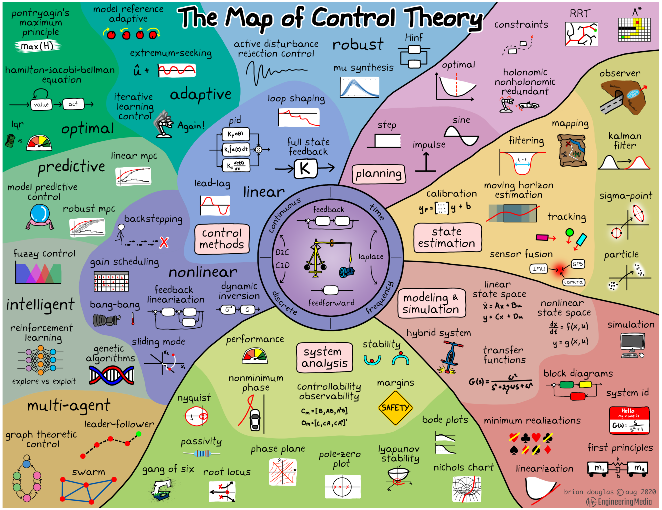
LUND UNIVERSITY

Control System Synthesis - Basics

PHD CLASS - FALL 2020

1. Introduction
2. Fundamentals: problem formulation
   - System representation and feedback basics
   - Specifications and performance limitations
3. Design techniques


The Map of Control Theory
pontryagin's maximum principle
max(H)
model reference adaptive
extremum-seeking
û +
hamilton-jacobi-bellman equation
value
act
iterative learning control
adaptive
Again!
lqr
vs.
optimal
predictive
linear mpc
model predictive control
robust mpc
active disturbance rejection control
robust
Hinf
mu synthesis
loop shaping
pid
$K_p e(t)$
$K_i \int e(t)\,dt$
$K_d \frac{de(t)}{dt}$
full state feedback
K
lead-lag
linear
backstepping
control methods
fuzzy control
gain scheduling
nonlinear
intelligent
feedback linearization
dynamic inversion
G^{-1}
G
bang-bang
reinforcement learning
explore vs exploit
genetic algorithms
sliding mode
multi-agent
leader-follower
graph theoretic control
swarm
constraints
RRT
A*
optimal
holonomic
nonholonomic
redundant
observer
step
sine
impulse
planning
filtering
mapping
kalman filter
calibration
$y_r = [\,]\,y + b$
moving horizon estimation
state estimation
tracking
sigma-point
sensor fusion
IMU
GPS
camera
particle
feedback
time
continuous
D2C
C2D
laplace
discrete
feedforward
frequency
modeling & simulation
linear state space
$\dot{x} = Ax + Bu$
$y = Cx + Du$
nonlinear state space
$\frac{dx}{dt} = f(x,u)$
$y = g(x,u)$
hybrid system
simulation
transfer functions
$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$
block diagrams
system id
Hello my name is
$G(s) = \frac{5}{s^2+1}$
minimum realizations
first principles
m_1
m_2
k
b
linearization
performance
system analysis
stability
nonminimum phase
controllability observability
$C_m = [B, AB, A^2B]$
$O_m = [C, CA, CA^2]'$
margins
SAFETY
nyquist
passivity
phase plane
pole-zero plot
lyapunov stability
bode plots
gang of six
root locus
nichols chart
brian douglas © aug 2020
Engineering Media

Content overview

Representation of feedback systems

Block diagrams

- graphical tool, essential due to the multidisciplinary aspect of control
- emphasize flow information and hide technological details
- causality

Two very different systems

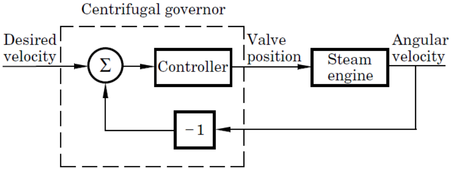


Figure: Centrifugal governor

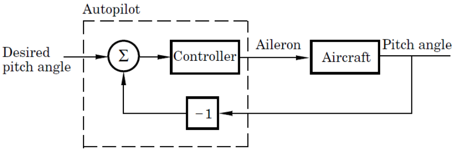


Figure: Autopilot

Representation of feedback systems

Block diagrams

- graphical tool, essential due to the multidisciplinary aspect of control
- emphasize flow information and hide technological details
- causality

Two very different systems, one block diagram!

