Friction Models and Friction Compensation

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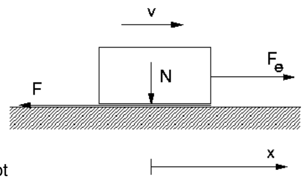
Introduction

- Essential in Motion Control
- Classics
  - Leonardo da Vinci ≈1452-1519
  - Amontons 1699
  - Coulomb 1785
- Tribology
- Control
- Physics AFM
  - Surface force appartus SFA, Atomic force microscope AFM
  - Nanopositioning
- Surface chemistry
- Geophysics - Earthquakes

Amontons's Paradox

Observations

- Friction is proportional to normal load
- Friction does not depend on the apparent area of contact



xgot

The classical friction law $F = \mu N$

Application Areas

- Robotics
- Machine tools
- Valves and actuators
- Automobiles
  - Tire-road interaction
  - ABS
  - Traction control
- Excavators
- Antennas
- Telescopes
- Mechatronics
- Micro-mechanical systems
- Geophysics
- Surface physics
- Physiology

Very Complex Phenomena

Reasonably well understood phenomenologically

- Stiction, elastic deformation

Some phenomena

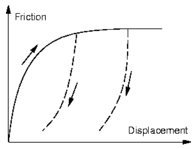
- Steady-state characteristics
- Pre-sliding displacement
- Hysteresis
- Varying break-away force
- Randomness—Repeatability

Much poorly explained

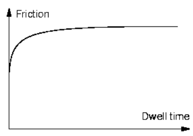
- Friction, surface roughness, and lubrication
- The Mica experiments

Some Friction Phenomena

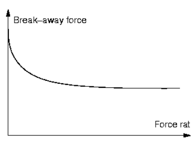
Pre-sliding displacement



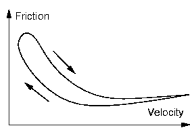
Dwell time dependence



Varying break away force

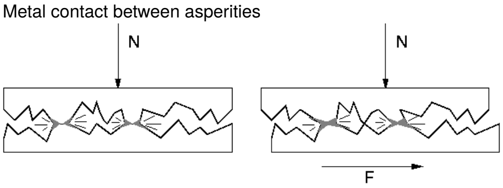


Hysteresis

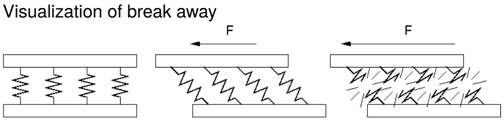


Simple Mechanisms

Metal contact between asperities

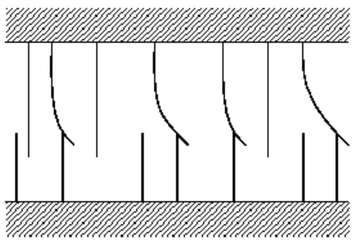


Visualization of break away



The Bristle Model

An abstraction of asperities



A Control Perspective

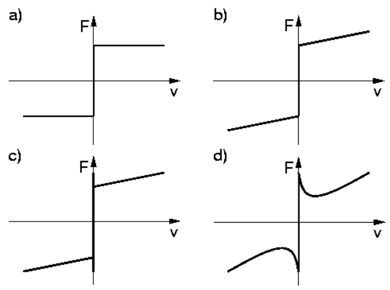
- Understand the effects on friction on control systems
- Typical phenomena
  - Limit cycles
  - Stick slip motion
  - Hunting
  - Lack of precision in tracking
- Modeling and Simulation
- Friction compensation
- Modeling for control

Friction Models and Friction Compensation

Friction Models

- Classical static models
  - Coulomb friction
  - Viscous friction
  - Stiction
- Mechanics and fluid dynamics
  - First principles
  - Microscopical contact
  - Viscosity
- Empirical phenomenological models
  - The Dahl model
  - The Bliman-Sorine model
  - LuGre model

Static Models



a) Coulomb b) Coulomb + viscous c) stiction d) Stribeck effect
In practice there are often asymmetries!

Rate and State Models

Friction has been studied extensively in the Physics and Earth Quake communities. The models are called rate and state models because friction is a function of velocity v and another variable which is dynamically related to velocity.
The Ruina-Rice model is a representative models it has the form

$$\mu = \mu_0 + A \log\left(1 + \frac{v}{v_0}\right) + B \log\left(1 + \frac{\theta/}{\theta_0}\right)$$
$$\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_0}$$

In steady state we have $\theta = d_0/v$ and

$$\mu = \mu_0 + A \log\left(1 + \frac{v}{v_0}\right) + B \log\left(1 + \frac{d_0}{v\theta_0}\right)$$

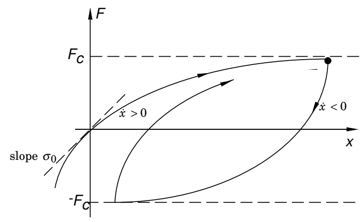
Dahl's Model

- P. Dahl Aerospace Corporation 1968
- Extensive use in simulations in military projects
- Inspired by solid friction

