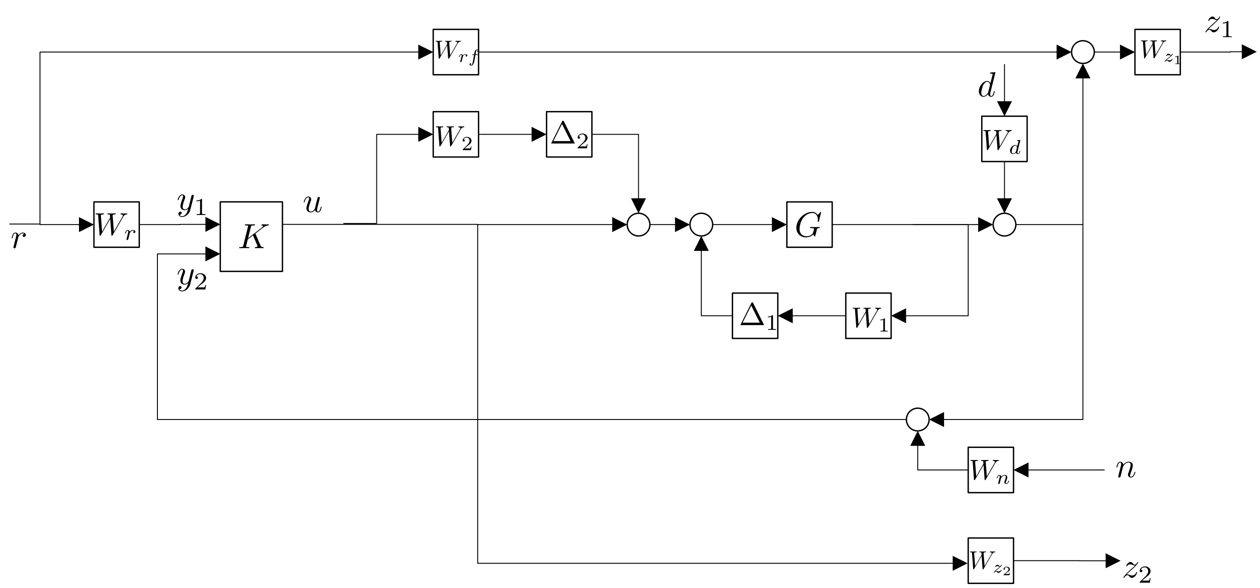
# Exercise 1

**1**. Use MATLAB to find the generalized plant of the following diagram:




Plant: $G = \begin{bmatrix} \frac{1}{s}, & \frac{1}{s} \end{bmatrix} \in \mathbb{C}^{1\times 2}$, controller: $K = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$, uncertainty: $\Delta = \begin{bmatrix} \Delta_1 & \\ & \Delta_2 \end{bmatrix} \in \mathbb{C}^{4\times 4}$ with $\Delta_1, \Delta_2 \in \mathbb{C}^{2\times 2}$

Controller input: $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in \mathbb{R}^2$, controller ouput $u \in \mathbb{R}^2$.

Controlled signal: $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \in \mathbb{R}^2$

Exogenous signal: $w = \begin{bmatrix} d \\ r \\ n \end{bmatrix} \in \mathbb{R}^3$

The weighting functions $W_r$, $W_{rf}$, $W_{z_1}$, $W_d$, $W_n$ are tunable real scalars, $W_1 \in \mathbb{R}^{2\times 1}$, $W_2 \in \mathbb{R}^{2\times 2}$, $W_{z_2} \in \mathbb{R}^{1\times 2}$ are tunable vectors (*Hint*: use the command `tunableGain` or `tunableSS`)

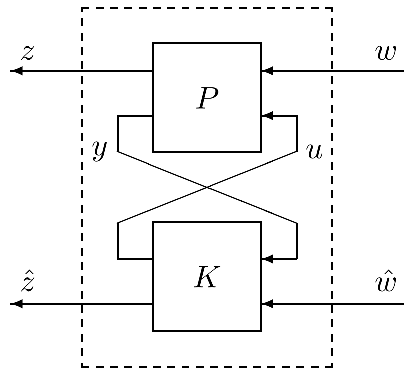
**2**. (1) Let $G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. When does $G^{-1}(s)$ exist? Derive a state space realization of $G^{-1}$ using parameters of the realization of $G$.

(2) Find a state space realization of

$$\begin{bmatrix} I & -K \\ -P & I \end{bmatrix}^{-1}$$

using state-space realization parameters of $P = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ and $K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$.

**3**. Show that the transfer matrix from $\begin{bmatrix} w \\ \hat{w} \end{bmatrix}$ to $\begin{bmatrix} z \\ \hat{z} \end{bmatrix}$ of the diagram




is

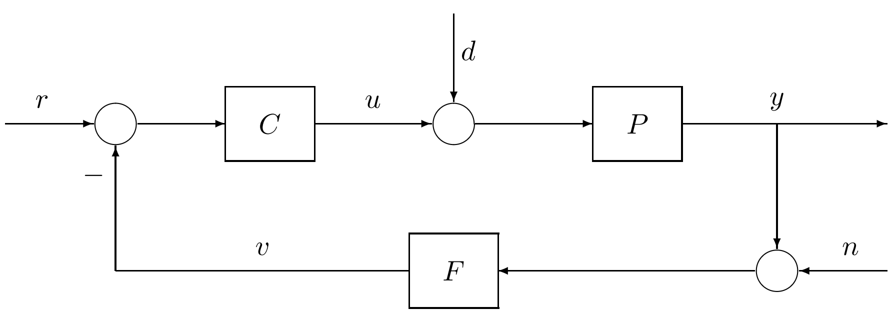
$$P * K = \begin{bmatrix} F_\ell(P, K_{11}) & P_{12}(I - K_{11}P_{22})^{-1}K_{12} \\ K_{21}(I - P_{22}K_{11})^{-1}P_{21} & F_u(K, P_{22}) \end{bmatrix}$$

$P * K$ is called the *star product.* LFTs are special cases of star product.

**4**. Let $G$ be proper and stable, prove that

$$\sup_{u \in L_2 \setminus \{0\}} \frac{\|Gu\|_2}{\|u\|_2} = \sup_{\omega \in \mathbb{R}} \{\text{largest signular value of } G(j\omega)\}$$

**5**. Consider the diagram




in which $P$, $C$, $F$ are proper scalar transfer functions.
(1) Show that the interconnection is well-posed if and only if $PCF(j\infty) \neq -1$.
(2) Assume there is no hidden unstable modes in $P$, $C$ and $F$. Let

$$P = \frac{N_P}{M_P},\ C = \frac{N_C}{M_C},\ F = \frac{N_F}{M_F}$$

in which all the denominators and numerators are real polynomials. In addition, the factorization of $P$ is coprime in the sense that $N_P$ and $M_P$ have no common divisor; same for $C$ and $F$. Show that the system is internally stable if and only if the polynomial

$$N_P N_C N_F + M_P M_C M_F$$

has no roots in the closed RHP.