# Control System Synthesis - PhD Class

Exercise session 1

24/09/2020

## 1 The X-29 aircraft

The X-29 aircraft has an unusual configuration, designed to enhance its maneuverability. It has a right half-plane pole at approximately $p = 6rad/s$ and a right half-plane zero at $z = 26rad/s$. The non-minimum phase factor then writes:

$$P_{nmp}(s) = \frac{z-s}{z+s}\frac{s+p}{s-p}.$$

**What are the fundamental limitations ?**

1. Apply the Maximum Modulus Principle to find a lower bound on $M_s = \max_{\omega>0} |S(\jmath\omega)|$ (consider $B_pS$ with $B_p(s) = \frac{s+p}{s-p}$)

2. Based on the Nyquist plot, prove that:

$$g_m \geq \frac{M_s}{M_s - 1}$$

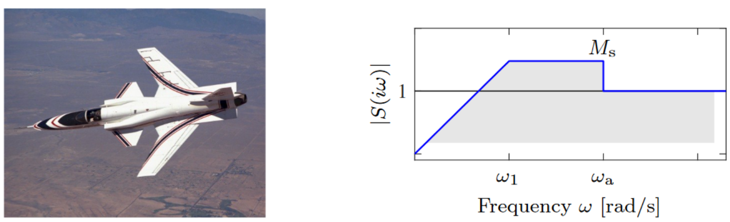
$$\varphi_m \geq 2arcsin\frac{1}{2M_s}$$

3. Apply gain crossover frequency inequality.

4. Assume that the sensitivity function is given by (see Figure 1):

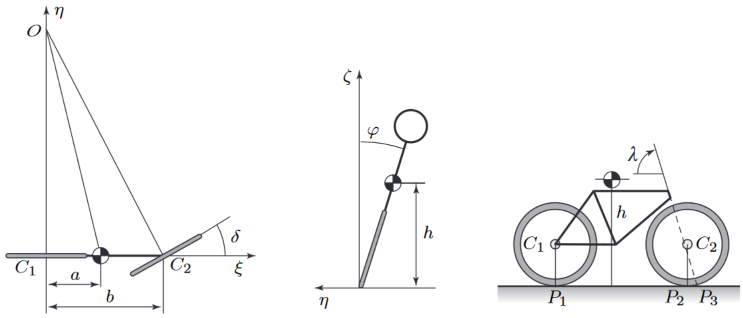
$$|S(\jmath\omega)| = \begin{cases} \frac{\omega}{\omega_1}M_s & if\ \omega_1 \leq \omega \\ M_s & if\ \omega_1 \leq \omega < \omega_a \\ 1 & if\ \omega_a \leq \omega \end{cases}.$$

Calculate $M_s$. Numerical application for $\omega_1 = 3rad/s$ and $\omega_a = 40rad/s$




(a) X-29 aircraft (b) Sensitivity analysis

Figure 1: Specifications




(a) Top view (b) Rear view (c) Side view

Figure 2: Schematic view of a bicycle.

## 2 Bicycle dynamics with rear-wheel steering

We assume that the bicycle rolls on the horizontal plane. A schematic view is given in Figure 2.

Let $v_0$ be the velocity of the bicycle at the rear wheel,$b$ the wheelbase,$\varphi$ the tilt angle, $\delta$ the steering angle and $m$ be the total mass of the system. The coordinate system rotates around the point $O$ with the angular velocity $\omega = v_0\delta/b$. Assuming that the steering angleis small, the equation of motion is:

$$J\frac{d\varphi}{dt} - \frac{Dv_0}{b}\frac{d\delta}{dt} = mgh\sin\varphi + \frac{mv_0^2 h}{b}\delta \tag{1}$$

where $J \sim mh^2$ is the moment of inertia with respect to the $\xi$-axis, and $D = mah$ the product of inertia with respect to the $\xi\zeta$ axes.

**Considering (1), write the associated transfer function and express the limitations on the stability margin $M_s$.**

(a) Unrideable bicycle (b) Rideable bicycle

## 3 Cruise control

Let's consider the cruise control problem.

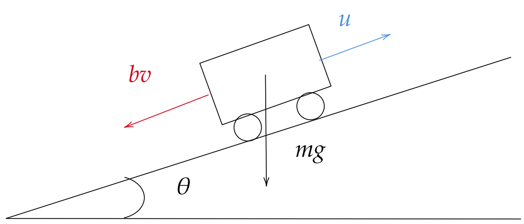



Figure 3: Physical setup

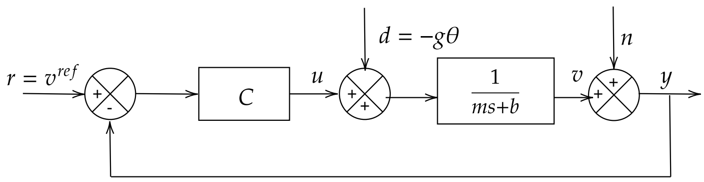



Figure 4: Schematic view of the considered control loop.

1. Consider a PID controller $C(s) = k_p + \frac{k_i}{s} + k_d s$. Write the transfer function between the disturbance $d$ and the output $v$.

2. What are the effects of $k_p$, $k_i$ and $k_d$ on rise time, overshoot, settling time and steady state error regarding disturbance rejection? Remember that for a second-order system $\frac{k}{\frac{1}{\omega_0^2}s^2+\frac{2\xi}{\omega_0}s+1}$, we have

| Property | Value |
|---|---|
| Steady-state value | $k$ |
| Rise time | $T_r = e^{\phi/tan\phi}$ with $\phi = arccos\xi$ |
| Overshoot | $D = e^{-\pi\xi\sqrt{1-\xi^2}}$ |
| Settling time 2% | $T_s \sim \frac{1}{4\xi\omega_0}$ |

3. Design a PID controller such that the response to a step disturbance satisfies
   - Rise time $T_r < 5s$
   - Overshoot $D < 10\%$
   - Steady-state error $e_{ss} < 2\%$

4. Simulate the results in Matlab for $m = 1000kg$ and $b = 50N.s/m$.