

# Physical modelling – AC Power systems

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# Outline

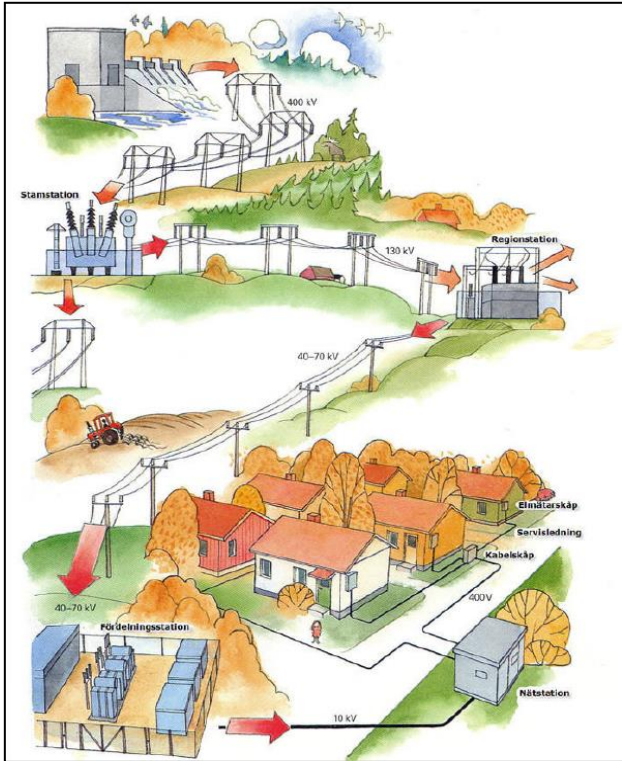
- The electric power system
- Electromagnetic transients
- Phasor model at steady state – power flow
- Electro-mechanical and mechanical oscillations
- Dynamic phasor simulation
- Linearized DAE and ODE
- Modal analysis
- Case study: Iceland

# The Nordic synchronous area

- Three main parts of a power system
  - Electricity **consumption** – demand
  - Electricity **generation** – power plants
  - Electricity **network** or grid
- Nordic area is one dynamic system
  - National borders just organizational boundaries
  - HVDC links permit trading but block most dynamics
  - Western Denmark in Continental Europe area



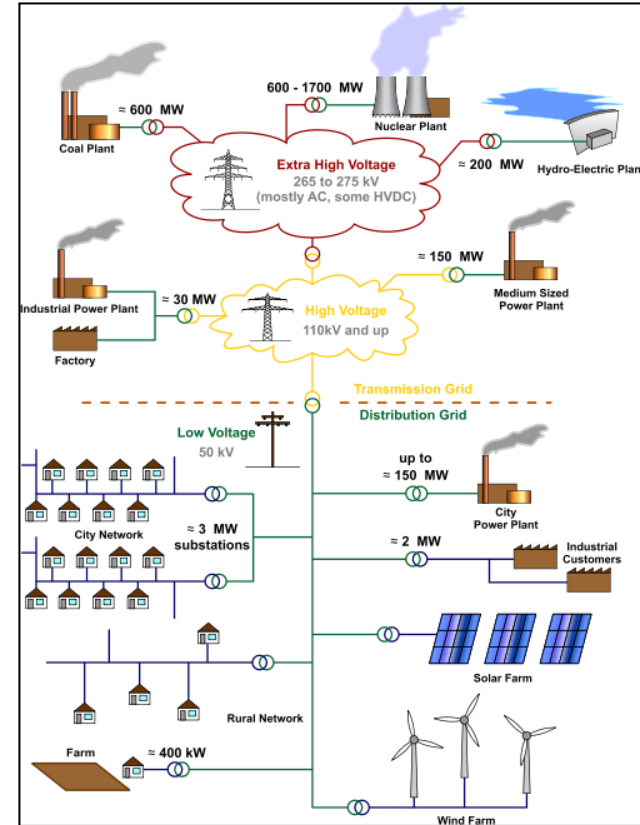
# Voltage levels



2022-05-03

- Transmission 420 kV and 230 kV
- **TSO** Svenska Kraftnät
- Meshed structure
  
- Subtransmission 145 kV
- **DSO** Vattenfall, E.ON, Ellevio + few
- Meshed structure
  
- MV distribution 10-70 kV
- **DNO** Göteborg Energi, Kraftringen, DSOs + hundreds
- Radial structure
  
- LV distribution 0.4 kV
- See MV distribution
- Radial structure

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# Inputs – What drives the system?

- Power flow variations
  - Consumer demand determines consumption
  - Electricity market determines production
  - Weather determines variable production
- Disturbances
  - Weather
  - Equipment failure
  - Human error

# Operation

- Goals
  1. Clear faults fast (safety) and selectively
  2. Voltage should be less than 10 % from nominal value
  3. Frequency should be less than 0.1 Hz from nominal 50 Hz
    - Schedule generation to balance consumption
    - Frequency control manages deviations in power balance
- Challenges
  - Many owners
  - Large distance
  - Many components
  - Many time scales
  - Limited observability
  - Limited controllability

# Power system automation by time scale

- Protection (50 ms – 3 s)
  - For each component: Detect abnormal situation and isolate fault
- Frequency control (s)
  - Turbine control
- Stability controls (1 Hz)
  - Power oscillation damping
- Voltage control (s-min)
  - Transformers, generators, capacitors
  
- Control room: Mainly monitoring + dispatch of repair crew

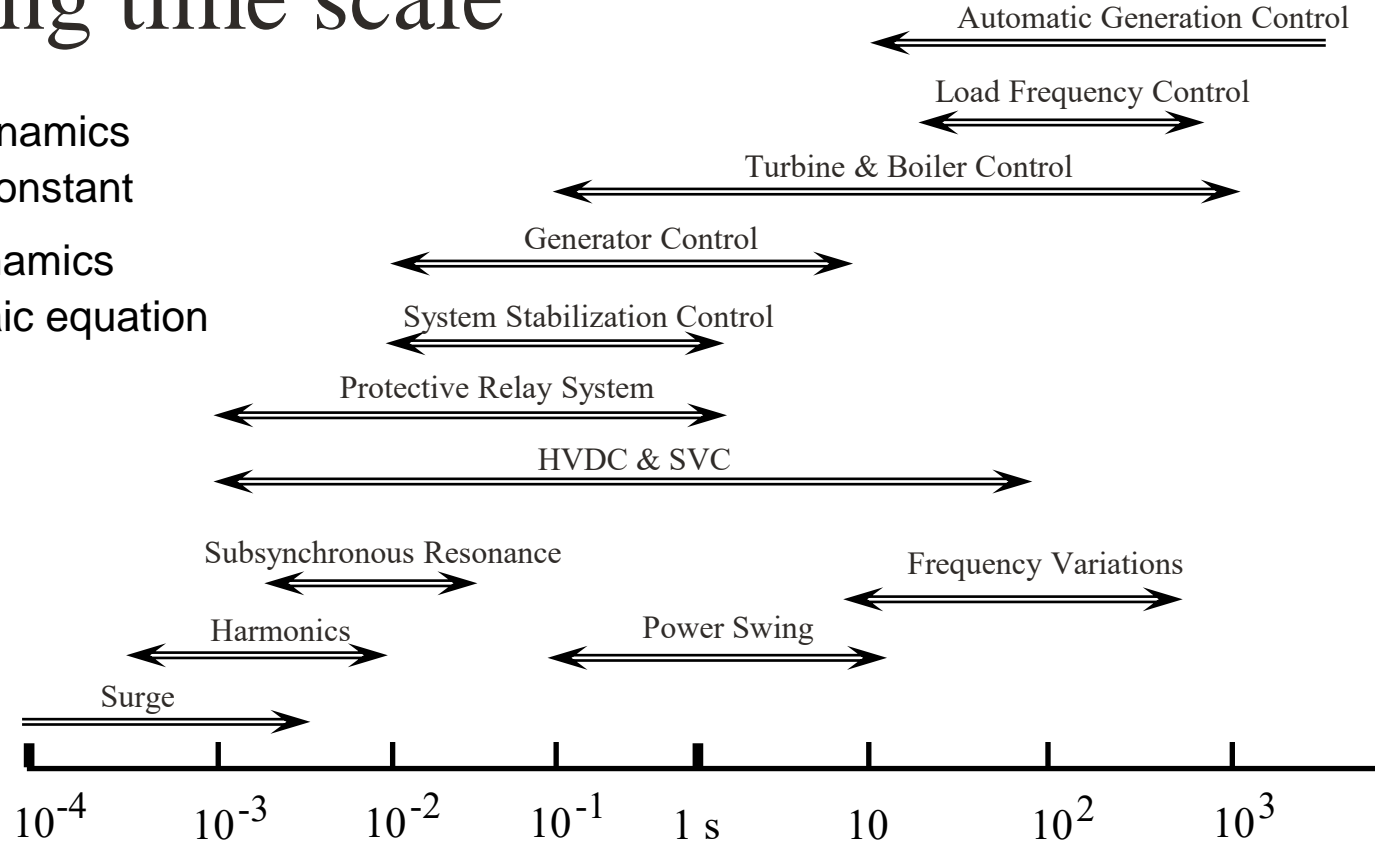
# Modelling challenges

- Model scope
  - Different models for different purposes
  - Models are geographically limited – extent and resolution
  - Models are temporally limited
- Keeping model valid
  - Svenska Kraftnät manages transmission+subtransmission model
  - Each DNO manages MV+LV distribution model (asset database, GIS)
  - Control room software manages network topology (breaker status)
  - Least square fitting of actual operating point to data + measurements



