LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRTF05

Exam January 13, 2021, 14–19

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminary grades:

Betyg	3:	12 points
	4:	17 points
	5:	22 points

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!

1. Your colleague Rhea has a system described by the following differential equation:

$$\ddot{y} - 4\dot{y} - 5y = u$$

a. Introduce the state variables $x_1 = y$ and $x_2 = \dot{y}$ and help her finding the state-space description of the system. (1 p)

b. Is the system asymptotically stable, stable, or unstable? (1 p)

c. Rhea has also an inverted pendulum, which type of stability does it have? (0.5 p)

2. The number of rabits $x_1(t)$ and foxes $x_2(t)$ in a forest can be described by the Lotka-Volterras equations

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 - \beta x_1 x_2 \\ \dot{x}_2 &= \delta x_1 x_2 - \gamma x_2 - u,\end{aligned}$$

where control signul u is the number of foxes killed during a week during the fox hunting, and α, β, γ and δ are positive real constants.

a. Determine all stationary points (x_1^0, x_2^0, u^0) for the system when $u^0 = 0$. (1 p)

b. Linearize the system around the stationary point where $x_1^0 > 0$ and $x_2^0 > 0$. Assume that the measurement signal is the number of foxes, i.e., $y = x_2$. (2 p)

c. Is the linearized system asymptotically stable, stable, or unstable? (1 p)

d. The number of foxes $y = x_2$ is at one occasion higher than the desired setpoint r. We want to determine the number of foxes that should be shot per week as a function of the actual populations of foxes and rabits, and the actual setpoint. We use the control law $u(t) = l_r r - l_1 x_1 - l_2 x_2$, where l_1, l_2 and l_r are constant coefficents. Determine l_1 and l_2 so that the closed linear system $Y(s) = G(s)R(s)$ get both poles in $s = -0.2$. (2 p)

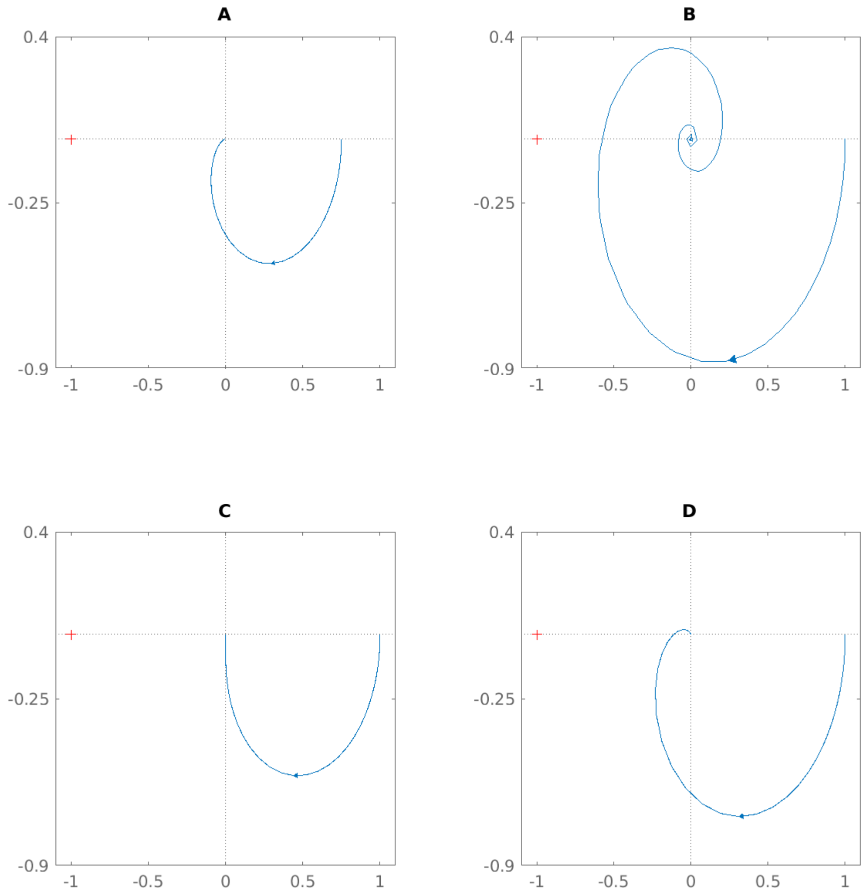
e. To hunt foxes without hunting rabits is perhaps not a good idea. Suppose that we, for a constant setpoint $r(t) = \varepsilon\alpha/\beta$, with $0 < \varepsilon < 1$, manage to choose $u(t)$ so that the control error becomes constant and zero, i.e., $x_2(t) = r(t) = \varepsilon\alpha/\beta$, while $x_1(t) > 0$. What will then happen with the number of rabits $x_1(t)$ when $t \to \infty$ according the the nonlinear Lotka-Volterra equations? (0.5 p)