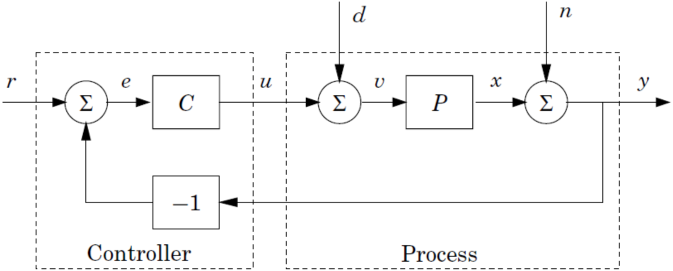


Figure: Generic problem

Representation of feedback systems

System description

- need to describe the process to be controlled

→ model-based/data-driven design

- importance of LTI systems = linear and time-invariant
  - homogeneity: $g(au) = ag(u)$
  - superposition: $g(u_1 + u_2) = g(u_1) + g(u_2)$
  - time-invariance: $g(u(t - T)) = y(t - T)$
  - allowed operations: $ax(t)$, $\int x(t)dt$, $\frac{dx}{dt}$, $x_1(t) \pm x_2(t)$

→ do not represent most real-world systems

→ “Linear systems are important because we can solve them”, Richard Feynman

→ possibility to approximate a real-world system as LTI over a region

Representation of feedback systems

System description

Time-domain

State-space model

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

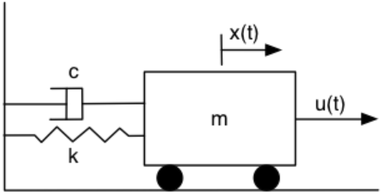
Frequency-domain

Transfer function

$$G(s) = \frac{b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Representation of feedback systems

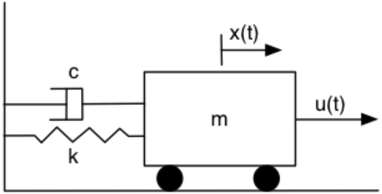
System description



$$m\ddot{x}(t) + c\dot{x} + kx(t) - u(t) = 0$$

Representation of feedback systems

System description



$$m\ddot{x}(t) + c\dot{x} + kx(t) - u(t) = 0$$

$$X = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$$

$$\dot{X} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{pmatrix} X + \frac{1}{m} u$$

Representation of feedback systems

System description

Time domain

Frequency domain

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{\jmath\omega t}d\omega \xleftrightarrow[\text{Inverse Fourier transform}]{\text{Fourier transform}} X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-\jmath\omega t}dt$$

Representation of feedback systems

System description

Time domain

Frequency domain

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The Laplace transform and the s-plane

$$e^{st} = e^{(\sigma+\jmath\omega)t} = e^{\sigma t}e^{\jmath\omega t}$$

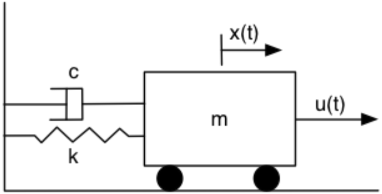
Time domain

s-plane

$$x(t) \xrightarrow{\text{Laplace transform}} X(s) = \int_{0}^{+\infty} x(t)e^{-st}dt$$

Representation of feedback systems

System description



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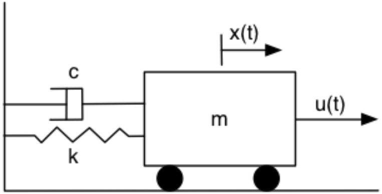
Representation of feedback systems