# Selecting time scale

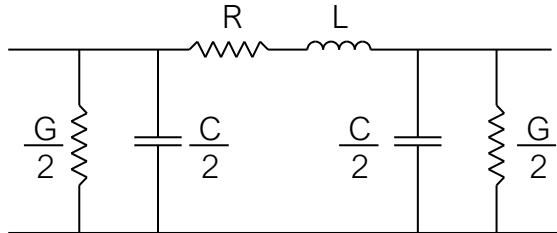
- Too slow dynamics
  - Make constant
- Too fast dynamics
  - Algebraic equation



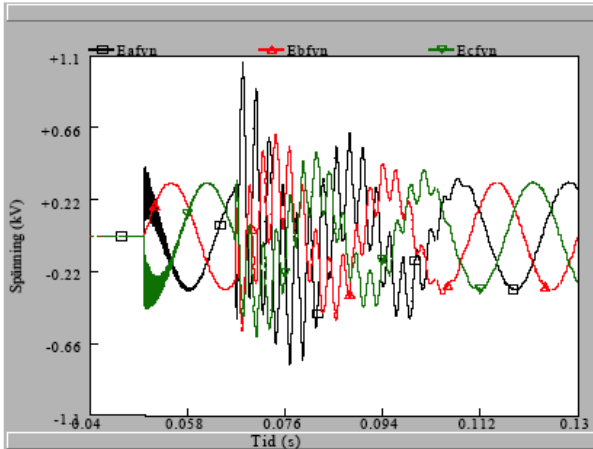
# Power line physics

- Magnetic field from current in conductors
  - Series inductance  $L$  in H/km
- Ohmic losses from current in conductors
  - Series resistance  $R$  in  $\Omega$ /km
- Electrostatic field between different potentials
  - Shunt capacitance  $C$  in F/km
- Ohmic losses from discharges to air
  - Shunt conductance  $G$  in  $\Omega^{-1}$ /km

$\pi$ -model of one phase



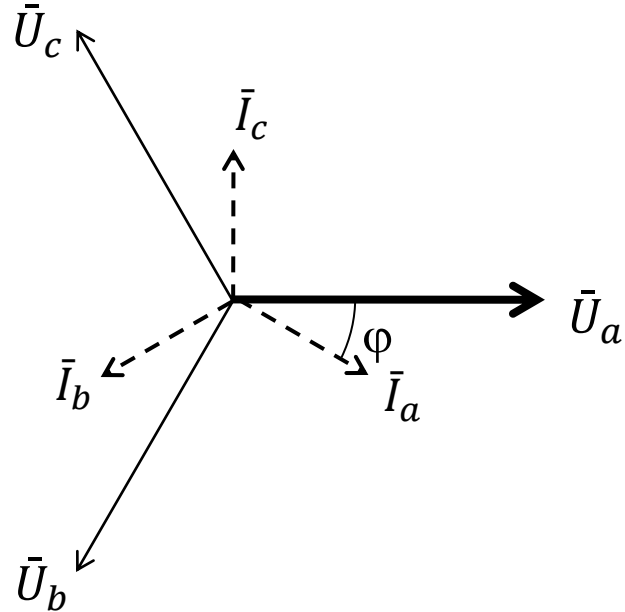
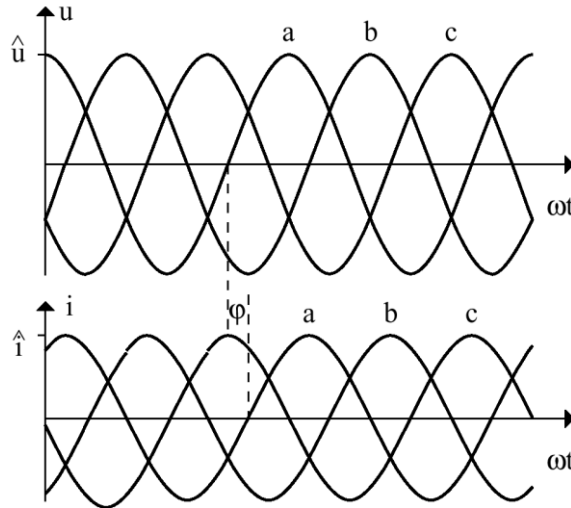
# Electromagnetic transients (EMT)



- Relevant waveform resolution: ns to  $\mu$ s
- Purpose: Matching with high voltage lab measurements
- Modelling
  - Explicit waveforms represent voltages and currents
  - Three-phase line  $\pi$ -model has nine dynamic states
- Simulation software
  - EMTP, PSCAD/EMTDC, RSCAD/EMTDC
  - Everything modelled as RLC-circuits with sources
  - Typically fixed time step ns to  $\mu$ s

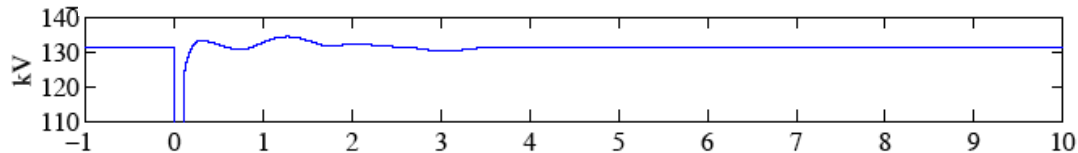
# Complex phasors

- During a cycle, an AC quantity can be represented by a complex number
- In power engineering, absolute value of phasor is rms value
- Frequency is implicit; typically nominal



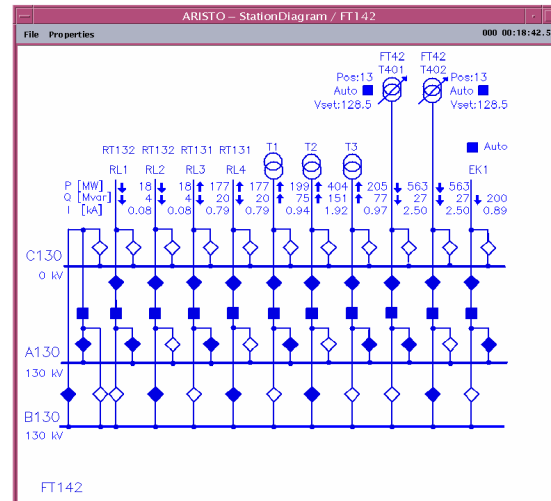
# Dynamic phasor simulation

- Relevant waveform resolution: few cycles and up
- Purpose: Analyse behaviour of entire system
- Modelling
  - Complex phasors represent voltages and currents
  - Three-phase line  $\pi$ -model has no dynamic states
- Simulation software
  - PSS/E, PowerFactory, EUROSTAG, ARISTO
  - Network modelled as complex impedances, dynamics in generators
  - Typically variable time step ms and up



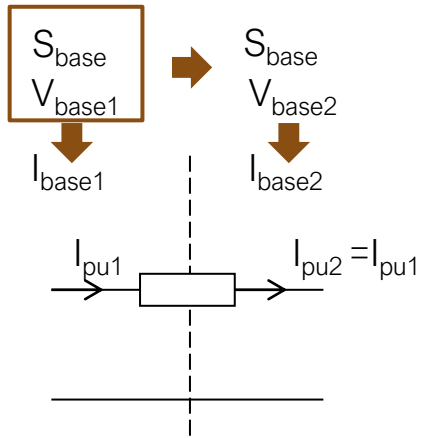
# ARISTO real-time simulator

- By Svenska Kraftnät/ABB
  - For operator training
  - EMS can use SCADA or ARISTO as data source
- Circuit breaker-based model
  - Connecting/sectionalizing bus bars changes topology
- Full Nordic model
  - 29 000 switches
  - 1500 generators
  - 3000 loads
  - 3200 switchyards →