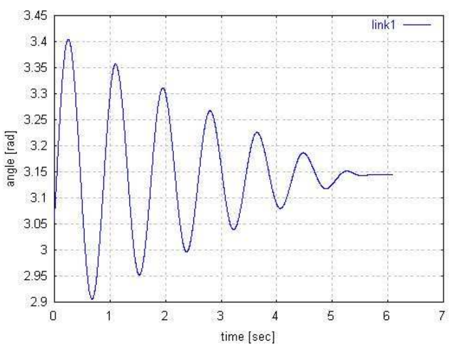
Stress-strain curve

$$\frac{dF}{dx} = \sigma\left(1 - \frac{F}{F_c}\operatorname{sgn} v\right)^{\alpha}$$

- σ stiffness
- α shape



Ball Bearing Friction is Similar to Solid Friction



Dahl's Model - Continued

The stress-strain curve

$$\frac{dF}{dx} = \sigma\left(1 - \frac{F}{F_c}\operatorname{sgn} v\right)^{\alpha}$$

Differentiate with respect to time

$$\frac{dF}{dt} = \frac{dF}{dx}\frac{dx}{dt} = \frac{dF}{dx}v = \sigma\left(1 - \frac{F}{F_c}\operatorname{sgn} v\right)^{\alpha} v.$$

For $\alpha = 1$

$$\frac{dF}{dt} = \sigma v - \sigma\frac{F}{F_c}|v|.$$

Introduce $F = \sigma z$ then

$$\frac{dz}{dt} = v - \frac{\sigma|v|}{F_c}z,$$
$$F = \sigma z.$$

Dahl's Model - Steady State Properties

$$\frac{dz}{dt} = v - \frac{\sigma|v|}{F_c} z,$$
$$F = \sigma z.$$

In steady state

$$z = \frac{F_c}{\sigma} \operatorname{sgn} v$$
$$F = F_c \operatorname{sgn} v$$

- The steady state version of Dahl's model corresponds to Coulomb friction.

Properties of The Dahl Model

- Simple dynamic model
- Used extensively in simulation studies
- Captures many phenomena
  - Zero slip displacement
  - Hysteresis
- Friction depends only on displacement
- Does not capture Stribeck effect
- Does not capture stick-slip
- Can we extend the model to include these effects?

The Bliman-Sorine Model

Idea:
- Generalize Dahl to obtain Stribeck effect
- Keep rate independence

The sliding variable

$$s = \int_0^t |v(\tau)| d\tau.$$

The model

$$\frac{dx_s}{ds} = Ax_s + Bv_s$$
$$F = Cx_s$$

$$A = \begin{pmatrix} -1/(\eta\varepsilon_f) & 0 \\ 0 & -1/\varepsilon_f \end{pmatrix}, \qquad B = \begin{pmatrix} f_1/(\eta\varepsilon_f) \\ -f_2/\varepsilon_f \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 \end{pmatrix},$$

Properties of the Bliman-Sorine Model

- Dahl models in parallel
- At least second order dynamics
- Captures many properties
- Captures stiction
- Friction depends on displacement
- Friction does not depend on velocity
- Related to hysteresis models

Friction Models and Friction Compensation

1. Introduction
2. Friction Models
3. The LuGre Model
4. Effects of Friction on Control Systems
5. Friction Compensation
6. Summary

The LuGre Model

Idea: Generalize Dahl to obtain Stribeck effect and stick-slip

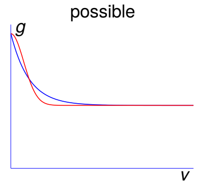
The Dahl model

$$\frac{dz}{dt} = v - \frac{\sigma|v|}{F_c} z,$$
$$F = \sigma z.$$

The LuGre Model

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z,$$
$$g(v) = l_c + (l_s - l_c)e^{-|v|/v_s}$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_3 v$$

Other forms of F and g possible

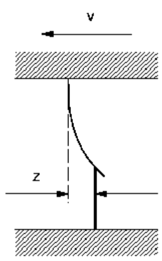


Bristle Interpretation

The variable z in the LuGre model can be interpreted as the average bristle deflection!