System description



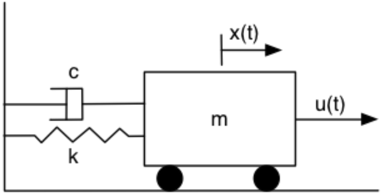
$$m\ddot{x}(t) + c\dot{x} + kx(t) - u(t) = 0$$

$$\mathcal{L}(x) = X(s)$$

$$\mathcal{L}(\dot{x}) = sX(s) - x(0)$$

Representation of feedback systems

System description



$$m\ddot{x}(t) + c\dot{x} + kx(t) - u(t) = 0$$

$$\mathcal{L}(x) = X(s)$$

$$\mathcal{L}(\dot{x}) = sX(s) - x(0)$$

$$ms^2X(s) + csX(s) + kX(s) - U(s) = 0$$

$$G(s) = \frac{X(s)}{U(s)} = \frac{1}{ms^2 + cs + k}$$

Representation of feedback systems

Other characterization of LTI systems

1 Impulse response

$$\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & t \neq 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t) dt = 1$$

$$y(t) = (u * h)(t) = \int_{-\infty}^{+\infty} u(t-\tau)h(\tau)d\tau$$

$\mathcal{L}(\delta(t)) = 1 \rightarrow$ the transfer function is the Laplace transform of the impulse response

Representation of feedback systems

Other characterization of LTI systems

1. **Impulse response**

$$\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & t \neq 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t) dt = 1$$

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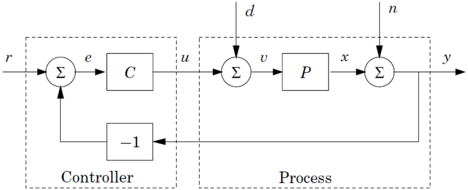
$\mathcal{L}(\delta(t)) = 1 \rightarrow$ the transfer function is the Laplace transform of the impulse response

2. **Frequency response** = $\{G(\jmath\omega)\}$
A system's frequency response is the Fourier transform of its impulse response

Feedback fundamentals

Static analysis

→ Static model = instantaneous input-output relation



$$y = x + n$$

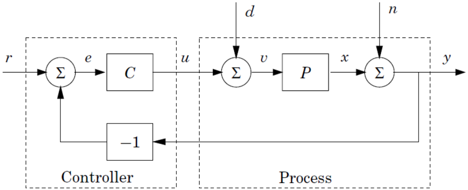
$$x = k_p(u + d)$$

$$u = k_c(r - y)$$

Feedback fundamentals

Static analysis

→ Static model = instantaneous input-output relation



$$y = x + n$$

$$x = k_p(u + d)$$

$$u = k_c(r - y)$$

$$x = \frac{k_p}{1 + k_p k_c} d - \frac{k_p k_c}{1 + k_p k_c} n + \frac{k_p k_c}{1 + k_p k_c} r$$

$$y = \frac{k_p}{1 + k_p k_c} d + \frac{1}{1 + k_p k_c} n + \frac{k_p k_c}{1 + k_p k_c} r$$

$$u = -\frac{k_p k_c}{1 + k_p k_c} d - \frac{k_c}{1 + k_p k_c} n + \frac{k_c}{1 + k_p k_c} r$$

Feedback fundamentals

Static analysis

- disturbance rejection ($r = 0$ and $n = 0$): $x = \frac{k_p}{1+k_p k_c} d$

Feedback fundamentals

Static analysis

- disturbance rejection ($r = 0$ and $n = 0$): $x = \frac{k_p}{1+k_pk_c}d$
- reference tracking ($d = 0$ and $n = 0$): $x = \frac{k_pk_c}{1+k_pk_c}r$

Feedback fundamentals

Static analysis

- disturbance rejection ($r = 0$ and $n = 0$): $x = \frac{k_p}{1+k_p k_c} d$
- reference tracking ($d = 0$ and $n = 0$): $x = \frac{k_p k_c}{1+k_p k_c} r$
- robustness to process variations ($d = 0$ and $n = 0$): $\frac{dx}{dk_p} = \frac{x}{k_p} \frac{1}{1+k_p k_c}$

Feedback fundamentals

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→ The loop gain k_pk_c should be high

Feedback fundamentals

Static analysis

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→ The loop gain $k_p k_c$ should be high

- Obtaining a linear behaviour through feedback: $y = f(u)$ and $u = k_c(r - y)$

$$y + \frac{1}{k_c} f^{-1}(y) = r$$

Feedback fundamentals

Static analysis

- disturbance rejection ($r = 0$ and $n = 0$): $x = \frac{k_p}{1+k_pk_c}d$
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- **This does not consider the dynamics of the system and the controller!**