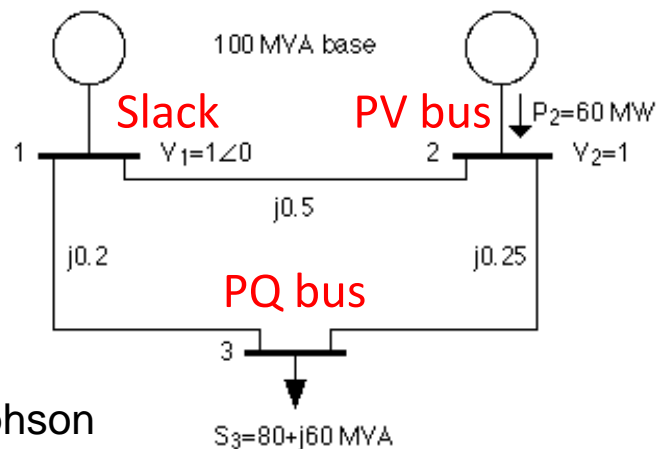
# Normalization eliminates transformers

- At each voltage level an admittance matrix captures all lines
- An admittance matrix capturing several voltage levels very complicated
- Working with normalized – “per unit” quantities
  - Normalizing to common MVA base eliminates transformers
  - Normalizing to MVA base of each component → parameters similar
- Normalized quantities is numerically advantageous
  - Power systems analysis key application of early computer analysis



# Steady state power flow

- Find  $V$  and  $\theta$  and based on that flows and losses
- Example:
- Unknowns:  $x=[\theta_2 \ \theta_3 \ V_3]^T$
- $y=f(x)$ :  
 $f(x)=[P_2(x) \ P_3(x) \ Q_3(x)]^T$ ,  
 $P_k(x)$  is  $P$  from bus  $k$  to rest of network  
 $y=[60 \ -80 \ -60]^T$
- Solve numerically often with Newton-Raphson
- Jacobian inherits structure of admittance matrix
- Power flow calculations basis for power system planning

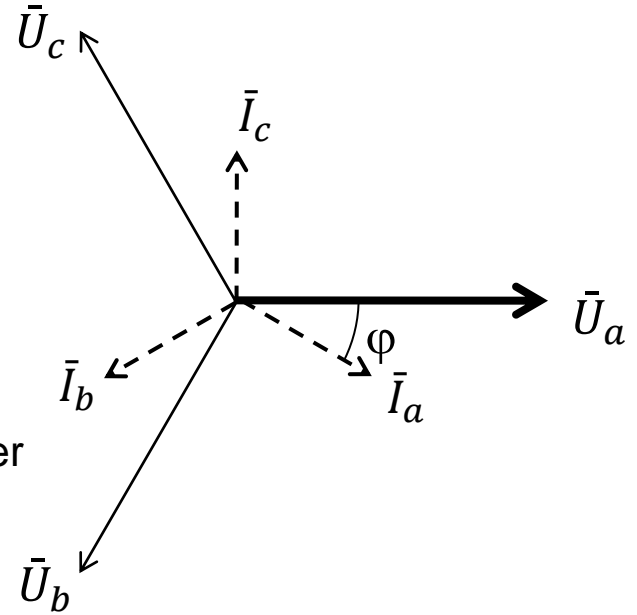


$$J = \frac{\partial f_i}{\partial x_j} = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}$$



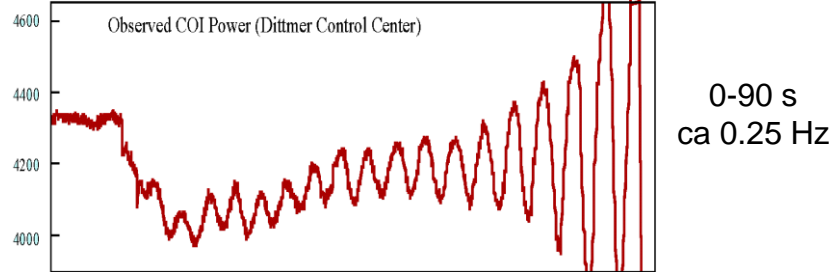
# Symmetry

- Symmetry = All phase quantities have same magnitude and  $\pm 120^\circ$  phase separation
  - Enough to compute one phase
  - Other phases only differ by phase angle
  - Three-phase power = 3 x single phase power
- Most faults are unsymmetric
  - Tree leaning on one phase conductor
  - Lightning strike
  - Object falling on two phase conductors etc.
- Unsymmetry is managed by change of coordinates using “sequence components”



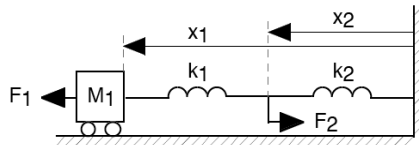
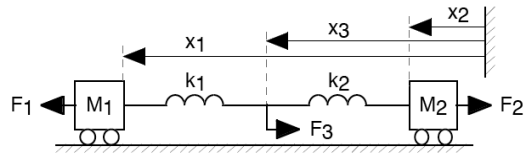
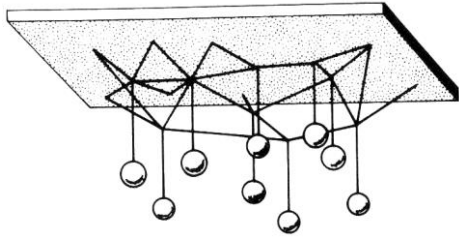
# Electro-mechanical oscillations

- Generator speeds
  - All tend towards 50 Hz, oscillation around 50 Hz (Americas 60 Hz)
  - Kinetic energy ( $J\omega^2$ )/2 varies, power oscillates
- Resonance!
  - 1-3 Hz single generator, <1 Hz generator groups



MW flow California-Oregon, 10 August 1996  
just before blackout of 7.5 million people

# Mechanical oscillations



- General system
  - Many resonances (modes)
  - Complex mode shapes
- Two-mass system
  - $M1$  vs  $M2$  = swing mode
  - $M1$  with  $M2$  = rigid body
  - Frequency dynamics
- Single-mass system
  - Only swing mode

# Dynamic phasor model

- Complex variables represented as  $[Re(\cdot) \quad Im(\cdot)]^T$  or  $[Abs(\cdot) \quad Arg(\cdot)]^T$
- Network equations with bus admittance matrix:  $I = Y_{bus}V$
- Nonlinear differential-algebraic equations for component  $i$ :

$$\begin{cases} \dot{x}_{d,i} = f_i(x_{d,i}, x_{a,i}, u_i) \\ 0 = g_i(x_{d,i}, x_{a,i}, u_i), \quad d=\text{dynamic}, \quad a=\text{algebraic}, \quad u=\text{inputs}, \quad y=\text{outputs} \\ y_i = h_i(x_{d,i}, x_{a,i}, u_i) \end{cases}$$

- Build system:

$$\begin{cases} \dot{x}_d = f(x_d, x_a, u) \\ 0 = g(x_d, x_a, u) \\ y = h(x_d, x_a, u) \end{cases} \text{ where } x_d = \begin{bmatrix} x_{d,1} \\ \vdots \\ x_{d,k} \end{bmatrix}, x_a = \begin{bmatrix} x_{a,1} \\ \vdots \\ x_{a,k} \\ y \\ V \end{bmatrix}, u = \begin{bmatrix} u_1 \\ \vdots \\ u_k \end{bmatrix}, y = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix}$$

# Synchronous generator

$$-r \frac{\omega_o r_f}{M_{dv}} E_{fd} + \frac{\omega_o r_f}{DET} \left[ \frac{L'_{MD} b^2}{DET} - (b + mrc) \right] \lambda_f + \frac{\omega_o r_f}{DET} \left[ mrc + \frac{L'_{MD} b l_f}{DET} \right] \lambda_D + \frac{\omega_o r_f}{DET} l_D L''_{MD} i_d - \frac{d\lambda_f}{dt} = 0 \quad (8-a)$$

$$\frac{\omega_o r_D}{DET} \left[ mrc + \frac{L'_{MD} l_D}{DET} \right] \lambda_f + \frac{\omega_o r_D}{DET} \left[ l_f^2 \frac{L'_{MD}}{DET} - (l_f + mrc) \right] \lambda_D + \frac{\omega_o r_D}{DET} l_f L''_{MD} i_d - \frac{d\lambda_D}{dt} = 0 \quad (8-b)$$

$$\frac{\omega_o r_{Q1}}{l_{Q1}} \left( \frac{L'_{MQ}}{l_{Q1}} - 1 \right) \lambda_{Q1} + \frac{\omega_o r_{Q1}}{l_{Q1}} \frac{L'_{MQ}}{l_{Q2}} \lambda_{Q2} + \frac{\omega_o r_{Q1}}{l_{Q1}} L'_{MQ} i_q - \frac{d\lambda_{Q1}}{dt} = 0 \quad (8-c)$$