3. In this problem, the following transfer functions are studied:

$$G_1(s) = \frac{4}{(s+2)^2(s+1)} \qquad G_2(s) = \frac{4}{(s+2)^2}e^{-s}$$
$$G_3(s) = \frac{3}{(s+2)^2} \qquad G_4(s) = \frac{s+4}{(s+2)^2}$$

a. Figure 1 shows four Nyquist plots. Pair each blot with the corresponding transfer function. Motivate your answer. (2 p)

b. Suppose that the systems are controlled with a P controller with gain $K > 0$. For every transfer function, determine the values of K that result in stable closed-loop systems. Motivate your ansers. (2 p)



Figur 1: Nyquist plots for problem 3.

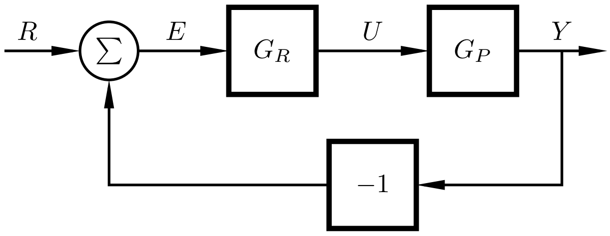
4. You are employed as a consultant at the company Energi AB that works with temperature control of offices. They have found a model that describes heating of a room that is given by the following transfer function

$$G_P(s) = \frac{1}{(1+2s)(1+3s)}$$

a. If you compare this process with the tank labs, does it have dynamics similar to the upper or the lower tank? Motivate your answer! (0.5 p)

b. Today, the company uses a PI controller to control the system. Determine the transfer function $G(s)$ from setpoint R to measurement signal Y for the closed-loop system (see Figure 2). Insert the expressions for G_P and G_R and ensure you get a nice expression for the transfer function (the dominator should be on the form $(s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n)$. The transfer function for a PI controller is: $G_R(s) = K(1 + \frac{1}{sT_i})$. (2 p)

c. Given that $T_i = 1$, which is the largest gain K that we can have in the controller and still have an asymptotically stable closed-loop system? (1 p)



Figur 2: Closed-loop system.

d. Is there another type of controller that you would recommend the company to use instead of a PI controller? Why/why not? (0.5 p)

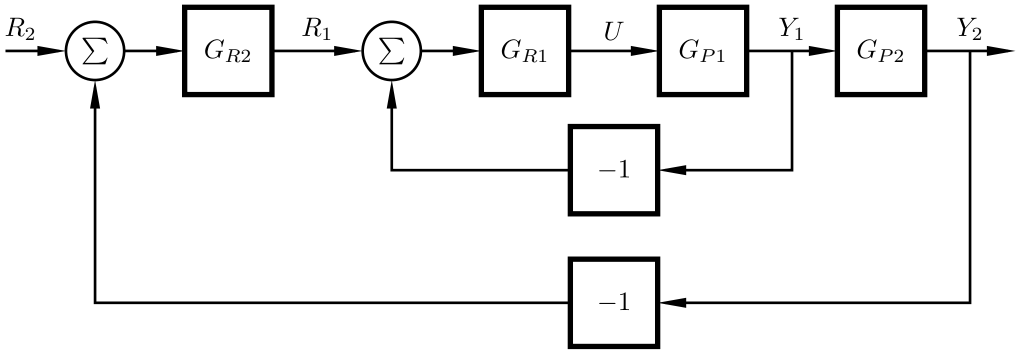
5. You and your colleague Abigail work with a system given on state-space form as:

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

a. Control the system using state feedback so that the characteristic polynomial becomes $s^2 + 4s + 9$. (1 p)

b. Describe, in words, why one often needs some variant of a Kalman filter when using state feedback. (0.5 p)

6. You and your friend Yasmin work on a home project and have the following two block diagrams as candidates to solve your problem (see Figure 3 and 4).