Feedback fundamentals

Static analysis

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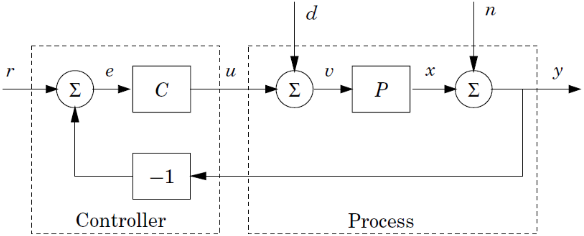
- Obtaining a linear behaviour through feedback: $y = f(u)$ and $u = k_c(r - y)$

$$y + \frac{1}{k_c}f^{-1}(y) = r$$

- **This does not consider the dynamics of the system and the controller!**
- **High loop gains lead to instability**

Feedback fundamentals

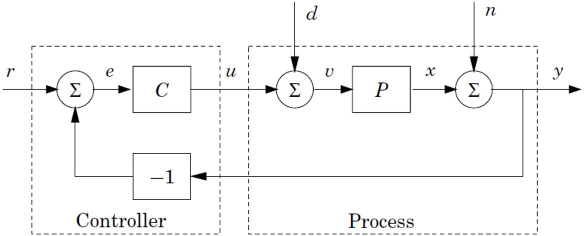
Design issues



- Stability

Feedback fundamentals

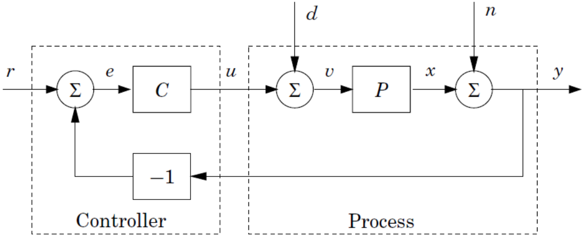
Design issues



- Stability
- Reference tracking

Feedback fundamentals

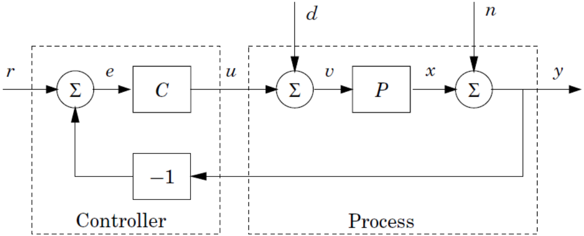
Design issues



- Stability
- Reference tracking
- Disturbance rejection

Feedback fundamentals

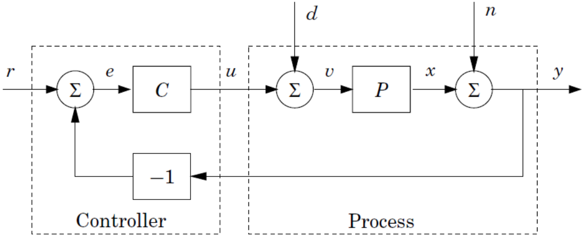
Design issues



- Stability
- Reference tracking
- Disturbance rejection
- Noise attenuation

Feedback fundamentals

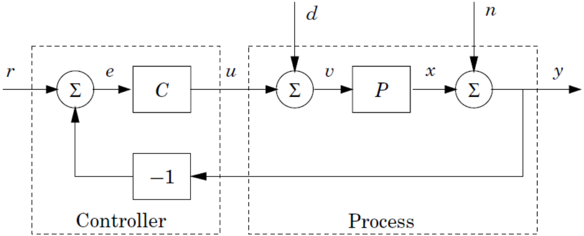
Design issues



- Stability
- Reference tracking
- Disturbance rejection
- Noise attenuation
- Robustness to process variations