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z,$$
$$g(v) = l_c + (l_s - l_c)e^{-|v|/v_s}$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$

Variable z has dimension length



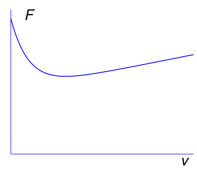
Steady State Properties of the LuGre Model

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z,$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$

In steady state $z = 0$ and $v = v_0$ constant

$$z_0 = \frac{g(v_0)}{\sigma_0} \operatorname{sgn} v_0$$
$$F = g(v) \operatorname{sgn} v_0 + \sigma_2 v_0$$

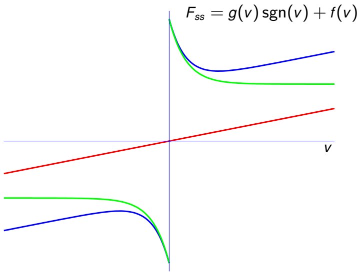
The term $\sigma_2 v$ represents viscous friction



Functions f and g Give Flexibility

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z,$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + f(v)$$



- Functions f and g can be chosen so that F_{ss} matches measured steady state friction
- Function F_{ss} is often asymmetrical, easy to deal with

Theorem 1 - Boundedness of z

Assume that $0 < g(v) < a$. Then $\Omega = \{z : |z| \le a\}$ is an invariant set for the LuGre model. If $|z(0)| < a$ then $|z(t)| \le a$ for all $t \ge 0$.

Proof

We have

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z$$

For $z = a$ we have

$$\frac{dz}{dt} = v - \frac{|v|}{g(a)} a \le v - |v|$$

dz/dt is thus either constant or negative and z cannot be larger than a. Applying the same argument at $z = -a$ gives the result.

Theorem 2 - Dissipativity

Consider the equation

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z$$

The map $\varphi : v \to z$ is dissipative with respect to the energy function $V = z^2/2$.

Proof

Along trajectories of the differential eqution we have

$$V(t) - V(0) = \int_0^t z \frac{dz}{dt} dt = \int_0^t z\Big(v - \frac{|v| z}{g(v)}\Big) dt = \int_0^t \Big(zv - \frac{|v| z^2}{g(v)}\Big) dt \le \int_0^t z$$

Hence

$$\int_0^t z v dt \ge V(t) - V(0)$$

Application of Constant Force

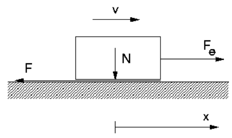
Consider a mass m and apply a constant force F_d

$$m\frac{dv}{dt} = F_d - F$$

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z,$$

$$g(v) = l_c + (l_s - l_c) e^{-|v|/v_s}$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$



Hence

$$m\frac{dv}{dt} = F_d - \sigma_0 - \sigma_1 \frac{dz}{dt} = -\sigma_1 v - \sigma_0 z + \sigma_1 \frac{z|v|}{g(v)} + F_d$$

$$\frac{dz}{dt} = v - \frac{z|v|}{g(v)}$$

Application of Constant Force

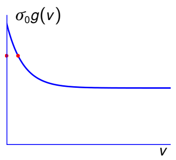
$$m\frac{dv}{dt} = -\sigma_1 v - \sigma_0 z + \sigma_1 \frac{z|v|}{g(v)} + F_d = -\sigma_1 v - \sigma_0 z + z f(v) + F_d$$

$$\frac{dz}{dt} = v - \frac{z|v|}{g(v)} = v - z f(v)$$

Equilibria exists only for $|F_d| \le \sigma_0 l_s$

$$z = \frac{F_d}{\sigma_0}$$

$$v = \frac{F_d |v|}{\sigma_0 g(v)}, \quad \Rightarrow F_d = \sigma_0 g(v)\,\mathrm{sgn}(v)$$



Two equilibria $v = 0$ or $F_d = \sigma_0 g(v_0)\,\mathrm{sgn}(v_0)$, with $F_d = \sigma_0 z_0$ in both cases. The second only for $\sigma_0 l_c \le |F_d| \le \sigma_0 l_s$

Linearization

Linearization gives the dynamics matrix

$$A = \begin{bmatrix} -\dfrac{\sigma_1(1 - zf')}{m} & \dfrac{-\sigma_0 + \sigma_1 f}{m} \\ 1 - zf' & -f \end{bmatrix}$$

where $f(v) = |v|/g(v)$. We have

$$\operatorname{trace} A = -(1 - zf')\frac{\sigma_1}{m} - f$$

$$\det A = (1 - zf')\frac{\sigma_0}{m}$$

At the equilibrium $v = 0, z = FD/\sigma_0$ we have $f = f' = 0$ and we get $\operatorname{trace} A = -\sigma_1/m$ and $\det A = \sigma_0/m$. The characteristic polynomial of the A-matrix is

$$s^2 + \frac{\sigma_1}{m} s + \frac{\sigma_0}{m}$$

Model Parameters

For a mass m which moves subject to Coulomb friction we have

$$\sigma_0 l_c = \mu m g$$

If we start by specifying l_c we thus find that $\sigma_0 = \mu m g / l_c$. To find the parameter σ_1 we make the assumption that the polynomial

