LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRTF05

Exam October 25, 2021, 14–19

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminary grades:

Betyg 3: 12 points
4: 17 points
5: 22 points

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!

1. A system is given on state-space form as

$$\dot{x} = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

a. Determine the transfer function of the system. (1,0 p)

b. Give the poles and the zeros of the system, if they exist. (0,5 p)

c. Is the system asymptotically stable, stable, or unstable? (0,5 p)

d. Describe the system as a differential equation. (1,0 p)

2. A system is described by the following differential equation:

$$\ddot{y} + \dot{y}^2 + \frac{1}{y+1} = u$$

a. Introduce the state variables $x_1 = y$ and $x_2 = \dot{y}$ and write the system on state-space form. (1,0 p)

b. Determine all stationary points (x_1^0, x_2^0, u^0) for the sysem. (0,5 p)

c. Linearize the system around the point where $u^0 = 1$. (1,5 p)

3. We have a process

$$G_P(s) = \frac{1}{s+2}$$

with control signal $u(t)$ and measurement signal $y(t)$ that we want to control with a P controller $u(t) = K(r(t) - y(t))$, where $r(t)$ is our reference signal.

a. Determine the controller gain K so that the closed-loop system (from $r(t)$ to $y(t)$) get the pole polynomial $(s+4)$. (1,0 p)

b. Determine the stationary value of $y(t)$, i.e. $y(t)$ when $t \to \infty$, when the reference signal $r(t)$ is a unit step. (1,0 p)

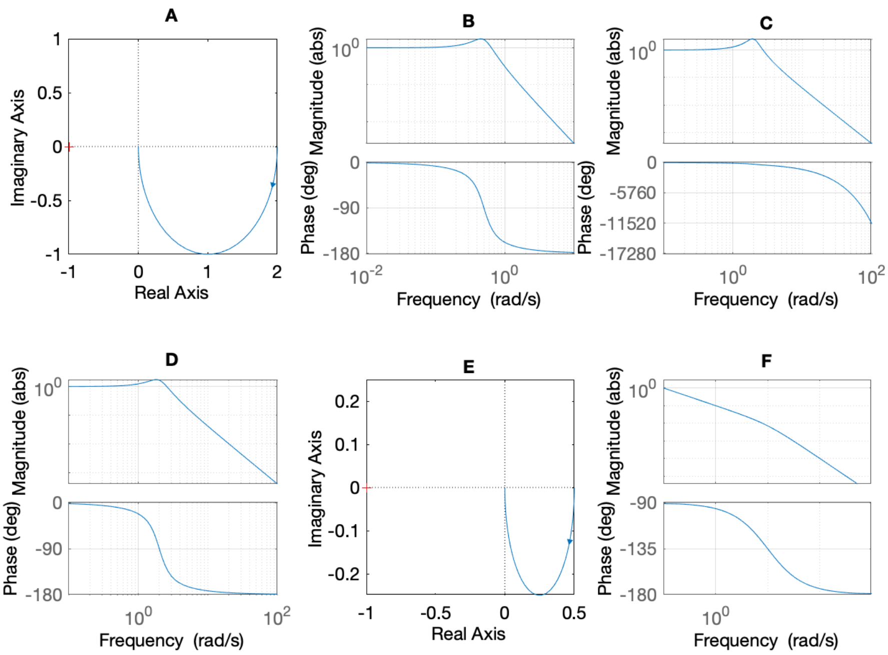
c. Determine the measurement signal $y(t)$ when the reference signal is $r(t) = \sin(3t)$. (1,0 p)

d. We want the stationary value of $y(t)$ to be equal to $r(t)$, when the reference signal is constant. Suggest a controller type that accomplishes this, and describe which property of the controller that makes this possible. (1,0 p)

4. Consider the following four systems:

$$G_\alpha(s) = \frac{1}{2s+2}, \quad G_\beta(s) = \frac{1}{s^2+10s},$$
$$G_\gamma(s) = \frac{0.25}{s^2+0.3s+0.25}, \quad G_\delta(s) = \frac{4}{s^2+1.2s+4}.$$

a. Determine which Bode or Nyquistd plot A-F in Figure 1 that corresponds to each of the four transfer functions (for each transfer function there is either a Bode or a Nyquist plot). The answers must be motivated. (2,0 p)



Figur 1 Bode-/Nyquist plot for problem 4.

b. Determine which of the step responses 1-6 in Figure 2 that corresponds to each of the transfer functions. The answers must be motivated. (2,0 p)

5. A system is given on state-space form

$$\begin{aligned} \dot{x} &= \underbrace{\begin{pmatrix} -5 & 1 \\ 0 & -3 \end{pmatrix}}_{A} x + \underbrace{\begin{pmatrix} 1 \\ \beta \end{pmatrix}}_{B} u \\ y &= \underbrace{\begin{pmatrix} 1 & 0 \end{pmatrix}}_{C} x \end{aligned}$$

a. What does it mean that a system is controllable? (0,5 p)

b. For which values of parameter β is the system not controllable? (1,0 p)