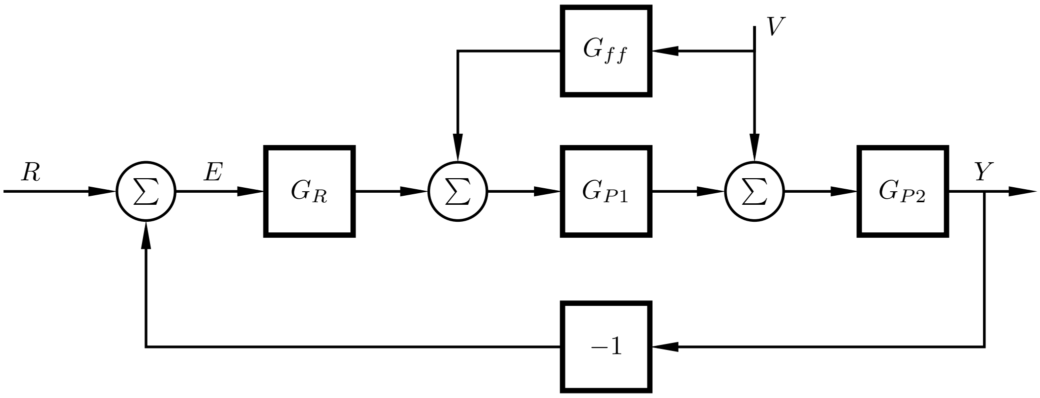


Figur 3: Block diagram A

a. What are the two control structures called? (0.5 p)

b. You have a measurable load disturbance that influences your measurement signal, which you would like to avoid. Which of the two structures, A or B, is best suited to solve the problem? (Motivation required!) (0.5 p)

c. Your goal is to control the temperature in a room. If you decide to use structure A, is it R_1 or R_2 that will be the reference for the temperature? (Motivation required!) (0.5 p)



Figur 4: Block diagram B

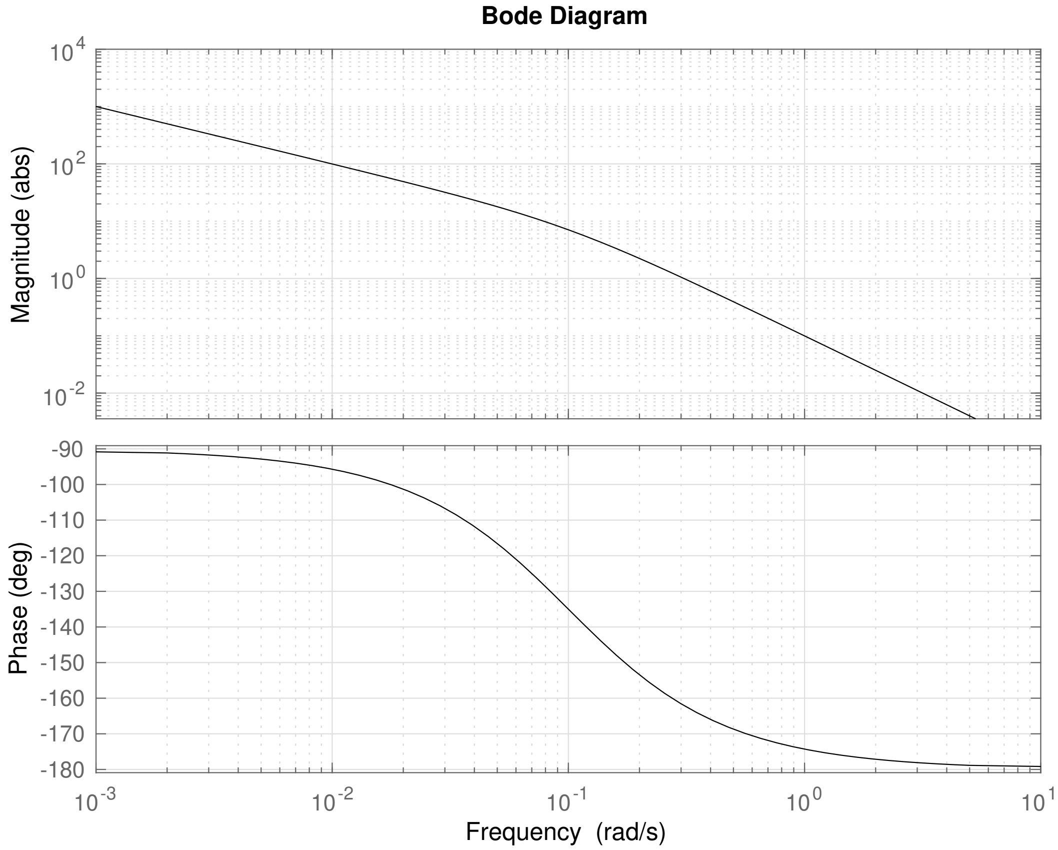
d. Mention a problem, and a solution to this problem, that you must handle if you choose control structure B. (0.5 p)