Feedback fundamentals

The Gang of Four



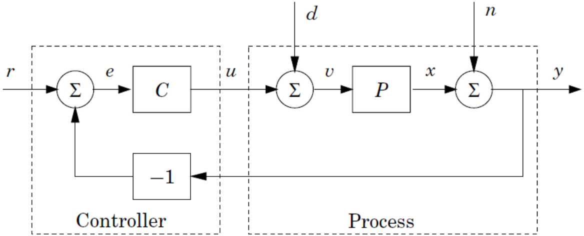
$$X(s) = \frac{P}{1+PC}D(s) - \frac{PC}{1+PC}N(s) + \frac{PC}{1+PC}R(s)$$

$$Y(s) = \frac{P}{1+PC}D(s) + \frac{1}{1+PC}N(s) + \frac{PC}{1+PC}R(s)$$

$$U(s) = -\frac{PC}{1+PC}D(s) - \frac{C}{1+PC}N(s) + \frac{C}{1+PC}R(s)$$

Feedback fundamentals

The Gang of Four



- The sensitivity function : $S(s) = \frac{1}{1+PC}$
- The complementary sensitivity function : $T(s) = \frac{PC}{1+PC}$

→ $S(s) + T(s) = 1$

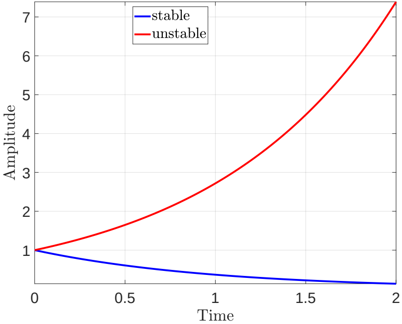
- The noise sensitivity function $\frac{C}{1+PC}$
- The disturbance sensitivity function $\frac{P}{1+PC}$

Feedback fundamentals

Stability and Nyquist plots

BIBO stability

$$G(s) \text{ stable} \quad \Longleftrightarrow \quad \text{poles in the LHP (transfer function)}$$



$$G_1(s) = \frac{1}{s+1}$$

$$G_2(s) = \frac{1}{s-1}$$

Feedback fundamentals

Stability and Nyquist plots

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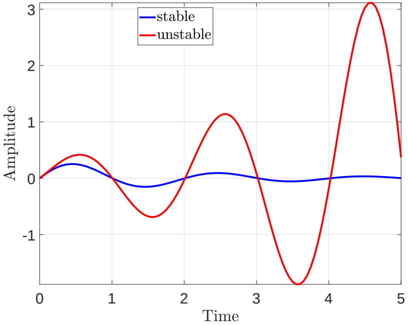


$$G_3(s) = \frac{1}{s^2+s+10}$$

$$G_4(s) = \frac{1}{s^2-s+10}$$

Feedback fundamentals

Stability and Nyquist plots

BIBO stability

$$G(s) \text{ stable} \iff \text{poles in the LHP (transfer function)}$$
$$\iff \text{eigenvalues of } A \text{ in the LHP (state-space model)}$$

Feedback fundamentals

Stability and Nyquist plots

BIBO stability

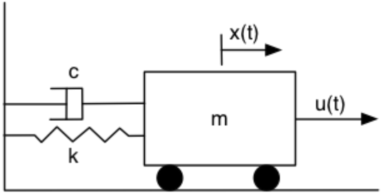
$$
\begin{aligned}
G(s) \text{ stable} \quad &\Longleftrightarrow \quad \text{poles in the LHP (transfer function)} \\
&\Longleftrightarrow \quad \text{eigenvalues of } A \text{ in the LHP (state-space model)} \\
&\Longleftrightarrow \quad \text{roots of the characteristic polynomial in the LHP (ODE)}
\end{aligned}
$$

Feedback fundamentals

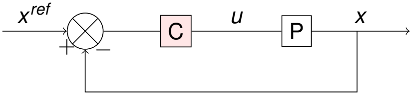
Stability and Nyquist plots

Nyquist criteria

$$L(s) = C(s)P(s)$$



$$m\ddot{x}(t) + c\dot{x} + kx(t) - u(t) = 0$$



Feedback fundamentals

Stability and Nyquist plots

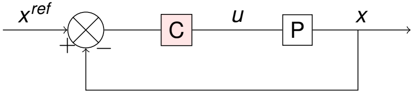
Nyquist criteria

$$L(s) = C(s)P(s)$$

$$P(s) = \frac{1}{ms^2 + cs + k}$$

$$C(s) = k_c$$

$$H(s) = \frac{k_c}{ms^2 + cs + k + k_c}$$



Nyquist criteria

$$P(s) = \frac{1}{ms^2 + cs + k}$$

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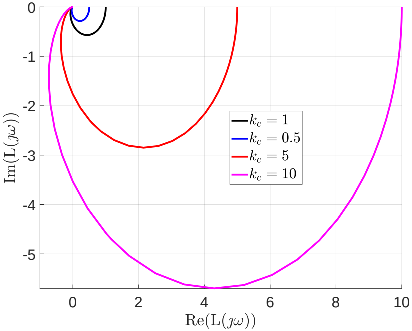


Figure: Nyquist plot.

