Professor Stodola's Contribution to Control Theory

BY P. PROFOS[1]

Summary

After giving a brief survey on the development of control theory in the 19th century, Stodola's direct contributions to mathematical treatment of control problems are shown and his role as a stimulator of Hurwitz's work on stability is exhibited.

Former Theoretical Work. Whenever one talks about the rudiments of control theory the name Stodola crops up. What did this Slovakian engineer and later professor at the Federal Institute of Technology, Zurich really contribute to the development of control theory and control techniques?

His achievements can of course only be adequately appreciated in the light of work performed by other pioneers of control theory before his time. Therefore the initial developments of this theory will first be outlined with the help of some particularly significant dates.

As in many other fields of technology the constructive developments preceded the theoretical ones; only the advent of difficulties normally gave rise to theoretical examinations. The various types of controllers (proportional action controller, integral controller, proportional-integral controller, proportional-derivative controller, etc.) had already been invented in the early ages of control engineering (before 1900) by practitioners without any theoretical basis. Later on it was the theorists' task to clarify the fundamental characteristics of these controllers and of the automatic control systems thereby devised.

Jean Poncelet made a substantial contribution to this problem when in 1830 he mathematically investigated the steady state behaviour of a fly ball governor. The first well known theoretical investigation of its dynamic behavior was published by *George B. Airy* in 1840. He described the system examined (fly ball governor with hydraulic damping) by means of a linear differential equation of second order, for which he also gave the solutions. In 1865 *Linders* tried to mathematically describe a whole speed control system—he could however not solve the equations he had derived. Nevertheless he was able to make interesting qualitative statements.

In 1868 *James Maxwell* made a fundamental contribution by describing a control loop by means of a linear differential equation of third order and by giving the solutions to the equation. Maxwell was probably the first who completely understood the connection between the real control system and its mathematical description. Furthermore he recognized that a precise description of more complex systems would lead to differential equations of higher order and he realized that difficulties would arise in the assessment of stability. He pointed out that for stability all the characteristic roots had to have negative real parts. Consequently, he posed for the first time the *problem* of interpreting the stability conditions in terms of coefficients of the differential equation.

Wischenegradsky's (1872) study, which also dealt with the linear differential equation system of third order, was of almost equal importance as Maxwell's publication which was hardly known in German speaking countries. Apart from his various solutions, Wischnegradsky was the first to give mathematical stability criteria in the form of coefficient-inequalities, which enabled a quick decision on the usefulness of a set of coefficients.

The English mathematician *Edward J. Routh* succeeded in finding the first general solution to the problem of stability. Based on the studies of Airy and Maxwell, in 1877 he disclosed an algorithm, by which the question of stability could be settled on the basis of the coefficients of the differential equation, independent of its order. Routh based his study, among other things, on the work done by the French mathematicians A. L. Cauchy (1831) and C. Sturm (1836).

In 1879 *Lincke*, on the occasion of his investigations of the dynamic behavior of a hydraulic servo-motor with mechanical feedback, made the first indications of analogies between the human muscular and nervous system on the one hand and automatic control systems on the other.

Stodola's Contributions. It was at this point in the history of control theory that the 33 year old engineer *Aurel Boleslaw Stodola* was appointed professor at the Federal Institute of Technology in Zurich (1892). This appointment came after his having studied in Budapest, Zurich, Charlottenburg and Paris, and after having approximately 10 years experience in industry. Stodola was in charge of thermal power engines in his sector, but at first he directed his attention to problems of control theory.

A first publication in 1893 "Ueber die Regulierung von Turbinen" [1][2] ("About the Regulation of Turbines") deals with the dynamics of the regulation of a high-pressure-water turbine taking into consideration the moving water masses in the penstock and the effect of an air vessel. The controller is assumed to be without inertia, the servo-motor to be operating instantaneously. The mathematical description of this system leads to a linear differential equation system of third order. Stodola gives the solutions to these equations and by making use of Wischnegradsky's stability criteria, he also gives stability conditions.

Compared to previous studies in this field, the progress achieved consists mainly in the fact that Stodola substantially simplified and reduced the mathematical presentation, thus making it clearer and easier to grasp. He achieved this by examining only the deviations of the variable magnitudes from their steady-state values, and by defining dimensionless variables while standardizing these deviations with technically meaningful reference magnitudes. Furthermore his suggestion to reduce the coefficients of his differential equations to so-called time constants[3] has proved to be most fruitful; a physical interpretation of these time constants can be given in relation to the operation of the control system elements.

Owing to these first merely formal but very important measures, Stodola succeeded in substantially simplifying the equations describing the dynamic behavior of the system. At the same time relations of general validity arose, as they were completely detached from the technical means of realization of the system described. They consisted merely of statements which are of importance to the dynamic behavior of the system. Thus, the conditions for the transferability of the equations and their solutions to other similar cases were given.

In this manner Stodola laid the foundation for a general control theory as defined by the "classical" control theory. Of great importance to the practical significance of this theory was the fact that Stodola's observations, despite thier abstract nature, always remained physically perceptible and transparent. In this way he not only made control theory accessible to mathematicians but particularly also to engineers, by providing them with an elegant theoretical tool which up to this day has only been supplemented or completed but never been replaced.

The far-reaching significance of these formal simplifications is demonstrated in the publication Stodola made in 1894 "Zur Frage der Regulierung hydraulischer Motoren" [2] ("The Question of the Regulation of Hydraulic Motors"). Stodola no longer maintains at this point certain simplifying assumptions which he had made earlier, but considers now the inertia and the

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[1] Eidenössische Technische Hochschule Zürich, Institut für Mess-und Regeltechnik, Sonneggstrasse, Switzerland.

