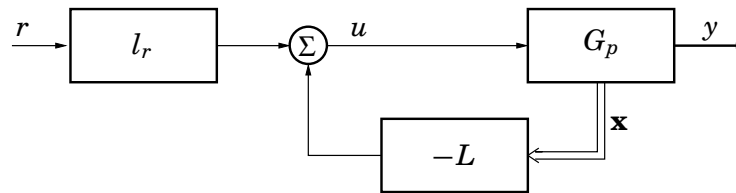
- 7.** A spring-mass-damper system shown in Figure 6 can be described by the following differential equation:

$$m\ddot{x} + c\dot{x} + kx = f$$

where  $x$  is the position of the mass,  $f$  an external force,  $c$  the damping coefficient of the system, and  $k$  is the spring constant. The position of the mass is the measurement signal.



**Figur 6** The spring-mass-damper system in Problem 7.



**Figur 7** Block diagram for the state feedback in Problem 7.

- a.** Introduce the states  $x_1 = x$ ,  $x_2 = \dot{x}$  and write the system on state-space form.

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du(1p)\end{aligned}$$

- b.** Determine the poles of the system when  $c = 0$ ,  $k = 1$  and  $m = 1$ . Is the system asymptotically stable, stable or unstable? Describe how this can be seen just by inspecting Figure 6 with the given parameter values. (1 p)
- c.** Design a state feedback  $u = -Lx + l_r r$  so that the closed-loop system gets a static gain equal to one, and poles in  $-2$ . (2 p)
- d.** In practice, it is not possible to measure  $\dot{x}$ , which means that it is only possible to make a feedback from one state,  $x_1$  i.e.  $u = -l_1 x_1 + l_r r$ . Modify the block diagram in Figure 7 according to this. What kind of controller is obtained now? (1 p)
- e.** Show that it is not possible to place the poles in  $-2$  using this kind of controller. (1 p)