Optimal Control Introduction

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Lund University

Optimal Control

K. J. Åström

1. Introduction
2. Calculus of Variations
3. Optimal Control
4. Computations
5. Stochastic Optimal Control
6. Conclusions

Theme: Subspecialities

A Brief History

Early beginning: Bernoulli, Newton, Euler, Lagrange

The Golden Era 1930-39: Department of Mathematics at Univeristy of Chicago

- Refine, polish, streamline - done

Emerging interest in control

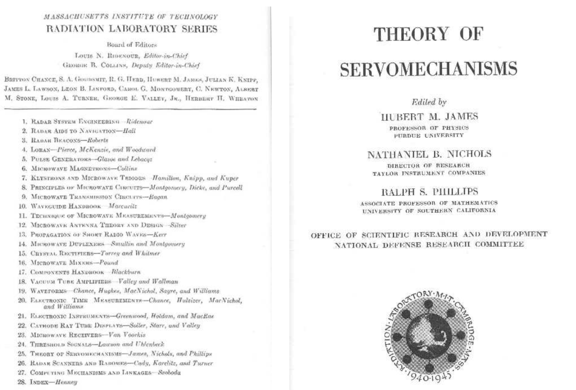
- The classic field calculus of variations became vitalized
- The space race (Sputnik 1957)
- Dynamic programming, Bellman 1957 - key idea recursion
- Pontryagin's maximum principle, 1962 - key idea new perturbation
- LaSalle's bang-bang principle
- Numerical solutions
- Model predictive control

Development often Outside Traditional Academia

Results were often obtain outside traditional academic groups because of commercial and political influences due to the Second World War.

- UK Tizard Mission
- MIT Radiation Laboratory
- MIT Instrumentation Laboratory - Draper
- Rand Corporation
- Swedish Defense Industry
- FOA
- Saab R-System
- Lars Erik Zachrisson
- TTN Group KTH

The Radiation Lab - MIT



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NATIONAL DEFENSE RESEARCH COMMITTEE

RADIATION·LABORATORY·M·I·T·CAMBRIDGE·MASS·1940-1945

The RAND Corporation

Set up as an independent non-profit research organization (Think Tank) for the US Airforce by Douglas Aircraft Corporation in 1945.

- Richard Bellman
- Georg Danzig LP
- Henry Kissinger
- John von Neumann
- Condolezza Rice
- Donald Rumsfeld
- Paul Samuelson

Swedish Defense Industry

- *Alliansfri i fred och neutral i krig - Non-aligned in peace neutral in war*
- Stril 60, JA37 Viggen, (Gripen)
- FOA 1945
  - Chemistry, Physics Electronics, Operations research
  - Bäckebobomben (Boestad, Luthander)
  - TTN Gruppen KTH Bengt Joel Andersson
- Aeronautics KTH Prof Luthander
- The Army, Navy and Air force Procurement Agencies (Arme-, flyg- och marinförvaltningarna)
  - Avionics Bureau
  - Missile Bureau
- Saab
  - Saab R-System
- Bofors - Gun-sights
- Volvo Flygmotor
- The Electronics Industry
  - AGA, Arenco, Ericsson, Philips, TUAB

FOA

Missile guidance

- Thorvald Persson
- Lars Erik Zachrisson
- Syftbäringsprincipen

Inertial navigation

- Philips, AGA, Saab
- MIT Draper
- KJ learns automatic control (Philips)

Analog simulation

- Jonas Agerberg
- SAMS, ADA

Nuclear reactors and weapons Grindsjön

Brodin, Persson and Jahnberg

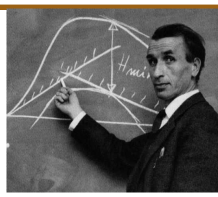
RB 04 early sea missile

Saab R-System

- Airplanes changed from weapon carriers to systems
- R-System formed at Saab 1954, inspired by Rand Corporation
- Hans Olov Palme - aerospace engineer from KTH
  - Enthusiastic, charismatic, visionary leader
- Recruited a fantastic talent pool 75 persons in 1955
  - Strong creativity, broad range and deep knowledge
  - Tore Gullstrand, Bengt Gunnar Magnusson, Gösta Hellgren, Gösta Lindberg, Lars Erik Zachrisson, Viggo Wentzel
- Three groups: Systems, avionics, special projects
- Airborn computers, missile guidance, inertial navigation, simulation, operations analysis
- Electronics industry stated TUAB as a competitor