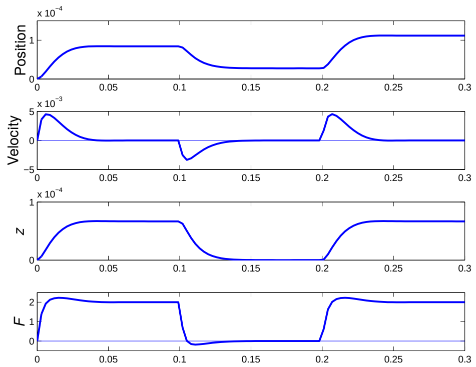
$$s^2 + \frac{\sigma_1}{m} s + \frac{\sigma_0}{m}$$

has roots with critical damping. Hence $\sigma_1^2 = 4 m \sigma_2$ or

$$\sigma_1 = 2\sqrt{m\sigma_0} = 2m\sqrt{\frac{\mu g}{l_c}}$$

Parameter l_s is typically 50 to 100 % larger than l_c. The friction model is characterized by only two parameters μ and l_c.

Simulation of Application of Force



Properties of LuGre Model

- Almost as simple as the Dahl model
- Captures many aspects of friction
  - Stiction
  - Stick slip
  - Stribeck
  - Hysteresis
  - Zero slip displacement
- But not all
  - Some hysteresis related phenomena
  - Item for research!
- Is passive if damping is velocity dependent
  - Very important for control design

Friction Models and Friction Compensation

Effects of Friction in Motion Control

Friction can has both benefits and drawbacks

- Essential part in many drive mechanisms *Capsubot*
- Friction can give rise to oscillations and poor precision

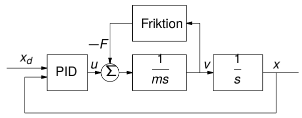
Stick slip can occure whenever the mechanisms

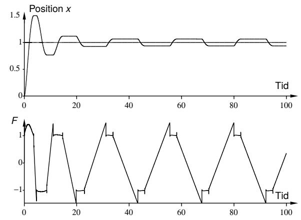
- Stiction
- Instability

are present. Typical examples are

- Traditional stick-slip, spring is the instability mechanism
- Inverted pendulum, gravity is the instability mechanism
- Servo systems, integral is instability mechanism.

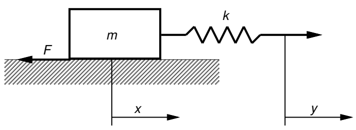
Hunting





Stick-slip Motion

A classic phenomena



Equations of motion

$$m\frac{d^2x}{dt^2} = k(y - x) - F$$
$$\frac{dl}{dt} = v_d - v$$
$$m\frac{dv}{dt} = kl - F$$

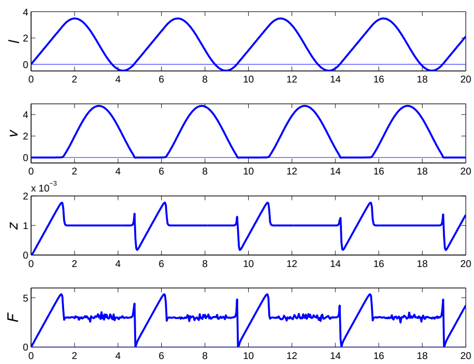
Simulation of Stick Slip

$$m\frac{d^2x}{dt^2} = k(y - x) - F$$
$$y = v_o t$$
$$l = y - v$$
$$\frac{dl}{dt} = v_0 - v$$
$$m\frac{dv}{dt} = kl - F$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$
$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z$$
$$g(v) = l_c + (l_s - l_c)e^{-v/v_s}$$

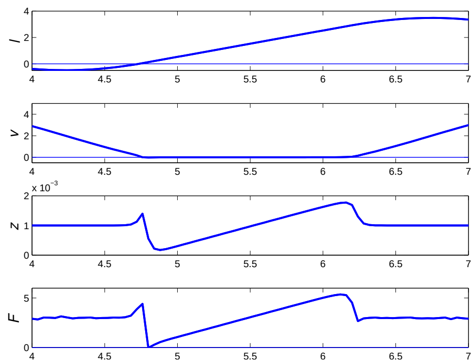
Parameters

$$m = 1$$
$$k = 2$$
$$\mu = 0.3$$
$$l_c = 0.003$$
$$l_s = 2l_c$$
$$v_s = 0.1$$
$$v_{d0} = 2$$
$$k = 2$$
$$\sigma_0 = \frac{\mu mg}{l_c}$$
$$\sigma_1 = 2\sqrt{m\sigma_0}$$
$$\sigma_2 = 0.0$$