[2] Numbers in brackets designate References at end of paper.

[3] Stodola was probably stimulated by his former master Helmholtz. It is said that Helmholtz was the first who operated with "time constants."

hydraulic damping of the controller as well as the delayed actions of the servo-motor. Owing to the simplified manner of presentation of the thusly created complex differential equation system of seventh order, Stodola was able to make some important qualitative statements even though he could not solve the equation by the mathematical means of the time. Independent of the studies of Maxwell and Routh (which he undoubtedly did not know at the time) Stodola presumed, however, that at least the question of stability, which had been considered most important, could probably be answered without the complete solution of the differential equation system. He consequently presented this "Maxwell-Stodola-problem" to the mathematician *A. Hurwitz*. It is characteristic of the parallelism of this intellectual development that Hurwitz also did not know the corresponding English publications and based his work on the studies of the French mathematician *Hermite* (1854). The solution to this problem was published by Hurwitz in 1895 in his well-known essay "Ueber die Bedingungen, unter welchen eine Gleichung nur Wurzeln mit negativen reellen Teilen besitzt" ("About the Conditions on Which an Equation Only Has Roots With Negative Real Parts"). Stodola, with Hurwitz's permission, evaluated them in his above mentioned essay.

Apart from his studies of the so-called inertia-controller (1899), Stodola's subsequent studies in the field of automatic control [3, 4, 5, 6, 7] were more in the range of application and did not bring new findings of fundamental importance. But Stodola was probably the first to realize that this type of controller not only forms and uses the control error x but also its temporal deviation dx/dt. He was also aware of the additional stabilizing effect of derivative action and of the possibility of reducing offset.

At the beginning of the 20th century *Tolle* compiled a good part of the existing findings of control theory and described them in great detail in his book *Regelung der Kraftmaschinen* (*Control of Power Machines*). As far as the formal treatment is concerned, Tolle made use of Stodola's methods, which largely contributed to the fact that this book (with two further editions) was considered as one of the most important textbooks in the field for over two decades.

So, Stodola was not the first to examine the mathematical side of control actions. With his studies, however, he took the final and decisive plunge from a complex, unmethodical calculation to a formally elegant, relatively simple and generally applicable calculation method. Furthermore by the dimensionless representation of signals he has, although not literally, but nevertheless unmistakably drawn our attention to the importance of control signals as information carriers.

List of Stodola's Publications on the Subject

1 Stodola, A. B., "Ueber die Regulierung von Turbinen," *Schweiz. Bauzeitung*, 1883, pp. 113; 121; 126.

2 Stodola, A. B., "Zur Frage der Regulierung hydraulischer Motoren," *Schweiz. Bauzeitung*, 1894, pp. 71, 80, 89.

3 Stodola, A. B., "Die amerikanischen Inertie-Regulatoren," *Schweiz. Bauzeitung*, 1899, p. 178.

4 Stodola, A. B., "Das Siemenssche Regulierprinzip und die amerikanischen Inertie-Regulatoren," *Zeitschrift des VDI*, 1899, pp. 506, 573.

5 Stodola, A. B., "Die neue hydraulische Regelung der Sulzer-Dampfmaschine und Versuche an der 2000 KW-Turbine des Basler Elektrizitätswerkes," 1911, pp. 1709, 1794, 1846.

6 Stodola, A. B., "Regulierversuche am Einrohr-Dampferzeuger der Gebruder Sulzer AG, Winterthur," *Schweiz. Bauzeitung*, 1934, p. 6.

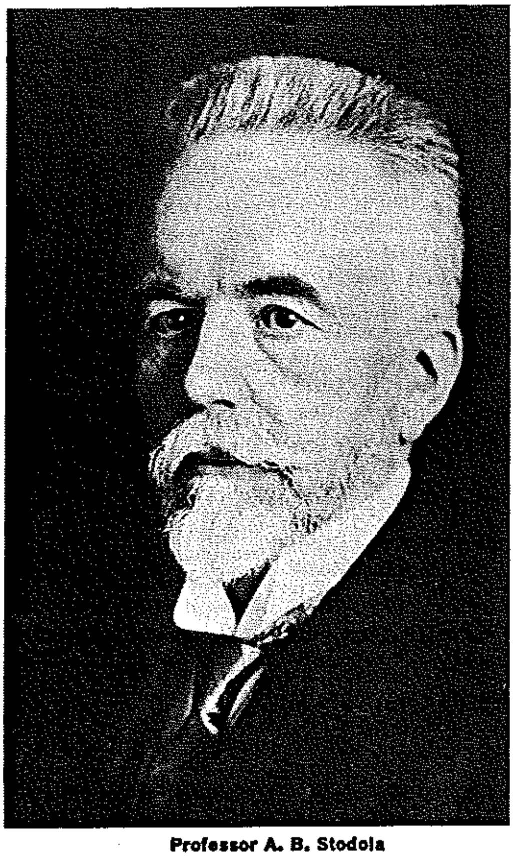
7 Stodola, A. B., "Leistungs- und Regelversuche an einem Velox-Dampferzeuger," *Zeitschrift des VDI*, 1935, p. 429.

Historical Summary-Representation

Röhrentrop, K.k *Entwickling der modernen Regelungstheorie*, Verlag Oldenburg, 1971.

Zur Megede, W., *Am Wege zur Automation - Antiker Traum - moderne Wirklichkeit*, Verl. Siemens AG, 1974.

Fuller, A. T., ed., *Stability of Motion*, Taylor and Francis Ltd., London, 1975.



Professor A. B. Stodola