Nyquist criteria

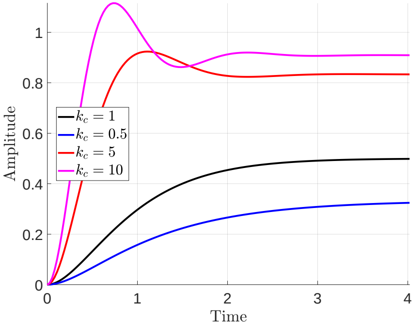


Figure: Step response.

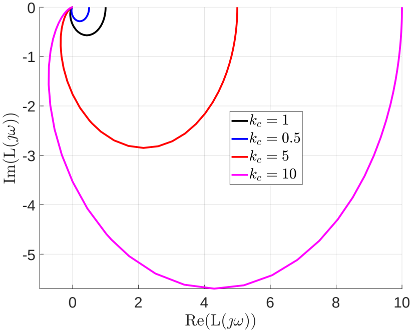


Figure: Nyquist plot.

Feedback fundamentals

Stability and Nyquist plots

Nyquist criteria $C(s) = \frac{k_c}{s}$

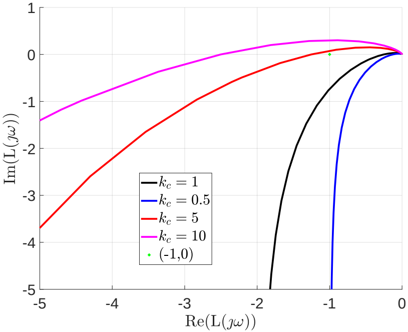


Figure: Nyquist plot.

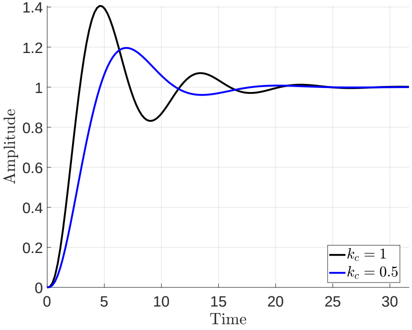


Figure: Step response.

Feedback fundamentals

Stability and Nyquist plots: introducing margins

Nyquist criteria

- the open-loop is simpler to understand (causal reasoning)

Feedback fundamentals

Stability and Nyquist plots: introducing margins

Nyquist criteria

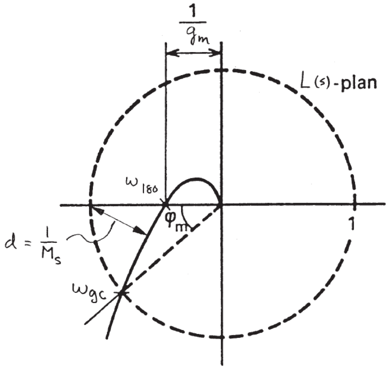
- the open-loop is simpler to understand (causal reasoning)
- easier to understand the influence of the controller on stability

Feedback fundamentals

Stability and Nyquist plots: introducing margins

Nyquist criteria

- the open-loop is simpler to understand (causal reasoning)
- easier to understand the influence of the controller on stability
- stability becomes more than a binary property



Margins visualisation on a Nyquist plot

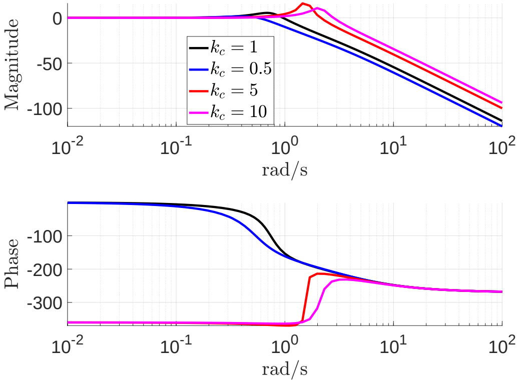
- Gain margin g_m (2-5) and stability margin s_m
- Phase margin φ_m (30-60°)
- Shortest distance d to $(-1, 0)$ (0.5-0.8)