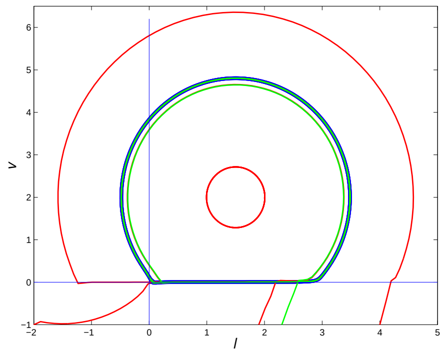
Simulation of Stick Slip



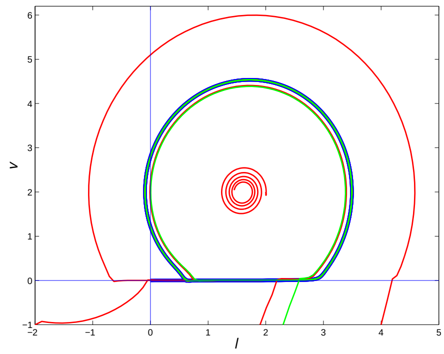
Simulation of Stick Slip - Details



Projection on l, v Plane $\sigma_2 = 0$



Projection on l, v Plane $\sigma_2 = 0.2$



Effects of k and s_2

Equations of motion

$$\frac{dl}{dt} = v_0 - v$$
$$m\frac{dv}{dt} = kl - F$$
$$\frac{dz}{dt} = v - \frac{|v|}{g(v)}z$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$
$$g(v) = l_c + (l_s - l_c)e^{-v/v_s}$$

z has little influence except when v is very small

$$\frac{dl}{dt} = v_0 - v$$
$$m\frac{dv}{dt} = kl - \sigma_0 l_c \operatorname{sgn}(v) - \sigma_2 v$$

Equilibrium

$$l = \frac{\sigma_0 l_c \operatorname{sgn}(v_0) + \sigma_2 v_0}{k}$$
$$v = v_0$$

Characeristic polynomial

$$s^2 + \frac{\sigma_2}{m}s + \frac{k}{m}$$

Relative damping $\zeta = \frac{\sigma_2}{2\sqrt{mk}}$

Effects of Parameter Variations

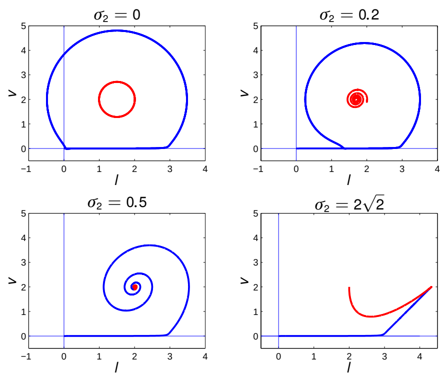
Equilibrium

$$l = \frac{\sigma_0 l_c \operatorname{sgn}(v_0) + \sigma_2 v_0}{k} = \frac{F_c + \sigma_2 v_0}{k}$$
$$v = v_0$$

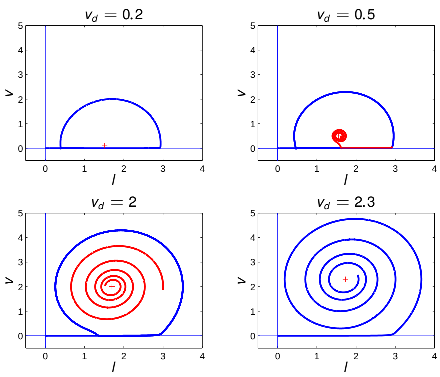
Relative damping $\zeta = \dfrac{\sigma_2}{2\sqrt{mk}}$

- σ_2 shifts equilibrium horizontally, influences damping and existence of limit cycle
- k shifts equilibrium horizontally, influences damping and existence of limit cycle
- v_0 shifts equilibrium horizontally and vertically, influences and existence of limit cycle

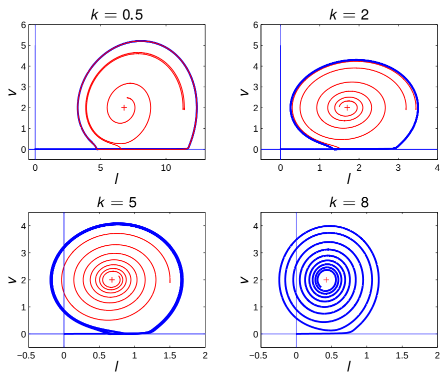
Effect of Viscous Friction σ_2



Effect of Pulling Velocity v_d



Effect of Spring Coefficient k



Stability of Sliding Equilibrium

Equations of Motion

$$\frac{dl}{dt} = v_0 - v$$
$$m\frac{dv}{dt} = kl - F$$
$$\frac{dz}{dt} = v - \frac{|v|}{g(v)}z$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$
$$g(v) = l_c + (l_s - l_c)e^{-v/v_s}$$

Equilibrium

$$v = v_0$$
$$z = g(v_0)\operatorname{sgn} v_0$$
$$l = (\sigma_0 z_0 + \sigma_2 v_0)/k$$