$$\frac{\omega_o r_{Q2}}{l_{Q2}} \left( \frac{L'_{MQ}}{l_{Q2}} - 1 \right) \lambda_{Q2} + \frac{\omega_o r_{Q2}}{l_{Q2}} \frac{L'_{MQ}}{l_{Q1}} \lambda_{Q1} + \frac{\omega_o r_{Q2}}{l_{Q2}} L'_{MQ} i_q - \frac{d\lambda_{Q2}}{dt} = 0 \quad (8-d)$$

$$-\omega (i_q + X_T) i_q - \omega \left( \frac{L'_{MQ}}{l_{Q1}} \lambda_{Q1} + \frac{L'_{MQ}}{l_{Q2}} \lambda_{Q2} + L'_{MQ} i_q \right) + (r + R_T) i_d + \sin \theta U_R - \cos \theta U_i = 0 \quad (8-e)$$

$$\omega (i_d + X_T) i_d + \omega \left( \frac{L'_{MD}}{DET} l_D \lambda_f + \frac{L'_{MD}}{DET} l_f \lambda_D + L'_{MD} i_d \right) + (r + R_T) i_q + \cos \theta U_R + \sin \theta U_i = 0 \quad (8-f)$$

in which:  $DET = l_D b + mrc (l_f + l_D)$

$$\frac{1}{L'_{MD}} = \frac{1}{M_d} + \frac{l_f}{DET} + \frac{l_D}{DET}$$

$$\frac{1}{L'_{MQ}} = \frac{1}{M_q} + \frac{1}{l_{Q1}} + \frac{1}{l_{Q2}}$$

$$\frac{P_N}{SNREF} Cm + D(\omega_{ref} - \omega) + \frac{L'_{MD}}{DET} l_D i_q \lambda_f + \frac{L'_{MD}}{DET} l_f i_q \lambda_D - \frac{L'_{MQ}}{l_{Q1}} i_d \lambda_{Q1} - \frac{L'_{MQ}}{l_{Q2}} i_d \lambda_{Q2} + (L'_{MD} - L'_{MQ}) i_d i_q - 2H \frac{d\omega}{dt} = 0 \quad [13]$$

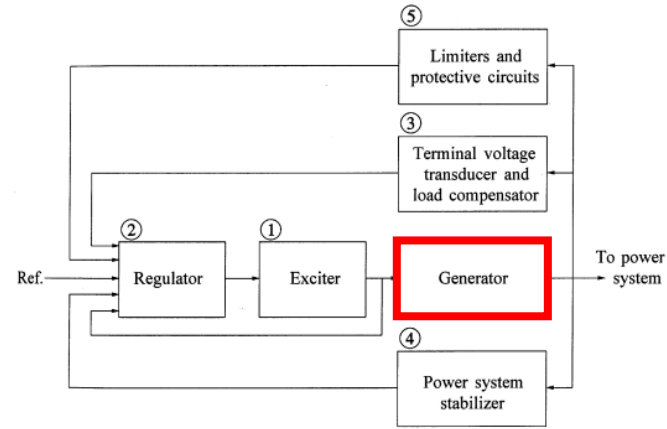
$$-\omega_o \omega_{ref} + \omega_o \omega - \frac{d\theta}{dt} = 0 \quad [14]$$

From EUROSTAG  
reference manual

6<sup>th</sup> order model

3<sup>rd</sup> order model

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$$-r \frac{\omega_o r_f}{M_{dv}} E_{fd} + \frac{\omega_o r_f}{l_f} \left( L'_{MD} i_d + \left( \frac{L'_{MD}}{l_f} - 1 \right) \lambda_f \right) - \frac{d\lambda_f}{dt} = 0 \quad (8-a')$$

$$-\omega (i_q + X_T) i_q - \omega M_q i_q + (r + R_T) i_d + \sin \theta U_R - \cos \theta U_i = 0 \quad (8-e')$$

$$\omega (i_d + X_T) i_d + \omega L'_{MD} \left( i_d + \frac{\lambda_f}{l_f} \right) + (r + R_T) i_q + \cos \theta U_R + \sin \theta U_i = 0 \quad (8-f')$$

in which  $\frac{1}{L'_{MD}} = \frac{1}{M_d} + \frac{1}{l_f}$

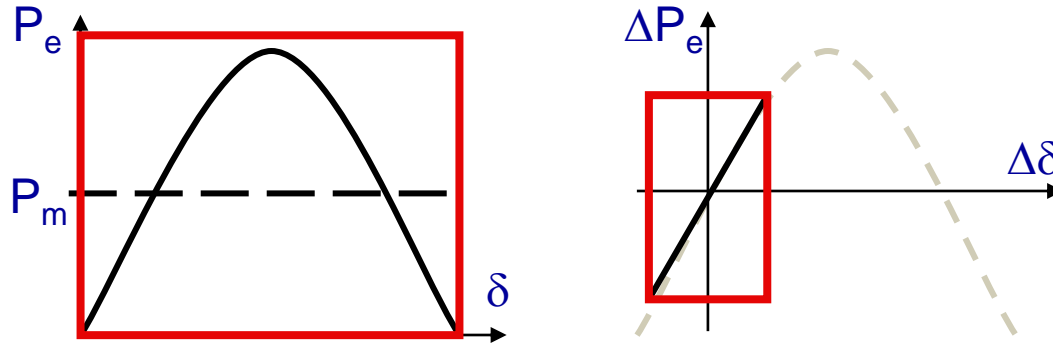
$$\frac{P_N}{SNREF} Cm + D(\omega_{ref} - \omega) + \frac{1}{l_f} i_q \lambda_f + (L'_{MD} - M_q) i_d i_q - 2H \frac{d\omega}{dt} = 0 \quad [13']$$

$$-\omega_o \omega_{ref} + \omega_o \omega - \frac{d\theta}{dt} = 0 \quad [14']$$

# Analysis methods

- Non-linear **time simulations**
  - Anything can be simulated
  - Operating point must be selected
  - Disturbance must be selected – only excites some dynamics
- **Modal analysis** of linearized model
  - Steady state operating point must be selected
  - Only valid near linearization point
  - Reveals all dynamics
- Best to **combine!**

# Linearization simplifies



- Size of disturbance  
Nonlinear model for large changes  $P_e = K_1 \cdot \sin \delta$   
Linearized model for small changes  $\Delta P_e = K_2 \cdot \Delta \delta$
- How small is small?  
"Small" is when linear model is valid 😊

# Linearized DAE

$$\Delta x_d = x_d - x_d^0$$

$$\Delta x_a = x_a - x_a^0$$

$$\Delta u = u - u^0$$

$$\Delta y = y - y^0$$

- Linearize around operating point

- Then

$$E_{dae} \frac{d}{dt} \begin{bmatrix} \Delta x_d \\ \Delta x_a \end{bmatrix} = A_{dae} \begin{bmatrix} \Delta x_d \\ \Delta x_a \end{bmatrix} + B_{dae} \Delta u$$

$$\Delta y = C_{dae} \begin{bmatrix} \Delta x_d \\ \Delta x_a \end{bmatrix} + D_{dae} \Delta u$$

where

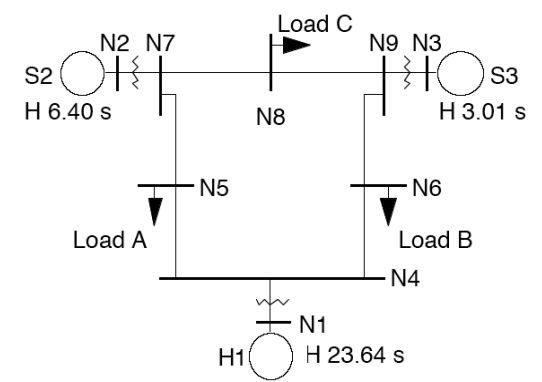
$$E_{dae} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \quad A_{dae} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x_d} & \frac{\partial f}{\partial x_a} \\ \frac{\partial g}{\partial x_d} & \frac{\partial g}{\partial x_a} \end{bmatrix} \quad B_{dae} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial u} \\ \frac{\partial g}{\partial u} \end{bmatrix}$$

$$C_{dae} = [C_1 \ C_2] = \begin{bmatrix} \frac{\partial h}{\partial x_d} & \frac{\partial h}{\partial x_a} \end{bmatrix} \quad D_{dae} = D_1 = \frac{\partial h}{\partial u}$$

- Collect all vectors in one:  $\Delta x = \begin{bmatrix} \Delta x_d \\ \Delta x_a \\ \Delta u \\ \Delta y \end{bmatrix}$  then  $E \Delta \dot{x} = A \Delta x$