Feedback fundamentals

Performances and Bode plots

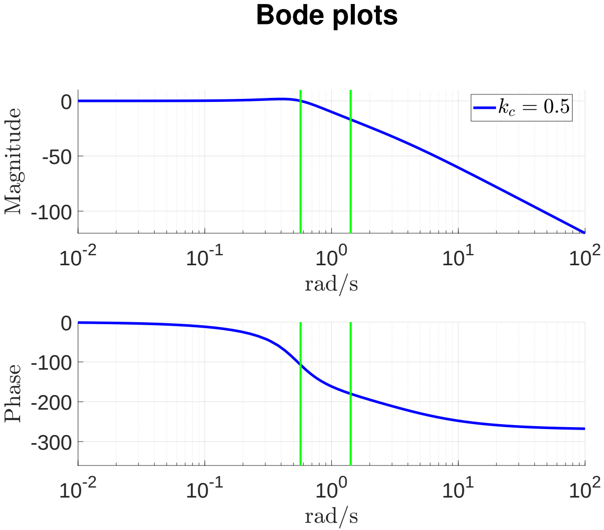
Bode plots

- good overview of performance and robustness of the feedback loop



Feedback fundamentals

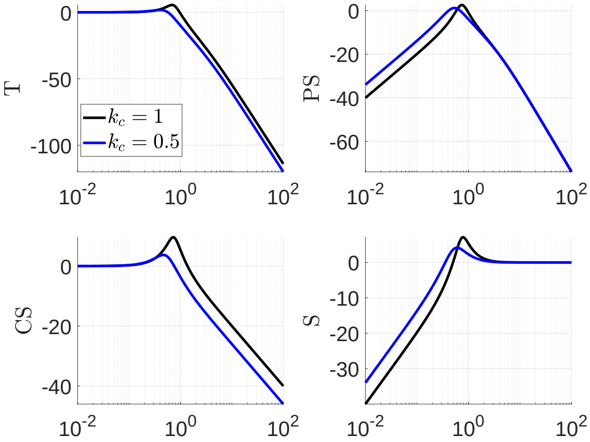
Performances and Bode plots



Feedback fundamentals

Performances and Bode plots

Bode plots



About feedforward

- Limitations of feedback systems: waiting for an error to occur to take corrective actions

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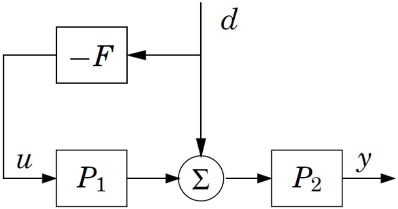
- Concept of feedforward: use the information about the disturbance to counteract it before it affects the system



About feedforward

- Limitations of feedback systems: waiting for an error to occur to take corrective actions
- Concept of feedforward: use the information about the disturbance to counteract it before it affects the system
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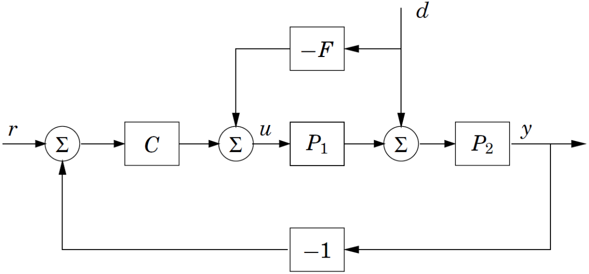


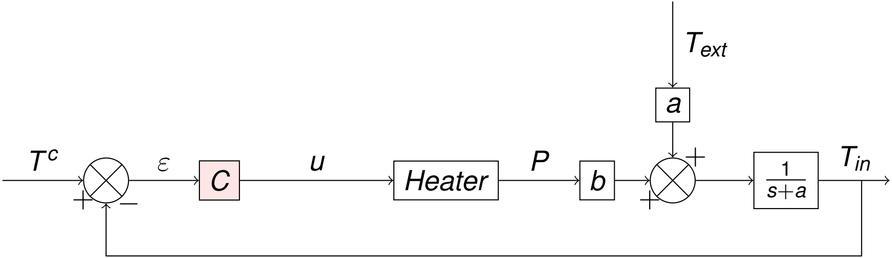
About feedforward

- Limitations of feedback systems: waiting for an error to occur to take corrective actions
- Concept of feedforward: use the information about the disturbance to counteract it before it affects the system
- Usually combined with feedback
- Requires to invert the system (impossible for RHP zeros and time delays)