Dynamics matrix

$$A = \begin{bmatrix} 0 & -1 & 0 \\ \frac{k}{m} & -\frac{\sigma_1(1-zf')+\sigma_2}{m} & -\frac{\sigma_0-\sigma_1 f}{m} \\ 0 & 1-zf' & -f \end{bmatrix}$$

Characteristic polynomial

$$s^3 + a_1 s^2 + a_2 s + a_3$$

$$f(v) = \frac{|v|}{g(v)}$$
$$a_1 = \frac{\sigma_1(1-zf') + \sigma_2}{m} + f$$
$$a_2 = \frac{\sigma_0(1-zf') + \sigma_2 f + k}{m}$$
$$a_3 = \frac{kf}{m}$$

Specifics

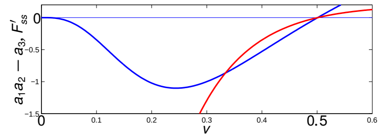
For large v we have $g = l_c$, $f = \frac{|v_0|}{l_c}$ and $f' = \frac{sgn(v_0)}{L_c}$ then

$$a_1 = \sigma_2 + f$$
$$a_2 = k + \frac{\sigma_2 |v|}{l_c}$$
$$a_3 = \frac{k|v_0|}{l_c} \quad a_1 a_2 - a_3 = \sigma_2^2 f + \sigma_2 f^2 + \sigma_2 k$$

The sliding equilibrium is thus stable when the velocity is large. Stick-slip oscillation requires intial conditions sufficiently far from the equilibrium.

Specifics ...

Instability for small v for the simulation example



Parameters

$l_c = 0.001$
$l_s = 0.002$
$\mu = 0.3$
$v_s = 0.1$
$g = 10$
$m = 1$
$k = 1$
$\sigma_0 = \frac{\mu m g}{l_c}$
$\sigma_1 = 2\sqrt{m s_0}$
$\sigma_2 = 0.2$

The red curve shows the derivative of the static friction function $F'_{ss}(v)$ for $v > 0$

$$F_{ss}(v) = \sigma_0 z + \sigma_2 v = \sigma_0 \Big(l_c + (l_s - l_c) e^{-v/v_s} \Big) + \sigma_2 v$$
$$F'_{ss}(v) = -\sigma_0 \frac{l_s - l_c}{v_s} e^{-v/v_s} + \sigma_2$$

Stable for $v > 0.501$

Analysis

Assuming that z is much faster than the other states the model can be approximated by the singularly perturbed system

$$\frac{dl}{dt} = v_0 - v$$
$$m\frac{dv}{dt} = kl - F_{ss}(v)$$
$$F_{ss} = \sigma_0 g(v)\,\mathrm{sgn}\, v + \sigma_2 v$$

The linearization of this system has the dynamics matrix

$$A = \begin{bmatrix} 0 & 1 \\ \frac{k}{m} & -\frac{F'_{ss}}{m} \end{bmatrix}$$

This system is stable if

$$F'_{ss}(v) = \frac{\sigma_0 g'(v)\,\mathrm{sgn}(v) + \sigma_2}{m} > 0$$

Summary

The stick slip behavior of the LuGre model is complex.

- Useful to approximate by neglecting z gives a crude picture which allows projection on the l, v plane.
- The zone around the strip $v = 0$ and $0 \le< l \le F_s/k$ acts like an attractor.
- Solutions can pass through the strip because the problem is really three dimensional.
- There is an equilibrium where the velocity and the length are constant. The stability of this equilbrium depends on the parameters.
- The limit cycle will typically disappear when v or k are large.

Friction Models and Friction Compensation

1. Introduction
2. Friction Models
3. The LuGre Model
4. Effects of Friction on Control Systems
5. Friction Compensation
   - Exploiting passivity
   - Servo systems
   - The Furuta Pendulum
6. Summary

Friction Compensation

Methods to reduce effects of friction

- Dither
- Acceleration feedback
- Model based friction compensation
- Adaptive friction compensation

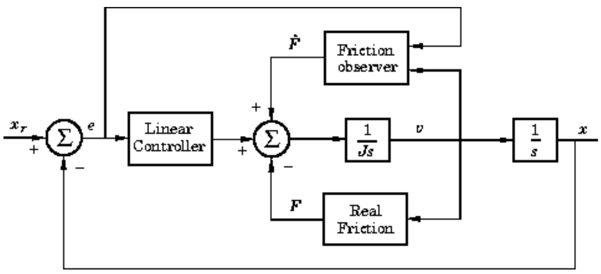
Requirements on system and computations

- System structure
- Velocity measurements and estimates
- Computational requirements

Control design methods

- Passivity based designs

Friction Compensation

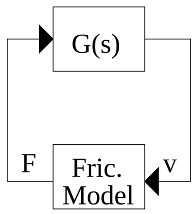


Friction Compensation

$$u = u_{fb} - \hat{F}$$
$$\hat{F} = \sigma_0 \hat{z} + \sigma_1 \frac{d\hat{z}}{dt} + \sigma_2 v$$
$$\frac{d\hat{z}}{dt} = v - \sigma_0 \frac{|v|}{g(v)} \hat{z}$$
$$g(v) = Fc + (F_s - F_c) e^{-v^2/v_s^2}$$

