Session 4

Hybrid systems

Reading assignment

Check the main results and examples of these papers.

- Johansson/Rantzer, IEEE TAC, 43:4 (1998).
- Chizeck/Willsky/Castanon, Int. J. on Control, 43:1 (1986)

Exercise 4.1 Consider two pendula

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ 1 - x_1 \end{bmatrix} \qquad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -1 - x_1 \end{bmatrix}$$

which are swinging around $x_1 = 1$ and $x_1 = -1$ respectively.
a. Find a control law that brings the state to $(0, 0)$ by jumping between the two swings. In other words, define $\mu : \mathbf{R}^2 \to \{-1, 1\}$ so that regardless of the initial state $x(0)$, the hybrid system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \mu(x_1, x_2) - x_1 \end{bmatrix}$$

gives $x(T) = 0$ for some $T > 0$. Try to do it with a minimal number of jumps.
b. Consider the same problem as in **a**, but with a control law $\mu(x_1, x_2)$ that can take any value in the interval $[-1, 1]$. (This can be approximately achieved by jumping fast between the swings.) Find a piecewise linear control law that stabilizes the system.

Exercise 4.2
The system

$$\dot{x}_1(t) = -0.1x_1(t) - u_1(t) + w_1(t)$$
$$\vdots$$
$$\dot{x}_9(t) = -0.9x_9(t) - u_9(t) + w_9(t)$$

can be used to model nine queues, where $w_i(t)$ represent new arrivals, $u_i(t)$ are served customers and the first terms on the right hand side mean agents that leave the queue without being served. Assume that $\sum_i u_i(t) \leq 1$ and all terms except one is equal to zero at any given time. Suggest a piecewise linear control law aimed to reduce the number agents that leave the queues without being served.

Exercise 4.3
Consider the discrete time system

$$x(t + 1) = 2x(t) + u(t - T(t)) + w(t)$$

a. Let $w(t) \equiv 0$. The control law $u(t) = -2x(t)$ is obviously stabilizing if $T(t) = 0$ for all t, but not if $T(t)$ is constantly equal to 1. However, suppose that the network delay $T(t)$ is jumping between the two values with jump probabilities

$$\begin{bmatrix} q_{00} & q_{01} \\ q_{10} & q_{11} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}$$

i.e. if $T = 0$ at one time instance, then with 90% probability this is true also one time step later. Is the jump linear system stable?

b. Let w be zero mean white noise with unit variance. With jump probabilities as in **a**, find a control law that minimizes the variance of x.