# Example: IEEE 9-bus



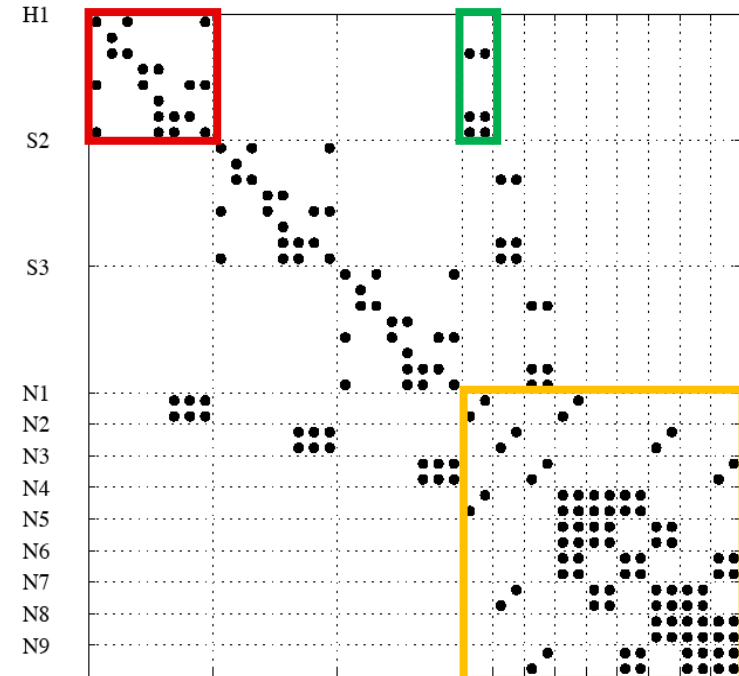
- One generator:  $x_{gen} = [\lambda_f \ V_{ref} \ E_{FD} \ T_m \ \omega \ \delta \ I_q \ I_d]^T$

$$E_{gen} \frac{dx_{gen}}{dt} = A_{gen} x_{gen} + B_{gen} V_{bus}$$

$$\text{diag} \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \frac{dx_{gen}}{dt} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} x_{gen} + \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \begin{bmatrix} V_{bus,real} \\ V_{bus,imag} \end{bmatrix}$$

- Full system:  $x^T = [x_{gen1}^T \ x_{gen2}^T \ x_{gen3}^T \ V_{bus1}^T \ \dots \ V_{bus9}^T]$

$$\text{diag} \begin{pmatrix} E_{gen1} \\ E_{gen2} \\ E_{gen3} \\ 0 \end{pmatrix} \frac{dx}{dt} = \begin{bmatrix} A_{gen1} & & B_{gen1} \\ & A_{gen2} & B_{gen2} \\ & & A_{gen3} & B_{gen3} \\ C_{gen1} & C_{gen2} & C_{gen3} & Y_{bus} \end{bmatrix} x$$



# Eliminate algebraic variables

- Inserting  $x_a = -A_{22}^{-1}(A_{21}x_d + B_2u)$
- gives the ODE
$$\begin{aligned}\dot{x}_d &= A_{ode}x_d + B_{ode}u \\ y &= C_{ode}x_d + D_{ode}u\end{aligned}$$
- where
$$\begin{aligned}A_{ode} &= A_{11} - A_{12}A_{22}^{-1}A_{21} \\ B_{ode} &= B_1 - A_{12}A_{22}^{-1}B_2 \\ C_{ode} &= C_1 - C_2A_{22}^{-1}A_{21} \\ D_{ode} &= D_1 - C_2A_{22}^{-1}B_2\end{aligned}$$
- Note:  $D_{ode}$  generally not zero

# Modal analysis ODE

- Right modal matrix holds the right eigenvectors with **mode observability** information of the dynamic states:

$$\begin{aligned}\Phi^{-1}A_{ode}\Phi &= \Lambda \\ A_{ode}\Phi &= \Phi\Lambda\end{aligned}$$

- Left modal matrix holds the left eigenvectors with **mode controllability** information of the dynamic states:

$$\begin{aligned}\Psi A_{ode}\Psi^{-1} &= \Lambda \\ \Psi A_{ode} &= \Lambda\Psi\end{aligned}$$

# Modal analysis DAE

- Right eigenvectors

$$A\Phi_{dae} = E\Phi_{dae}\Lambda$$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \Phi_{dae,d} \\ \Phi_{dae,a} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Phi_{dae,d} \\ \Phi_{dae,a} \end{bmatrix} \Lambda$$

- **Mode observability** in algebraic variables:

$$\Phi_{dae,a} = -A_{22}^{-1}A_{21}\Phi_{dae,d}$$

- Left eigenvectors

$$\Psi_{dae}A = \Lambda\Psi_{dae}E$$

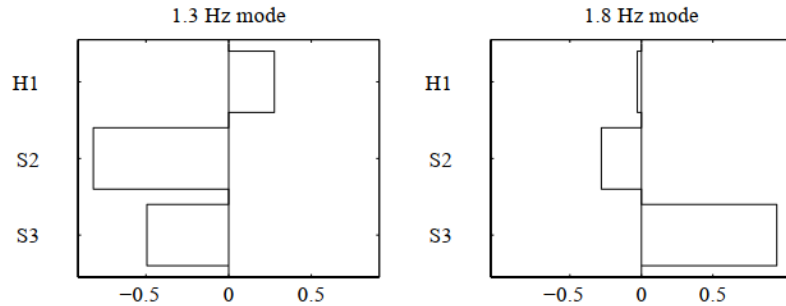
$$\begin{bmatrix} \Psi_{dae,d} & \Psi_{dae,a} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \Lambda \begin{bmatrix} \Psi_{dae,d} & \Psi_{dae,a} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

- **Mode controllability** in algebraic variables:

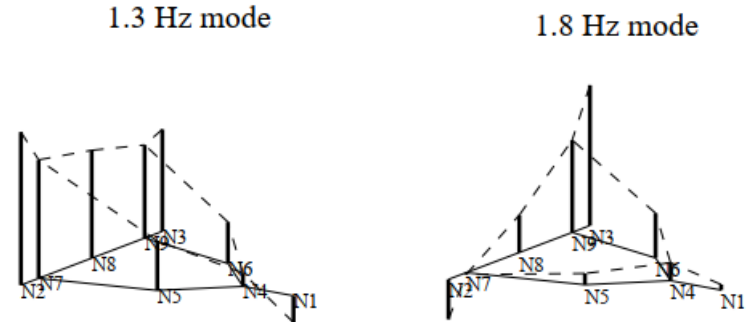
$$\Psi_{dae,a} = -\Psi_{dae,d}A_{12}A_{22}^{-1}$$

# Example: IEEE 9-bus

- ODE mode shapes  
= generator speed elements in right eigenvectors



- DAE mode shapes  
= phase angle information from algebraic parts of right eigenvectors



# Time-scale decomposition

- RL-circuit example:  $L \frac{di}{dt} + Ri = u \Leftrightarrow L \frac{di}{dt} = -Ri + u \Leftrightarrow \frac{L}{R} \frac{di}{dt} = -i + \frac{u}{R}$

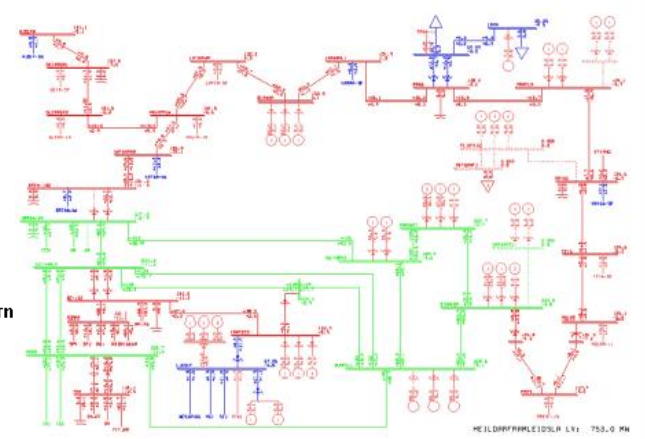
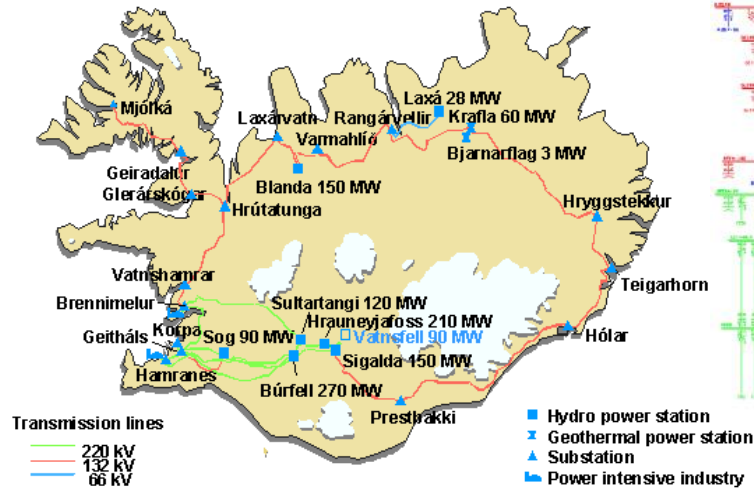
Time constant!

