Reading instructions and problem set 7

Feedback linearization, zero-dynamics, Lyapunov re-design, backtepping

Reading assignment

Khalil [3rd ed.] Ch 13. Khalil [3rd ed.] Ch.14.(1) 2-4 + "The joy of feedback" by P. Kokotović (handout)

(Extra reading:

- “Constructive Nonlinear Control” by R. Sepulchre *et al*, Springer, 1997)
- “Nonlinear & Adaptive Control Design” by M. Krstić *et al*, Wiley, (1995)

Comments on the text

Chapter 13 in Khalil covers the different concepts of feedback linearization (Input-Output linearization, full-state linearization etc), making use of the notation of Lie derivatives from previous lecture. Chapter 14 is devoted to nonlinear design methods. Chapter 14.1 covers sliding mode in more details than what was covered in FRTN05. We haven’t covered it in the lectures and you may just browse this section. Chapter 14.2 deals with the concept of Lyapunov redesign which is useful when you already know a stabilizing control law (and a Lyapunov function!!) for a nominal system. You use the redesign to cope with "matching disturbances", i.e., disturbances appearing at the same place as you control signal. The nonlinear damping in 14.2.2 is often instrumental in integrator backstepping. In addition to Ch. 14.3 I suggest that you also read the paper “The joy of Feedback” by Petar Kokotovic (Bode prize lecturer 1991). The passivity-based control presented in Ch. 14.4 makes use of the properties from Chapter 6.

Exercise 7.1 Show that the system is feedback linearizable and design a state-feedback controller to globally stabilize the origin

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 \\
\dot{x}_2 &= x_1 - x_2 - x_1 x_3 + u \\
\dot{x}_3 &= x_1 + x_1 x_2 - 2x_3
\end{aligned}
$$

Exercise 7.2 Sontag’s formula (see lecture notes):
Consider the system

$$\dot{x} = x^2 + u$$

For scalar, linearizable systems,

$$V_S(x) = \frac{1}{2}x^2$$

is always a *Control Lyapunov Function* (CLF).
Derive a stabilizing controller using Sontag's formula based on V_S.

Remark: Compare your result with the optimal controller-example in the lecture handouts (p.8, Lec 7). (Note that the functions V_S and V (from the lecture slides) are different with different interpretations.)

Exercise 7.3 Consider the systems

$$\dot{x}_1 = x_1 + x_2/(1 + x_1^2)$$
$$\dot{x}_2 = -x_2 + u$$

Design a globally stabilizing feedback controller for the origin using

(a) feedback linearization.

(b) backstepping

(c) Compare the two controllers from (a) and (b) in a simulation study

Exercise 7.4 [Khalil 14.40] Consider the system

$$\dot{x}_1 = -x_1 + x_1^2(x_2 + \delta(t))$$
$$\dot{x}_2 = u$$

where $\delta(t)$ is a bounded function of t, for all $t \geq 0$, but we do not know an upper bound of $|\delta(t)|$. Use backstepping and nonliner damping to design a state feedback controller that ensures global boundedness of the state x for all initial contitions $x(0) \in R^2$