7. You and your colleague Miyuki are going to tune a controller for a process. For your help, you have access to the Bode plots given in Figure 5.

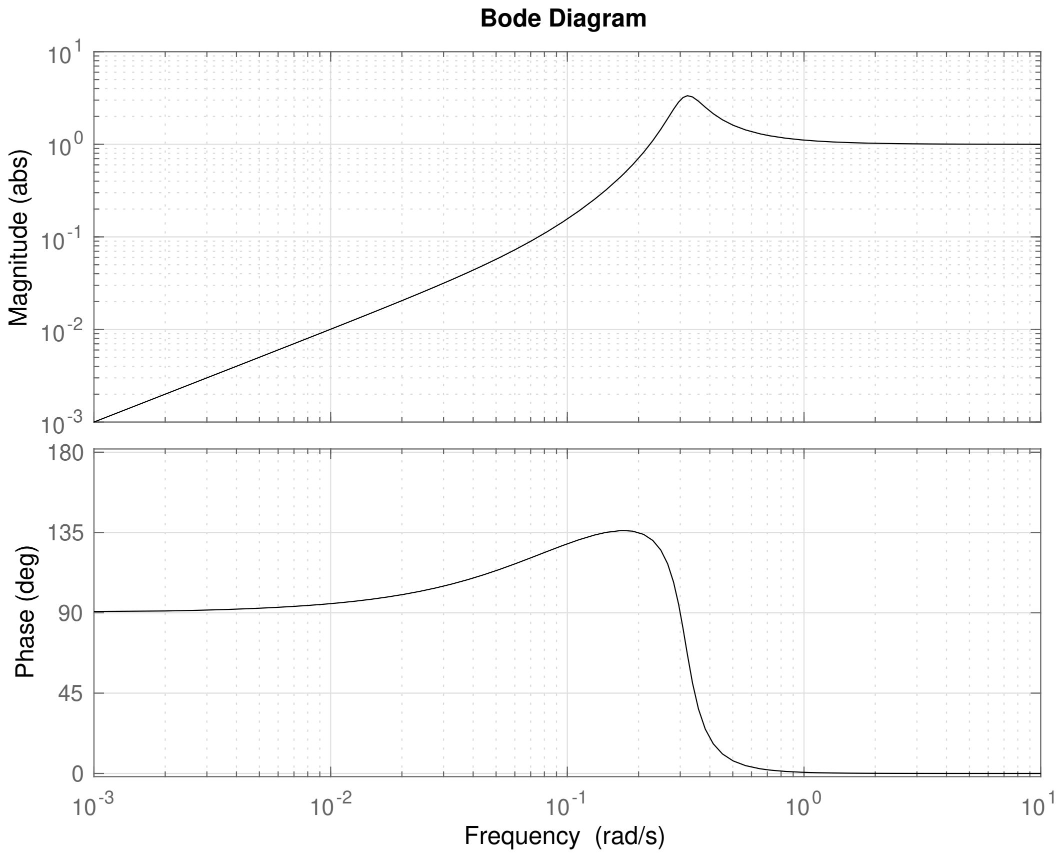
a. Determine the phase and amplitude margins. (1 p)

b. Determine the shortest distance between the Nyquist plot and the point -1. (1 p)

c. You decide to design a compensator so that the phase margin becomes $50°$ with a retained speed of the loop. Determine the transfer function of the compensator. (2.5 p)



(a) Bode plot for the system.



(b) Bode plot of the sensitivity function.

Figur 5: Bode plots for Problem 7.