- Scale row  $i$  of  $A$  and  $B$  matrices so that  $A(i,i)=-1$
- Then the factor in front of  $\dot{x}_{d,i}$  can be interpreted as a time constant
- If some time constants are small enough

$$\begin{bmatrix} I & 0 \\ 0 & \varepsilon \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \Delta x_{slow} \\ \Delta x_{fast} \end{bmatrix} = A \begin{bmatrix} \Delta x_{slow} \\ \Delta x_{fast} \end{bmatrix}$$

- setting them to zero yields algebraic equations, like for the network equations in a phasor model

# The Icelandic power system 2002



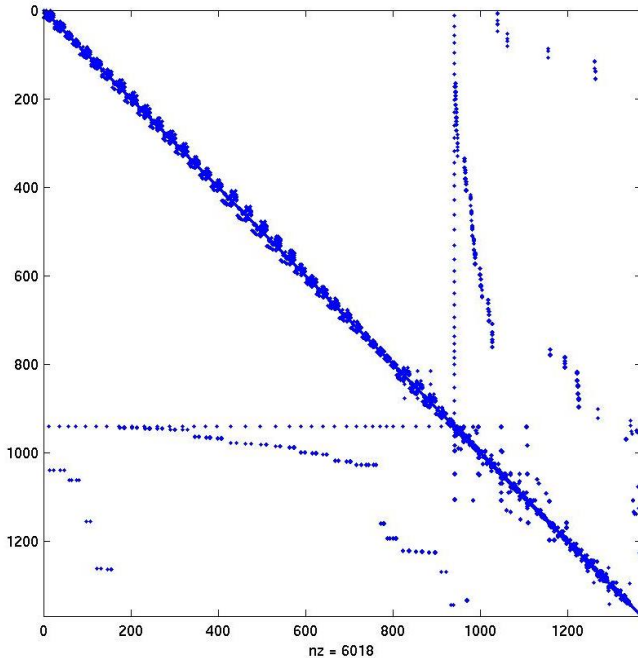
- Peak load 2002 1000 MW (Sweden 28 000 MW)
- New plant of 690 MW (!) on-line 2007

# Case study and solution structure

- Scope
  - Icelandic national power system with electro-mechanical oscillations
  - Find the difficult situations challenging stability
  - Suggest damping controller!
- Multi-dimensional problem
  - Many fault locations → many linearized models
  - Many possible control signals
  - Many possible measurement signals
  - Before, combine all with all, after



# Model of Icelandic power system



- 37 generators
- $\approx 590$  dynamic states
- 202 network nodes
- $\approx 810$  algebraic states
- $A_{dae} \approx 1400 \times 1400$
- $A_{ode} \approx 590 \times 590$

# Control design: Measurement signal

- What quantity is best to measure?
  - Generator speed or power
  - Voltage magnitude
  - Voltage phase angle by GPS-based PMU:s
- Where to measure?
  - Location of generator or network node
- If choice not optimum
  - Oscillation not visible
  - Oscillation less visible - need more gain

# Control design: Control signal

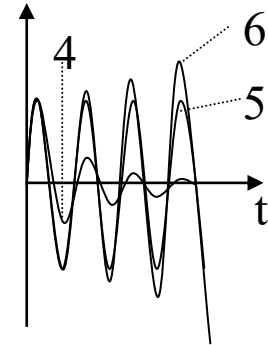
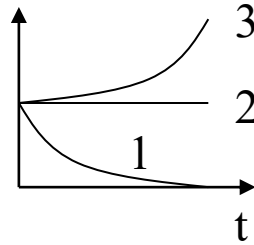
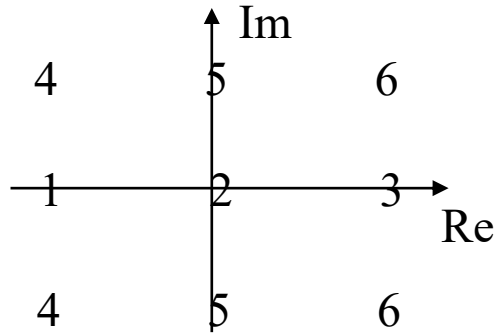
- What to control?
  - Generator voltage (for free)
  - Real power (requires new actuator)
- Where to control?
  - Location of generator or network node
- If choice not optimum
  - Less impact on oscillation
  - Actuator must be larger (expensive!)

# Control design: Closing the loop

- Control signal selected
- Measurement signal selected
- Local control or communication need
- Design controller
  - Phase characteristic
  - Gain
  - Root locus plot for both

# Eigenvalues $A\varphi = \varphi\lambda \quad |\lambda - A| = 0 \quad \lambda = \sigma \pm j\omega$

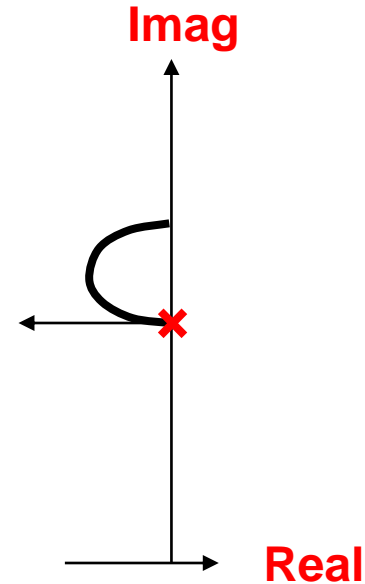
- Response of one mode  $\sim |\varphi_i| e^{\lambda t - \arg \varphi_i} = |\varphi_i| e^{\sigma t} \cos(\omega t - \arg \varphi_i)$
- $\sigma$  is damping and  $\omega$  is frequency



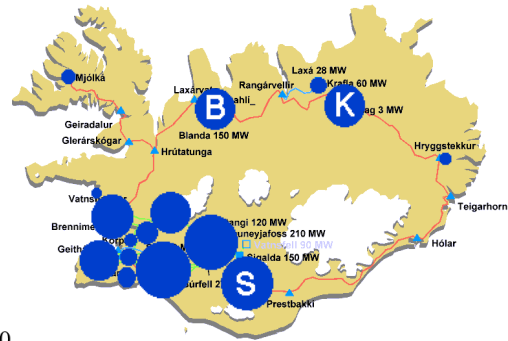
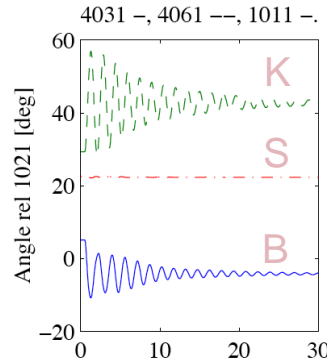
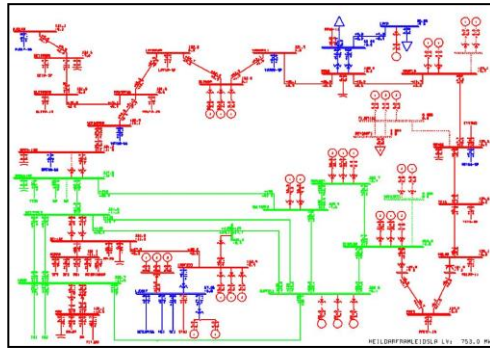
- Complex pair = resonance = *mode*

# Controller

- Increase gain from zero
- Plot eigenvalues
  - Root locus plot
- If direction is  $\alpha$  degrees wrong
  - Phase compensation of  $\alpha$  degrees at mode frequency



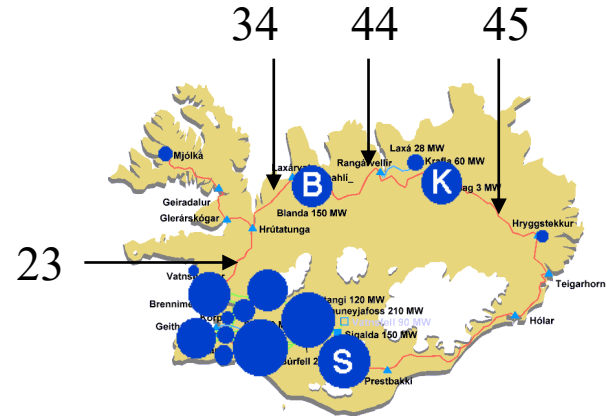
# Spring-mass model



- Circle area  $\sim$  kinetic energy in power plant
- Distance  $\sim$  impedance  $\sim$  spring length
- Think
  - One large mass and two small
  - Long springs