Lars Erik Zachrisson (Z) 1919-1980

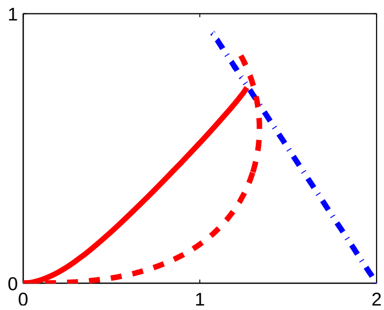
- Engineering Physics KTH 1945
- FOA 1947-57 missile guidance
- Proportional navigation 1946



- Markov Games 1955 (Isaac's 1965)
  - A tank duel with game theoretic implications, 1955, 1957
  - Markov games. In advances of Game Theory. Princeton University Press 1964. Isaacs bok 1965.
- Saab R-system 1957-63
- Docent in Automatic Control KTH 1959
- Professor Optimization and System Theory KTH 1963
- Anders Lindquist 1972 (Z:s first PhD student) intellectual grandfather of Anders Rantzer

Proportional Navigation - Syftbäringsprincipen

How to steer a missile to hit a target?
Constant angle between missile and line-of-sight to target!



TTN Group

- Goal: Understand inertial navigation and guidance

- Structure
  - FFV: Torsten Bergens
  - FOA: Thorvald Persson
  - KTH: Bengt Joel Anderson, Jahnberg, Åslund, KJÅ
  - Aga, Philips, Saab, Grindsjön
- Free-wheeling, chaotic, kulgyrot
- Free access to Besk (The only Swedish Computer)
- The MIT connection - The Instrumentation Laboratory, Draper, Markey
- Fantastic learning experience BUT many constraints (secrecy)

Optimal Control at Lund 1972-2022

- Krister Mårtensson (TD #2 1972)
- Torkel Glad (TD #11, 1976)
- Bengt Pettersson (TL #2, 1970)
- Bo Lincoln (TL #67, 2003)
- Sven Hedlund (TL #68 2003)
- Mattias Grundelius (TD #71 1995)
- Johan Åkesson (TD #81 2007)
- Andreas Wernerud (TD #82 2008)
- Per-Ola Larsson (TD #88 2011)
- Karl Mårtensson (TD #91 2012)
- Pontus Gisselson (TD #94 2012)
- Fredrik Magnusson (TD #115 2016)
- Martin Morin (TD #138 2022)
- Hamed Sadeghi (TD #139 2022)

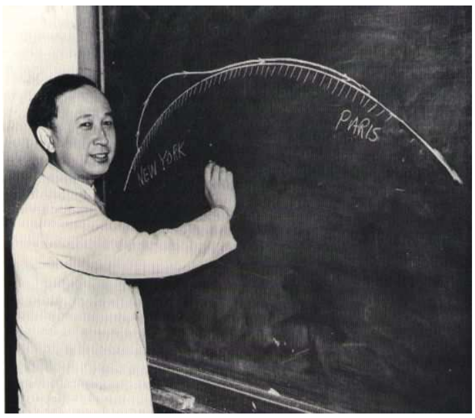
Books - Calculus of Variations

- Oskar Bolza - Lectures on the Calculus of Variations Based on Lectures from 1901 reprinted by Dover1906
- Gilbert A Bliss - Lectures on the Calculus of Variations AMS 1946 - Good symmary of Chicago School
- Marston Morse -The Calculus of Variations in the Large AMS 1934 - Excellent book
- Constantin Carathe'odory - Calculus of Variations and Partial Differential Equations of the First Order 1965- Good Classic
- I. M. Gelfand and S. V. Fomin - Calculus of Variations 1963 - Excellent
- Goldstine, Herman H. . A History of the Calculus of Variations from the 17th through the 19th Century 2012, Springer - Good coverage of history
- Young L C - Calculus of Variations Saunders 1969 - Good classic
- Rawlings J B, Mayne D and Diehl M M - Model Predictive Control: Theory, Computation, and Design 2nd Edition, Nob Hill 2022

Books - Optimal Control

- M. Athans, P. L. Falb 1966 - Optimal Control
- K. J. Åström 1970 Introduction to Stochastic Control Theory
- A. E. Bryson, Y.C. Ho 1979 - Applied Optimal Control: Optimization, Estimation and Control
- D. Liberzon 2012 - Calculus of Variations and Optimal Control Theory - Good mix chosen textbook for the course