About feedforward

Example: building temperature control

$$\frac{dT_{in}}{dt} = -a(T_{in} - T_{ext}) + bP$$



About feedforward

Example: building temperature control

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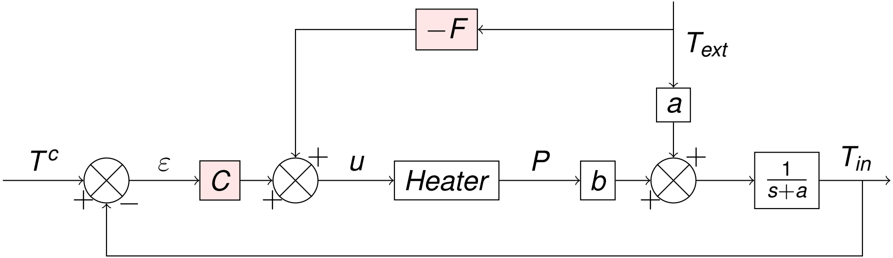


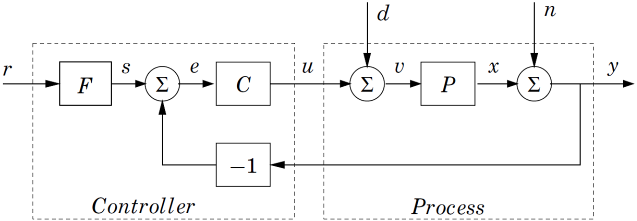
About feedforward

Feedback	**Feedforward**
Closed-loop	Open-loop
Reactive	Planning
Robust to modeling errors	Sensitive to modelling errors
Risk of instability	No risk of instability

About feedforward

Other architectures

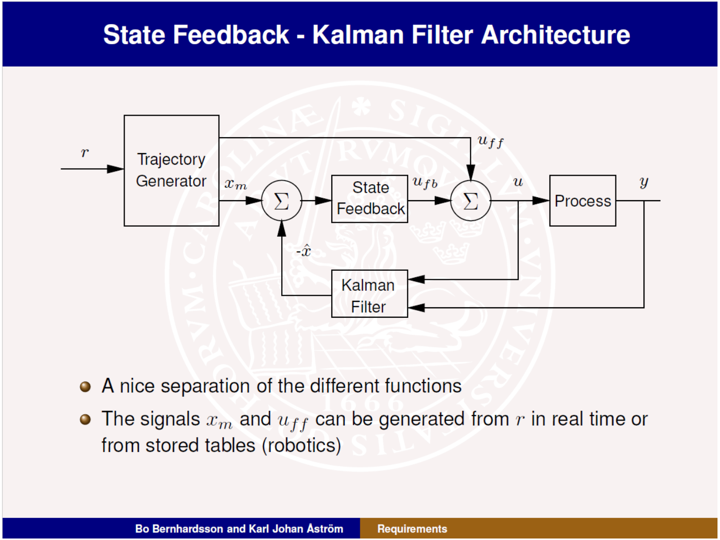
Two Degrees of Freedom architecture



- The feedforward F is designed for reference tracking
- C handles disturbance rejection, noise attenuation, robustness

About feedforward

Other architectures



State Feedback - Kalman Filter Architecture

- A nice separation of the different functions
- The signals x_m and u_{ff} can be generated from r in real time or from stored tables (robotics)

Bo Bernhardsson and Karl Johan Åström Requirements

Conclusions

- About the system
  - Different descriptions: physical models or experimental data
  - The system should not be taken for granted
- Requirements: stability, performance and robustness

→ properties of the Gang of Four

- Limit of actuators: rate and saturation
- Measurement noise: importance of filtering, sampling

Next week:

- More on process uncertainties and robustness
- Dynamic limitations