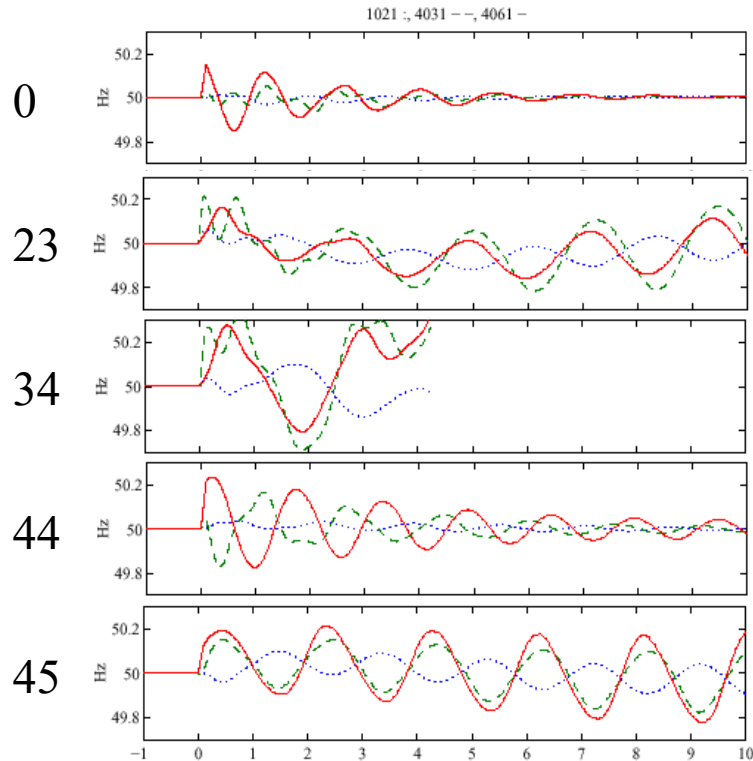
# Critical faults

- Scenarios
  - Intact system
  - Fault at 23
  - Fault at 34
  - Fault at 44
  - Fault at 45





# Time simulations



Olof Samuelsson

# Modes

Eigenvalue Rel. damping

$$-0.39 \pm j4.4 + 0.089$$

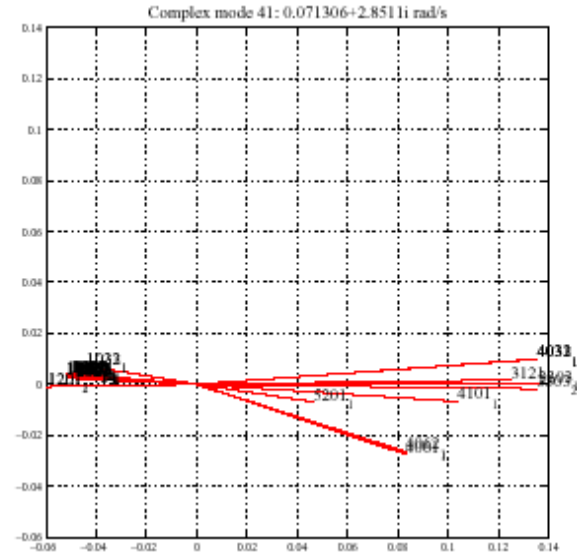
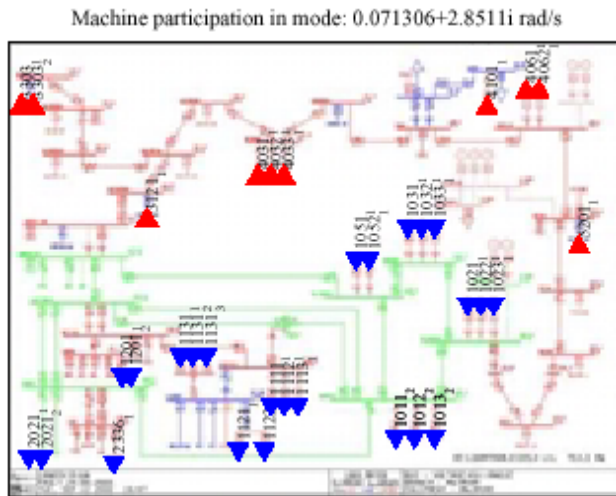
$$+0.07 \pm j2.9 - 0.025$$

$$+0.17 \pm j2.5 - 0.066$$

$$-0.12 \pm j4.1 + 0.030$$

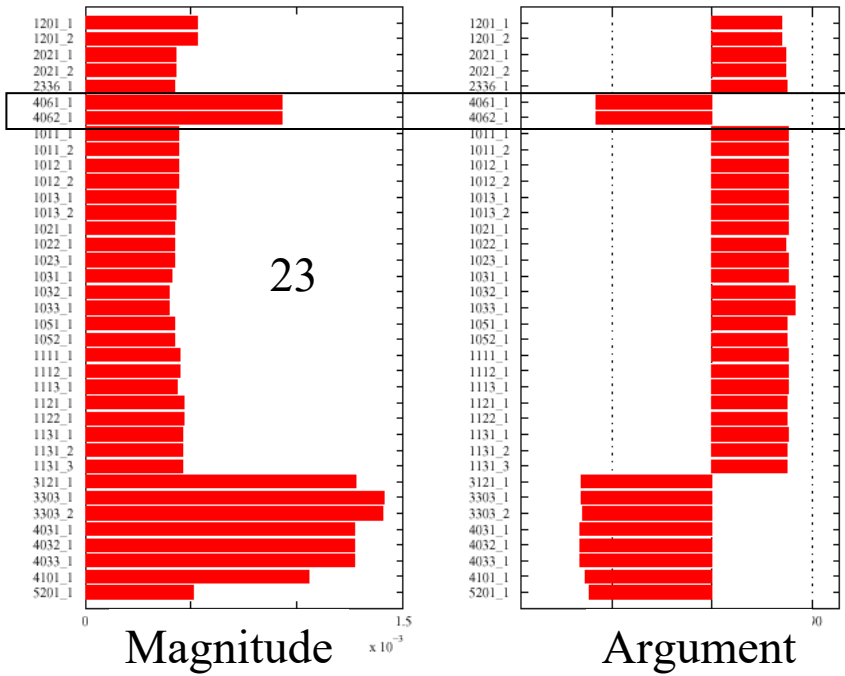
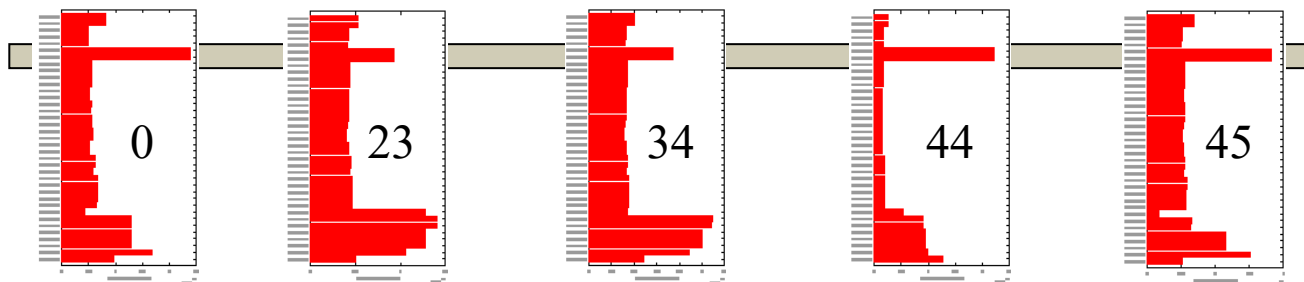
$$+0.04 \pm j3.3 - 0.012$$