H. S. Tsien - Engineering Cybernetics 1954



Tsien - Engineering Cybernetics

- Introduction
- Method of Laplace Transform
- Input Output and Transfer Function
- Feedback Servomechanism
- Noninteracting Control
- AC and Oscillating Control Servos
- Sampling servomechanisms
- Linear Systems with Time Lag
- Linear Systems with Random Inputs
- Relay Servomechanisms
- Nonlinear Systems
- Linear Systems with Variable Coefficients
- Control Design by Perturbation Theory
- Control Design with Specified Criteria
- Optimalizing Control
- Filtering of Noise
- Ultra- and Multi-stability
- Control of Error

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Theme: Subspecialities

Calculus of Variations - The Beginning

- John Bernoulli: The brachistochrone problem 1696
- Let a particle slide along a frictionless curve. Find the curve that takes the particle from A to B in the shortest time.

$$J(y) = \int_A^B \frac{\sqrt{1+y'^2}}{\sqrt{2gy}}dx = \int_A^B L(y, y')dx$$

- Solved by: John and James Bernoulli, Newton, l'Hospital
- Euler: Isoperimetric problems. Example largest area with given circumference.
- Functionals: Curve $\rightarrow R$
- Functional analysis
- Gilbert A. Bliss Lectures on the Calculus of Variations. University of Chicago Press 1946.

Typical Problem - Euler's Equation

Consider

$$J(y) = \int_0^1 L(y, y')dx$$

Find a function y(x) that minimizes $J(y)$.

The first variation

$$\begin{aligned}\delta J &= \int_0^1 \Big(\frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial y'}\delta y'\Big)dx \\ &= \frac{\partial L}{\partial y'}\delta y\Big|_0^1 + \int_0^1 \Big(\frac{\partial L}{\partial y} - \frac{d}{dx}\frac{\partial L}{\partial y'}\Big)\delta y\, dx\end{aligned}$$

Necessary conditions - Euler Lagrange equation:

$$\frac{\partial L}{\partial y} - \frac{d}{dx}\Big(\frac{\partial L}{\partial y'}\Big) = 0 \qquad \frac{\partial L}{\partial y'} = 0 \qquad \text{for } x = 0 \text{ and } x = 1$$

Two point boundary value problem for ODE

A Fundamental Difficulty

Assume that ... *exist* then ...

What if it does not exist!!!

Prove that the largest integer is N=1. ($N^2 \leq N, N \leq 1$)

Conjugate points - Ex shortest distance between north and south pole

Lagrange's mistake

See L. C. Young Lectures on the Calculus of Variations and Optimal Control Theory. W. B. Saunders 1969

Another View - the Hamilton-Jacobi Equation

$$J(y, t) = \int_0^t L(y, y')dt$$

For mechanical systems the function L can be interpreted as the sum of kintetic $T(\dot{x})$ and potential energy $V(x)$ energy.

We have

$$J(y, t) = \int_0^t L(y, y')dt, \qquad \frac{\partial J}{\partial t} = L(y, y') + \frac{\partial J}{\partial y}\frac{dy}{dt}$$

Introduce the Hamiltonian

$$H(y, y', p) = L(y, y') + py' \qquad H^0(y, p) = \min_{y'} H(x, y', p)$$

The function J satisfies the partial differential equation

$$\frac{\partial J}{\partial t} + H^0\Big(y, \frac{\partial J}{\partial y}\Big) = 0, \quad J(y, 0) = 0$$

the *Hamilton-Jacobi equation*, hence initial value problem for a PDE.

Calculus of Variations - Compact notation

Consider

$$J(y, t) = \int_0^t L(y, y')dt$$

Introduce

$$H(y, y', p) = L(y, y') + py' \qquad H^0(y, p) = \min_{y'} H(x, y', p)$$

Euler-Lagrange equation

$$\frac{dy}{dx} = \frac{\partial H^0}{\partial p} \qquad \frac{dp}{dx} = -\frac{\partial H^0}{\partial x}$$

Hamilton-Jacobi equation

$$\frac{\partial J}{\partial t} + H^0\Big(x, \frac{\partial J}{\partial x}\Big) = 0, \quad J(y, 0) = 0$$

Physics

- Formulate natural laws
- Optics - Light passes the shortest way
- Underwater acoustics
- Mechanics
- Strength of materials
- Light rays or waves
- Hamilton and Jacobi
- Special relativity theory
- The Lorenz metric
- The general theory of relativity
- Control

Optics

Law: Light passes between two points in the shortest time.