- More general observers
- Velocity measurements
- Little dynamics from u to F

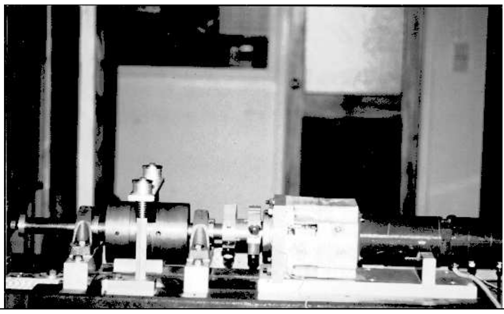
System Structure



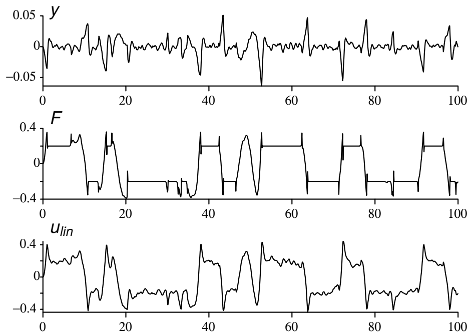
Much theory available

Model Based Friction Compensation

- Laboratory experiments
  - Servo systems
  - Inverted pendulums
- Industrial experiments
  - Hydraulic robots
  - Electric robots
  - Positioning systems



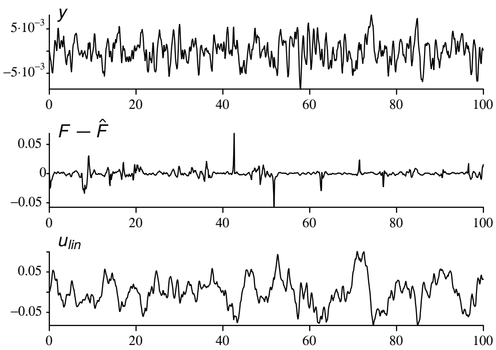
Performance Degradation due to Friction



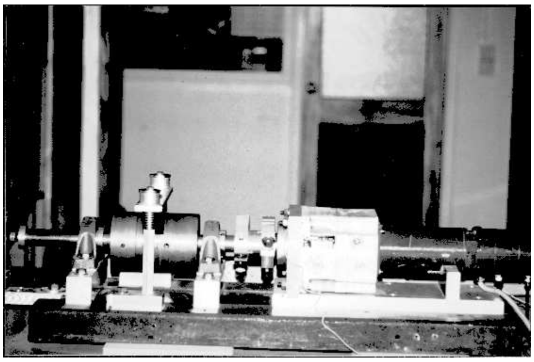
Probability Distributions

- Non-gaussian distributions
- Fat tails
- Measures such as variance is questionable