# Mode shape case 23



- Interarea mode involves all generators

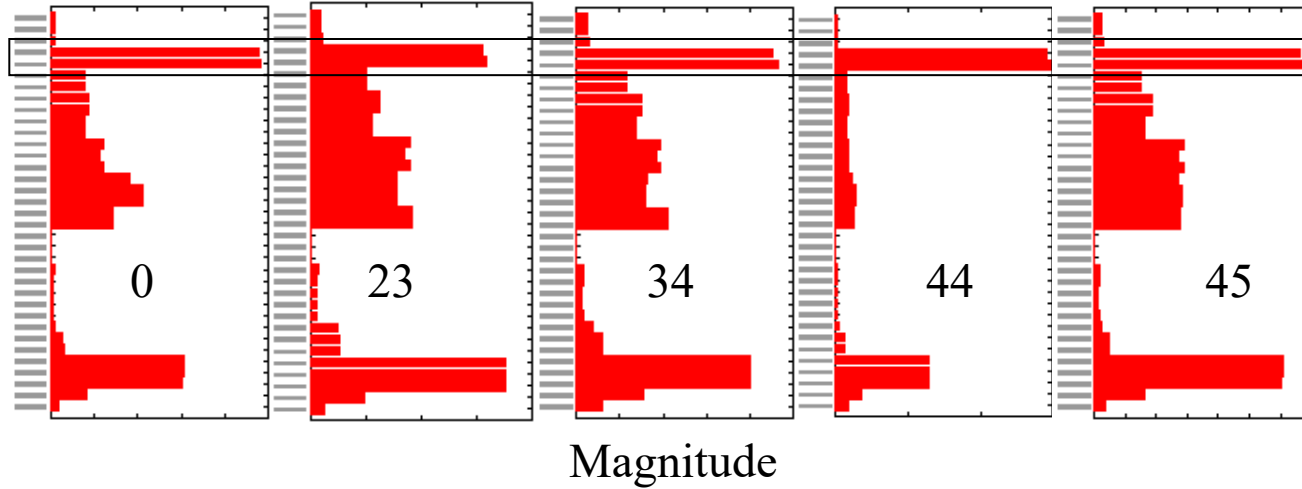
# Right eigenvector $\rightarrow$ Mode observability – speed



Interarea mode  
observable at  
4061/2

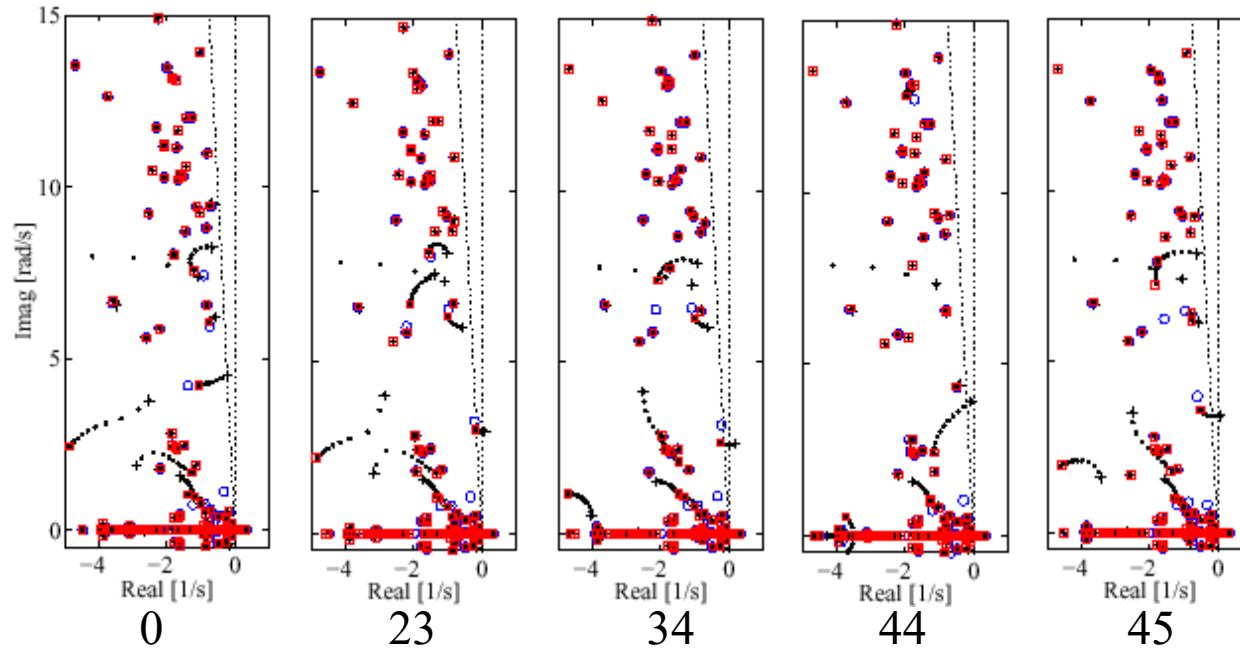
# Left eigenvector $\rightarrow$ Mode controllability

- Voltage controller setpoint



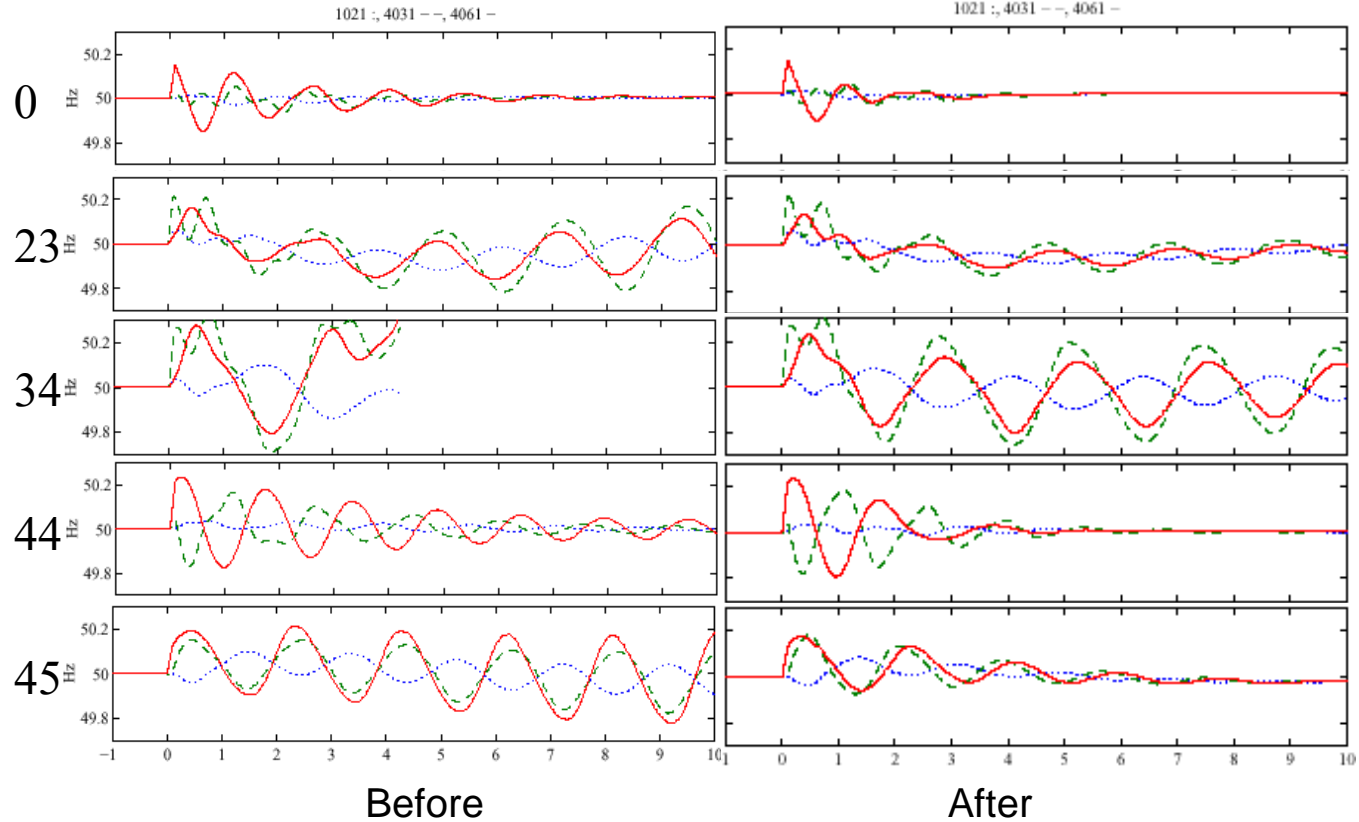
- Interarea mode controllable at 4061/2

# Root locus plots



- Interarea mode damped in all cases
- Gain selected

# Time simulations



# Power system modelling – summary

- Dynamics in time scales from  $\mu\text{s}$  to decades
- Active and reactive power as important as voltage and current
- Waveforms, one or three complex phasors represent three-phase AC quantity
- (Steady state) Operating point important – in practice one every hour
  - Time simulations – valid for one disturbance at one operating point
  - Modal analysis – valid for any disturbance at one operating point
- In modal analysis: Use information in algebraic variables of DAE
- Physical understanding necessary to debug and interpret simulations
- Great effort to keep large models up to date

*“The purpose of computing is insight, not numbers”*

Richard Hamming, Numerical Methods for Scientists and Engineers (1962)





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