Assume that velocity is space dependent

$$J(y) = \int_A^B \frac{ds}{v} = \int_A^B \frac{\sqrt{1+y'^2}}{v(x,y)} dx$$

Standard problem with

$$F(x,y,y') = \frac{\sqrt{1+y'^2}}{v(x,y)}$$

Examples

- v constant: straight line
- v piecewise constant: reflection law
- v affine in y: circular paths (submarine hunting)

Work out these cases!

Mechanics

Kinetic energy $T = T(x, \dot{x})$

Potential energy $V = V(x)$

The Lagrange function

$$L(x,\dot{x}) = T(x,\dot{x}) - V(x)$$

Equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

Compare with the Euler equations! Hence the *Euler-Lagrange Equation*.

Example: Mass on a spring.

$$L(x,\dot{x}) = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$$

The equation of motion

$$m\frac{d^2x}{dt^2} + kx = 0$$

Strong Impact on Physics and Engineering

- Natural way to formulate natural laws
- Optics
- Mechanics - Euler-Lagrange Equations
- Strength of materials
- Light rays or waves - Hamilton and Jacobi
- Special relativity theory - The Lorenz metric
- The general theory of relativity
- Optimal control

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Theme: Subspecialities

Optimal Control

$$\frac{dx}{dt} = f(x,u), \qquad \min_u J(u) = G(x(T)) + \int_0^T g(x(t),u(t))dt$$

Hamiltonian

$$H(x,p,u) = g(x,u) + p^T f(x,u), \qquad H^0(x,p) = \min_u H(x,p,u)$$

Euler-Lagrange-Pontryagin (particle view)

$$\frac{dx}{dt} = \frac{\partial H^0}{\partial p}, \qquad x(0) = a$$

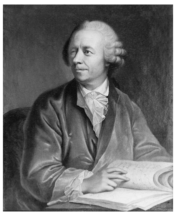
$$\frac{dp}{dt} = -\frac{\partial H^0}{\partial p}, \qquad p(T) = G'(x)$$

Hamilton-Jacobi-Bellman (wave view)

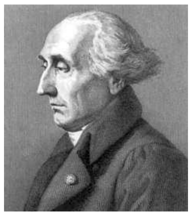
$$\frac{\partial J}{\partial t} + H^0\Big(x, \frac{\partial J}{\partial x}\Big), \qquad J(x,T) = G(x)$$

Three Giants

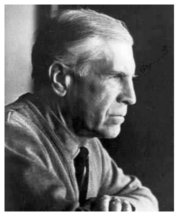
Leonhard Euler 1707-1783



Joseph-LouisLagrange 1706-1813



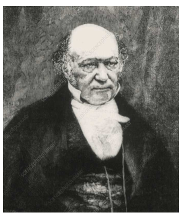
Lev Pontryagin 1908-1988



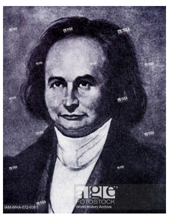
Particle View - ODE

Three More

William Rowan Hamilton 1805-1865



Karl Gustav Jacobi 1804-1851



Richard Bellman 1920-1984

Wave and energy views - PDE

Optimal Control and Calculus of Variations

- Very similar
- Different variations (norms)
- Classical: free choice of (optimization variable) u
- Optimal control: choice of u restricted $\frac{dx}{dt} = f(x,u)$
- No reason to deal with the special case $f(x,u) = u$
- Optimal control is a very natural formulation
- Numerical computing

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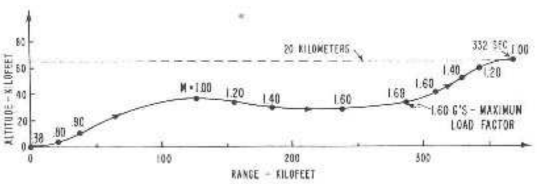
Theme: Subspecialities

Numerical Calculations

- Two point boundary value problem for ordinary differential equation or initial value problem for partial differential equation
- Shooting
- Special iteration for minimum time problems
- Iteration in the space of control signals
- First and second variations
- Parameterize the control signal
- Collocation methods

Brysons Flight Test

Bryson (Professor at Harvard) made major computations and a flight test: How to reach a given height in minimum time?



