Exercise 2

1. Consider a nominal plant

$$P(s) = \frac{1}{s+1}.$$

and a set of perturbed models

$$\Pi = \{\tilde{P} : \tilde{P} = (1 + w\Delta)P,\ \Delta \in H_\infty,\ \|\Delta\|_\infty \leq 1\}$$

in which

$$w = \frac{0.125s + 0.25}{(0.125/4)s + 1}.$$

Find the extreme parameter values in each of the plants (a) - (g) below so that each plant belongs to the set Π. All parameters are assumed to be positive.

(a) Neglected delay: find the largest θ for $P_a = Pe^{-\theta s}$;
(b) Neglected lag: find the largest τ for $P_b = P\frac{1}{\tau s+1}$;
(c) Uncertain pole: find the range of a for $P_c = \frac{1}{s+a}$;
(d) Uncertain pole (time constant form): find the range of T for $P_d = \frac{1}{Ts+1}$;
(e) Neglected resonance: find the range of ζ for $P_e = P\frac{1}{(s/70)^2+2\zeta(s/70)+1}$;
(f) Neglected dynamics: find the largest integer m for $P_f = P\left(\frac{1}{0.01s+1}\right)^m$;
(g) Neglected RHP-zeros: find the largest τ_z for $P_g = P\frac{-\tau_z s+1}{\tau_z s+1}$.

2. (a) Derive the 1) LFT structure and 2) the robust stability test of the uncertain models: $\tilde{P} = P(I + W_1\Delta W_2 P)^{-1}$, $\|\Delta\|_\infty \leq 1$;

(b) Derive the expressions for the nominal and uncertain plants In Figure 1. Uncertainty: $\|[\Delta_N\ \Delta_M]\|_\infty \leq \epsilon$. Derive the LFT structure and RS test. Is the small gain theorem true for non-square Δ and M? (see Theorem 8.1 of ZD)

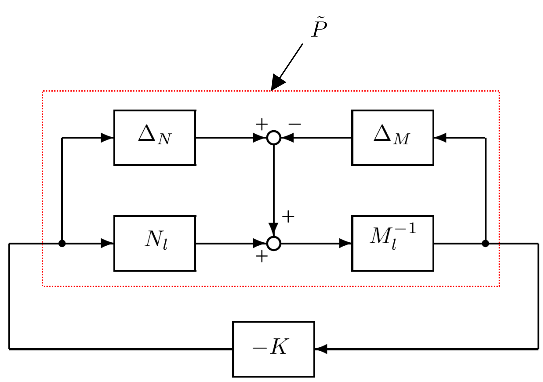

Figure 1: Coprime uncertainty

(c) In Figure 2, G is the nominal plant, K the controller, the uncertainties $\|\Delta_I\|_\infty \leq 1$, $\|\Delta_O\|_\infty \leq 1$. Derive the LFT structure of the system. Is it possible to derive a robust stability test for the system using the results in the lecture? What about when $\Delta_I = \Delta_O$?

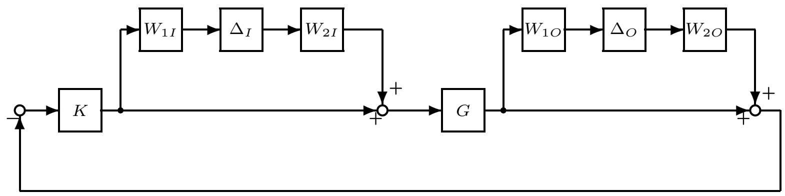

Figure 2: System with input and output multiplicative uncertainty

3. Let a unit feedback system with a controller $K(s) = \frac{1}{s}$ and a nominal plant model $P_o(s) = \frac{s+1}{s^2+0.2s+5}$. Construct a smallest destabilizing $\Delta \in RH_\infty$ in the sense of H_∞-norm for each of the following cases.

(a) $P = P_o + \Delta$;

(b) $P = P_o(1 + W\Delta)$ with $W = \frac{0.2(s+10)}{s+50}$;

(c) $P = \frac{N+\Delta_n}{M+\Delta_m}$, $N = \frac{s+1}{(s+2)^2}$,$M = \frac{s^2+0.2s+5}{(s+2)^2}$ and $\Delta = [\Delta_n \ \Delta_m]$.

4. (1) State and prove the small-gain theorem in the time domain. Assume that you do not know the frequency result (or the Nyquist theorem). In the time domain, the L_q gain of a system is defined as

$$\|G\|_q^q := \sup_{u \in L_q \setminus \{0\}} \frac{\|Gu\|^q}{\|u\|^q}$$

where $1 \leq q \leq \infty$.

(2) Can you extend it to nonlinear case? You can refer to:

- David J. Hill. "A generalization of the small-gain theorem for nonlinear feedback systems." *Automatica* 27.6 (1991): 1043-1045.

- Arjan Van der Schaft. *L2-gain and passivity techniques in nonlinear control*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2000.

5. (1) Read the paper: Soura Dasgupta. "Kharitonov's theorem revisited." *Systems & Control Letters* 11.5 (1988): 381-384.

(2) Explain the Kharitonov's theorem and summarize a proof.