c. Låt $\beta = 1$. Suppose that the system is controlled using the controller

$$u = -Lx$$

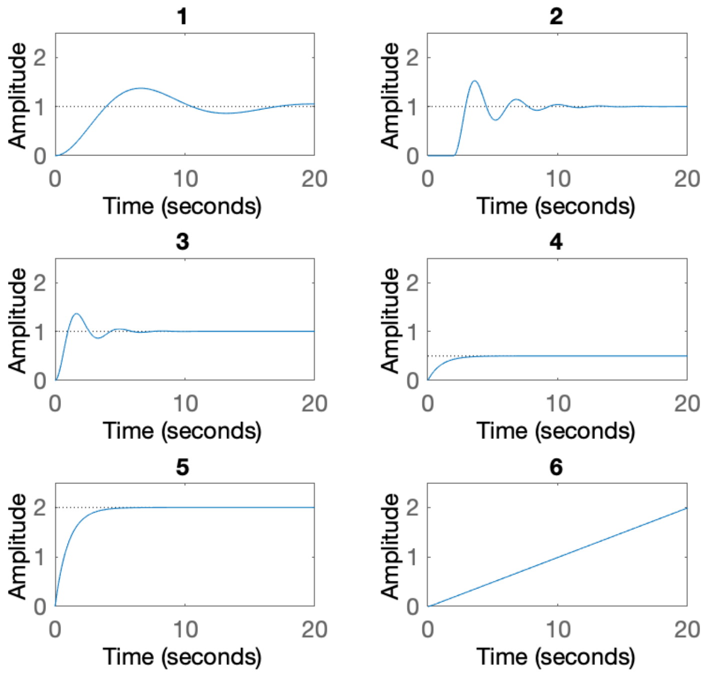
where $L = (l_1 \quad l_2)$. Determine L so that the closed-loop poles are placed in -5. (1,0 p)

d. Suppose that the system is controlled using the controller

$$u = -L\hat{x}$$

with the same L as above. Vector $\hat{x}$ is estimated using the Kalman filter

$$\dot{\hat{x}} = (A - KC)\,\hat{x} + Bu + Ky$$



Figur 2 Step responses for problem 4.

where $K = (k_1 \quad k_2)^\top$. Determine K so that the Kalman filter becomes twice as fast as the state feedback. (1,0 p)

e. Give a reason why the Kalman filter must not be too slow. (0,5 p)

f. Give a reason why the Kalman filter must not be too fast. (0,5 p)

6. The Bode plot of a linear time invariant process $G_P(s)$ is given in Figure 3.

a. Does the process have integral action? The answer must be motivated. (0,5 p)

b. Determine the gain margin, phase margin, and delay margin for the process. (1,5 p)

c. Determine the output from the system (when the transients have died out) when the input signal is $\sin(0.01t)$? (1,0 p)

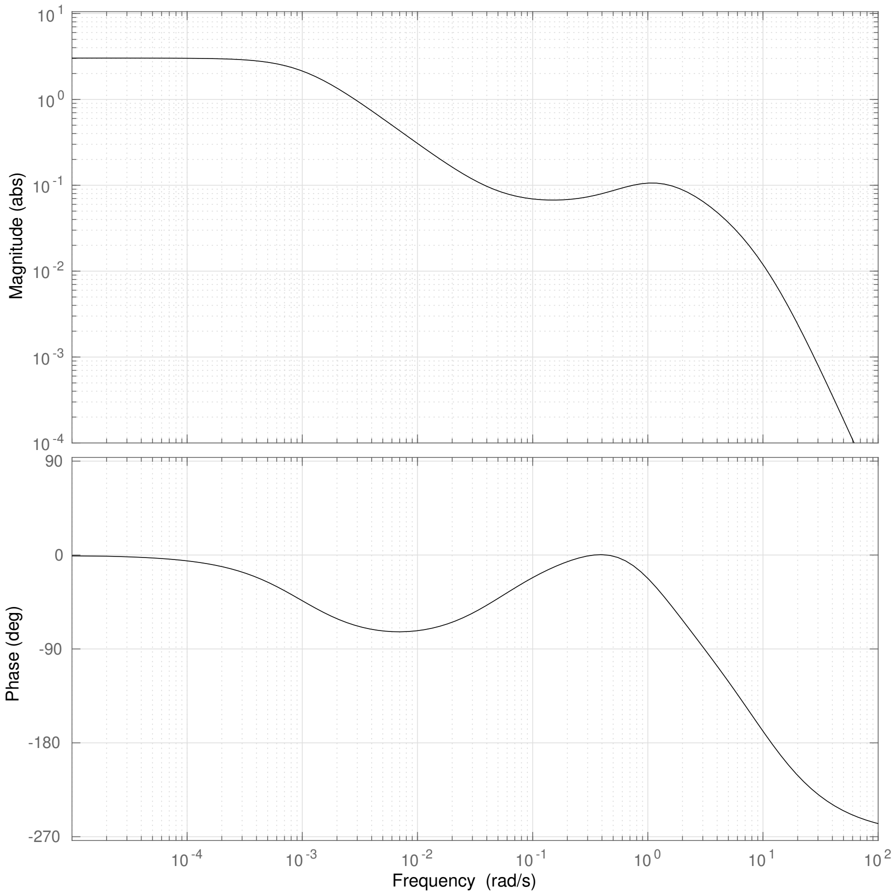
d. Use the Ziegler-Nichols frequency response method to determine the parameters of a PI controller. (1,0 p)

e. Instead of using the PI controller, reduce the stationary error that occurs when the reference value r is a unit step by a factor 2 using the lag compensator

$$G_K(s) = \frac{s+a}{s+a/M}.$$

The phase margin may be reduced by at most 6°. (2,0 p)

f. Let $M \to \infty$ in the lag compensator. Which type of controller does it correspond to? (0,5 p)



Figur 3 Bode plot for problem 6.