Session 1 — Readings and exercises

limit cycles, existence / uniqueness, Lyapunov, regions of attraction

Reading assignment

Khalil Chapter 1–3.1, (not 2.7), 4–4.6

Comments on chapter 2.6 The main topic is about existance of periodic orbits for planar systems and the most important subjects are the Poincaré-Bendixson Criterion and the Bendixson Criterion. Lemma 2.3 and Corollary 2.1 can also be used to rule out the existence of limit cycles.

Comments on chapter 3.1 The topics in Chapter 3.1 concerns (local) existence and uniqueness of solutions to differential equations, where the Lipschitz condition plays a major role.

Comments on chapter 4.1–4.4 This chapter is devoted to the study of equilibrium points of nonlinear autonomous systems. The main issues are the following.

- The use of Lyapunov functions and invariant sets for proofs of asymptotic stability (LaSalle's theorem). Consider in particular its application to the pendulum, Example 4.4.
- Lyapunov functions for proof of instability (Chetaev's theorem).
- Stability analysis by linearization.

Exercises on Chapters 2, 3.1, & 4

Exercise 1.1 = Kha. 2.20 (3,5)

Exercise 1.2 Kha 3.1 (1)

Exercise 1.3 = Kha. 3.2 (4)

Exercise 1.4 **(a)** = Kha. 4.8 (Radial boundedness)
(b) What is the region of attraction for the origin in **(a)**?
You may use simulation tools like e.g., *pplane* (see
`http://math.rice.edu/~dfield/`)

Exercise 1.5 = Kha 4.10 (Krasovskii's method). Under the same assumptions, prove also that $[x(t) - y(t)]^T P[x(t) - y(t)]$ decays exponentially whenever $\dot{x} = f(x)$ and $\dot{y} = f(y)$.

Exercise 1.6 = Kha. 4.19 (Robot manipulator)