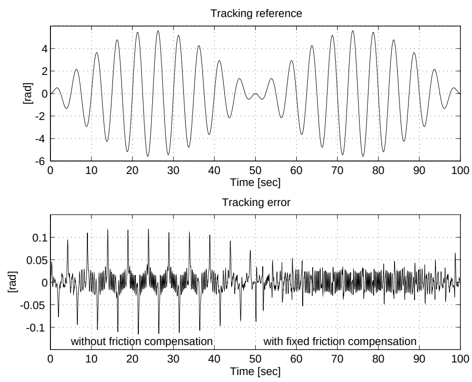
With Friction Compensation



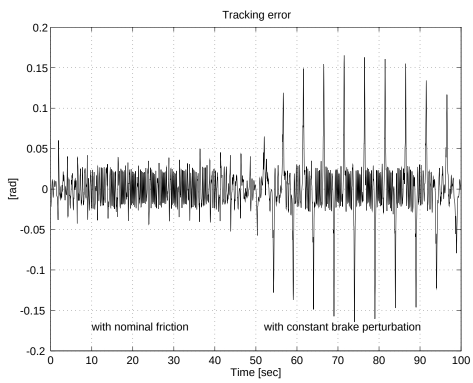
Laboratory Experiment



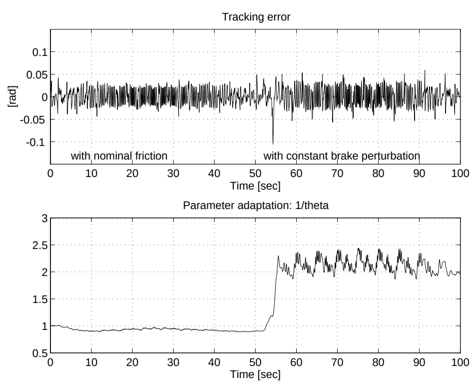
Effect of Friction Compensation



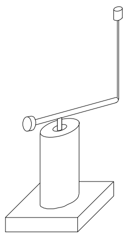
Parameter Sensitivity



Adaptive Friction Compensation

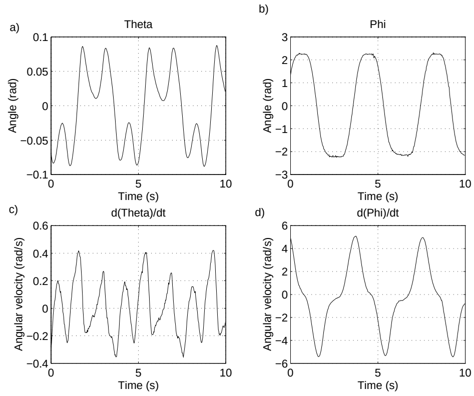


Experiments with the Furuta Pendulum

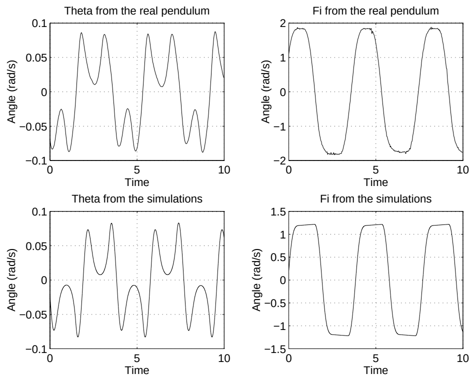


Arm angle φ, pendulum angle θ.

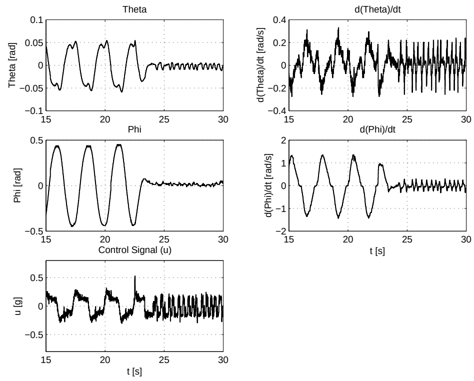
Effect of Friction



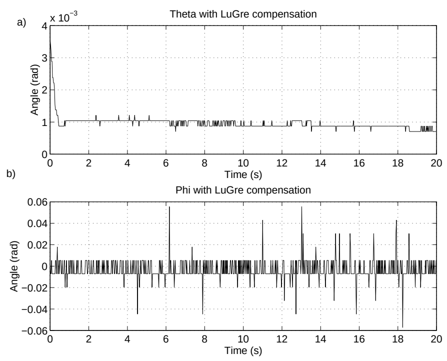
Comparison with Simulations



Experiment with Friction Compensation



Friction Compensation Based on Well Tuned LuGre Model



Friction Models and Friction Compensation

Summary

- A classical field
- Great interest in many disciplines
  - New measurement techniques and new sensors
- Essential in all motion control systems
- Particularly micro-mechanical systems
- Static and dynamic models
  - Dahl, Bliman-Sorine, LuGre
- Friction compensation
  - Model based
  - Accelerometer feedback
  - The need for adaptation
- The need for adaptation and better models

References

J. N. Bhushan, J. N. Israelachvili, and U. Landman. "Nanontribology: Friction, wear and lubrication at the atomic scale." Nature, **374**, pp. 607–616, 1995.

F.P. Bowden and D. Tabor.
The friction and Lubrication of Solids.
Oxford Univ. Press, Part I and II Oxford, 1950 and 1964.

P. Dahl.
A solid friction model.
Technical Report TOR-0158(3107–18)-1, The Aerospace Corporation, El Segundo, CA, 1968.

P.-A. Bliman and M. Sorine.
Friction modelling by hysteresis operators. application to Dahl, sticktion and Stribeck effects.
In *Proceedings of the Conference "Models of Hysteresis", Trento, Italy*, 1991.

References

Nam P. Suh. *Tribophysics*.
Englewood Cliffs, N.J.: Prentice-Hall, 1985.

Ernest Rabinowicz.
Friction and wear of materials.
New York: Wiley, second edition, 1995.

C. Canudas de Wit, H. Olsson, K. J. Åström, and P. Lischinsky.
"A new model for control of systems with friction." IEEE Transactions on Automatic Control, **40:3**, 1995.

Israelachvili, J.N. and A. D. Berman. (1999) Surface Forces and Microrheology of Molecularly Thin Liquid Films. Handbook of Micro/Nano Tribology. pp. 371–432.

Jianping Gaou, et al. (2004) Frictional Forces and Amontons' Law: From the Molecular to the Macroscopic Scle. J. Phys. Chem. B (108) 3410-3425.