Flew higher than ever before!
Reached the height twice as fast as with standard flight procedure!

Computations

Efficient computations of optimal control is a subspeciality of its own. Pontus at our department is a a very good researcher, he should give a guest lecture!

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Theme: Subspecialities

The Problem

Consider a system governed by the stochastic differential equation (SDE)

$$dx = f(x, u, t)dt + \sigma(x, u, t)dwh$$

Find a control law such that $u(t)$ is a function of $\mathcal{X}_y = \{x(\tau), 0 \le \tau \le t\}$, which minimizes the criterion

$$J(x, t) = \min_u E\Big(\int_0^{t_f} g(x(\tau), u(\tau), \tau)d\tau + G(x(t_f), t_f)\Big)$$

when the admissible controls are such that $u(t)$ is a function of $x(\tau), 0 \le \tau \le t$. Since the process x is a Markov process It follows that the optimal control is simply a function $u(t) = f(x(t))$, older values of x do not help!

Mean Value of Quadratic Form

Let x be normal $N(m, R)$ then

$$Ex^T Sx = m^T Sm + \operatorname{tr} SR$$

We have

$$\begin{aligned} Ex^T Sx &= E(x-m)^T S(x-m) + Em^T Sx + Ex^T Sm - Em^T Sm \\ &= E\operatorname{tr}(x-m)^T S(x-m) + m^T Sm \\ &= E\operatorname{tr} S(x-m)(x-m)^T + m^T Sm \\ &= \operatorname{tr} SE(x-m)(x-m)^T + m^T Sm \\ &= \operatorname{tr} SR + m^T Sm \end{aligned}$$

Dynamic Programming 1

Consider a system governed by the SDE

$$dx = f(x, u, t)dt + \sigma(x, u, t)dw$$

Introduce the cost to go

$$J(x, t) = \min_u E\Big(\int_t^{t_f} g(x, u, \tau)d\tau \Big| x(t) = x\Big)$$

and assume that it is sufficiently smooth. Bellman's principle of optimality becomes

$$\begin{aligned} J(x, t) &= \min_u E\bigg(\int_t^{t+h} g(x(\tau), u, \tau)d\tau + \int_{t+h}^{t_f} g(x(\tau), u(\tau), \tau)d\tau \Big| x(t) = x\bigg) \\ &= \min_{u(t,t+h)} E\bigg(\int_t^{t+h} g(x(\tau), u(\tau), \tau)d\tau + J\big(x(t+h), t+h\big)\big| x(t) = x\bigg) \end{aligned}$$

Dynamic Programming 2

We have

$$J(x,t) = \min_u \Big(E\int_t^{t+h} g(x(\tau),u(\tau),\tau)d\tau + V(x(t+h),t+h) \Big| x(t)=x \Big)$$

A series expansion of the right hand side gives (dw has magnitude $\sqrt{dt}$)

$$\min_u \Big(gdt + J_x^T dx + \frac{1}{2} E dx^T J_{xx} dx + V_t dt + O(dt^{3/2}) \Big) = 0$$

We have

$$\begin{aligned} E dx^T J_{xx} dx &= E\mathrm{tr}\, dx^T J_{xx} dx = E\mathrm{tr}\, J_{xx} dx dx^T \\ &= \mathrm{tr}\, J_{xx} E(fdt + \sigma dw)(fdt+\sigma dw)^T \\ &= \mathrm{tr}\, J_{xx}\sigma\sigma^T dt + o(dt) \end{aligned}$$

pHence

$$\min_u \Big(gdt + J_x^T fdt + \frac{1}{2}\mathrm{tr}\, J_{xx}\sigma\sigma^T dt + J_t dt + o(dt) \Big) = 0$$

Dynamic Programming 3

Summarizing we have

$$0 = \min_u \Big(gdt + V_x^T fdt + \frac{1}{2}\mathrm{tr}\, V_{xx}\sigma\sigma^T dt + V_t dt + o(dt) \Big)$$

Neglecting terms smaller than dt and dividing by dt gives

$$V_t + \min_u \Big(V_x^T f(x,u,t) + \frac{1}{2}\mathrm{tr}\, V_{xx}\sigma(x,u,t)\sigma^T(x,u,t) + g(x,u,t) \Big) = 0$$

which is called the Hamilton-Jacobi-Bellman equation. The boundary condition is

$$V(x,t_f) = G(x)$$

Compact Notation

Hamiltonian

$$H(x,p,Q,u) = g(x,u) + p^T f(x,u,t) + \frac{1}{2}\mathrm{tr}\, Q\sigma(x,u,t)\sigma^T(x,u,t)$$

