

Exercise 3

1. a) Let

$$M = \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$$

where $A, B \in \mathbb{C}^{n \times n}$ are two constant square matrices. Compute the structured singular value $\mu_S(M)$ for

$$\begin{aligned} S &= \{\Delta : \delta I_{2n}, \delta \in \mathbb{C}\} \\ S &= \{\Delta : \Delta \in \mathbb{C}^{2n \times 2n}\} \\ S &= \left\{ \Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} : \Delta_1, \Delta_2 \in \mathbb{C}^{n \times n} \right\} \end{aligned}$$

Hint: Use Schur's formula for determinant. The results can be expressed in terms of $\rho(AB)$, $\bar{\sigma}(A)$ and $\bar{\sigma}(B)$.

b) Given a structured uncertainty S . Show that

$$\begin{aligned} \mu_S(M) &= \frac{1}{\min\{\bar{\sigma}(\Delta) \mid \det(I - M\Delta) = 0 \text{ for structured } \Delta\}} \\ &= \frac{1}{\min\{k_m \mid \det(I - k_m M\Delta) = 0 \text{ for structured } \Delta, \bar{\sigma}(\Delta) \leq 1\}} \end{aligned}$$

unless no structured Δ makes $I - M\Delta$ singular, in which case $\mu_S(M) = 0$.

c) Let M be a 7×7 random complex matrix. Take the uncertain structure to be

$$\Delta = \left\{ \begin{bmatrix} \delta_1 I_3 & & \\ & \Delta_2 & \\ & & \delta_3 I_2 \end{bmatrix} : \delta_1, \delta_3 \in \mathbb{C}, \Delta_2 \in \mathbb{C}^{2 \times 2} \right\}$$

Compute $\mu(M)$ using MATLAB.

2. In Figure 1, G is the nominal plant, K the controller, uncertainties $\|\Delta_I\|_\infty \leq 1$, $\|\Delta_O\|_\infty \leq 1$. All transfer functions are scalar-valued.

a) Derive the robust stability test.

b) Derive the robust performance condition for $\|T_{zw}\|_\infty < \beta$.

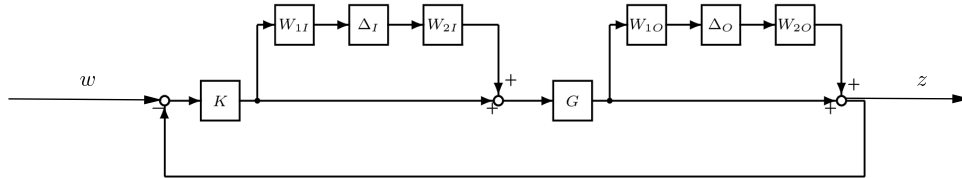


Figure 1: System with input and output multiplicative uncertainty

3. Consider a nominal plant P and a controller K

$$P(s) = \frac{1}{75s + 1} \begin{pmatrix} -87.8 & 1.4 \\ -108.2 & -1.4 \end{pmatrix}, \quad K(s) = \frac{75s + 1}{s} \begin{pmatrix} -0.0015 & 0 \\ 0 & -0.075 \end{pmatrix}$$

and a diagonal uncertainty $S = \text{diag}\{\delta_1, \delta_2\}$.

a) With the help of MATLAB calculate $\mu_S(T)$ ($= \min_D \bar{\sigma}(DTD^{-1})$, why?) and $\bar{\sigma}(T)$ at the frequency $\omega_0 = 0.2$ for $T = KP(I + KP)^{-1}$. Estimate the conservatism.

b) Analyze $T(j\omega_0)$ and $D_{\min}T(j\omega_0)D_{\min}^{-1}$ and indicate the property that you think most contributes to this difference.

c) Assume the multiplicative uncertainty model

$$\tilde{P} = P(I + W\Delta), \quad W(s) = \frac{s + 0.2}{0.5s + 1}, \quad \|\Delta\|_{\infty} \leq 1$$

and the performance criterion to be

$$\|W_p(I + \tilde{P}K)^{-1}\|_{\infty} < 1, \quad W_p(s) = \frac{s + 0.1}{2s}.$$

- i) Test stability robustness ignoring the structure of Δ .
- ii) Test stability robustness taking into account the structure of $\Delta \in S$.
- iii) Test nominal performance.
- iv) Test robust performance taking into account the structure of Δ .

4. Consider a unit feedback SISO system show in Figure 2 where

$$P = P_0(1 + W_1\Delta_1) + W_2\Delta_2, \quad \Delta_i \in \mathcal{RH}_{\infty}, \quad \|\Delta_i\|_{\infty} < 1, \quad i = 1, 2$$

and K is the controller. Suppose W_1 and W_2 are stable, and P and P_0 have the same number of poles in $\text{Re}\{s\} > 0$.

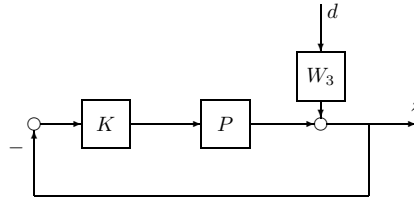


Figure 2: Plant with both additive and multiplicative uncertainties

a) Show that the feedback system is robustly stable if and only if K stabilizes P_0 and

$$\| |W_1T| + |W_2KS| \|_{\infty} \leq 1$$

where

$$S = \frac{1}{1 + P_0K}, \quad T = \frac{P_0K}{1 + P_0K}.$$

b) Show that the feedback system has robust performance; that is,

$$\|T_{zd}\|_{\infty} \leq 1$$

if and only if K stabilizes P_0 and

$$\| |W_3 S| + |W_1 T| + |W_2 K S| \|_\infty \leq 1.$$

5. Let M be a square matrix. Formulate the problem

$$\inf_{D \in \mathcal{D}_S} \bar{\sigma}(DMD^{-1})$$

as a convex optimization problem. *Hint:* A matrix commuting with all full square matrices $\mathbb{C}^{n \times n}$ must be of the form δI_n , and every matrix commutes with δI_n .