Minimal Hamiltonian

$$H_0(x,p,Q) = min_u H(x,p,Q.u)$$

Hamilton-Jacobi-Bellman equation

$$\frac{\partial V}{\partial t} + H_0(x,p,Q) = 0, \quad V_{t_f} = 0$$

Comparison Deterministic and Stochastic

Deterministic

$$\begin{aligned} &\frac{dx}{dt} = f(x,u,t) \\ &V(x,t) = \min \Big(\int_t^{t_f} g(x(\tau),u(\tau),\tau)d\tau + G(x(t_f),t_f) \Big) \\ &V_t + \min_u \Big(V_x^T f(x,u,t) + g(x,u,t) \Big) = 0 \end{aligned}$$

Stochastic

$$\begin{aligned} &dx = f(x,t)dt + \sigma(x,t)dw \\ &V(x,t) = \min E\Big(\int_t^{t_f} g(x(\tau),\tau)d\tau + G(x(t_f),t_f) \Big| x(t)=x \Big) \\ &V_t + \min_u \Big(V_x^T f(x,u,,t) + \frac{1}{2}\mathrm{tr}\, V_{xx}\sigma(x,u,t)\sigma^T(x,u,t) + g(x,u,t) \Big) = 0 \end{aligned}$$

One extra term appears in the Hamilton-Jacobi-Bellman equation in the stochastic case because of $dt \sim \sqrt{dt}$

Some Interesting Problems

- Bryson's record flight
- Satellite launch - enabling space exploration
- The container problem by Krister Mårtensson. How to move a hanging load, much work done later in Luleå.
- Mill wide control - Control of paper production by Bengt Pettersson. Interesting to formulate criteria that gives few changes of control variables.
- Tetrapak - how to fill milkboxes quickly. A method that utilize tilting of the container is developed. It enables faster movements with less slosh. The methods simultaneously calculates the horizontal and rotational acceleration references by solving a minimum energy optimal control problem. Experiments show that the method is successful if the maximum allowed surface elevation is not too large.

The Sloshing Problem

Grundelius, Mattias LU (2001) PhD Thesis TFRT-1062

The work presented is focused on development of systematic methods for calculation of acceleration references that move the container as fast as possible without too much slosh. The methods are based on a simple model of the slosh phenomenon which is derived from fluid dynamics and system identification. The acceleration reference is calculated both directly using optimal control techniques with various cost functions and constraints and iteratively using iterative learning control. To enable practical evaluation of the acceleration references and the use of iterative learning control an experimental setup has been used where it is possible to measure the surface elevation on one side of the container using an infrared laser displacement sensor. The experimental evaluations show that it is possible to achieve fast movements by solving a minimum energy optimal control problem and tuning of the model parameters. Iterative learning control methods are successful in finding good acceleration references in practice using only a simple model of the slosh phenomenon.

PRODUCTION CONTROL OF A PULP AND PAPER MILL

Bengt Pettersson lic-avhandling, report 7007 sept 1970. A mathematical model of the mill is developed in the form of an ODE with 10 states and 9 control variables. The scheduling problem is formulated as a deterministic optimization problem. A solution based on the Pontryagin maximum principle was developed, leading to a boundary value problem which is reformulated as a linear programming problem with 50 rows and 40 columns. The numerical algorithm developed is feasible to run on a process computer. The off-line execution time is about 20 minutes on an IBM 1800. The production control system was irnplemented at the Gruvon paper mill in November 1969. It has been in continuous operation since then. Experience from the first six months of operation is described.

Optimal Control

K. J. Åström

1. Introduction
2. Calculus of Variations
3. Optimal Control
4. Computations
5. Stochastic Optimal Control
6. Conclusions

Theme: Subspecialities

Conclusions

- Nice theory with many connections
- Elegant formalism in terms of the Hamiltonian function
- Two approaches
  - Euler-Lagrange-Pontryagin - ODE
  - Hamilton-Jacobi-Bellman - PDE
- No reason to study classical calculus of variations and optimal control separately
- Nice software available
- Computational power is available
- Model Predictive Control - a recent addition