Adaptive Control

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Adaptive Control

Introduction

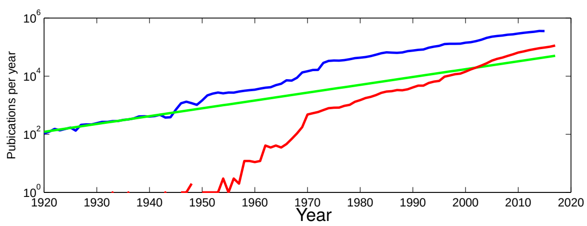
Adapt to adjust to a specified use or situation
Tune to adjust for proper response
Autonomous independence, self-governing
Learn to acquire knowledge or skill by study, instruction or experience
Reason the intellectual process of seeking truth or knowledge by infering from either fact of logic
Intelligence the capacity to acquire and apply knowledge
In Automatic Control

- Automatic tuning - tuning on demand
- Gain scheduling - adjust controller parameter based on direct measurement of environmental parameters
- Adaptation - continuous adjustment of controller parameters based on regular measured signals

A Brief History of Adaptive Control

- Adaptive Control: Learn enough about a process and its environment for control – restricted domain, prior info
- Development similar to neural networks
  Many ups and downs, lots of strong egos
- Early work driven adaptive flight control 1950-1970.
  The brave era: Develop an idea, hack a system, simulate and fly!
  Several adaptive schemes emerged no analysi
  Disasters in flight tests - the X-15 crash nov 15 1967
  Gregory P. C. ed, Proc. Self Adaptive Flight Control Systems. Wright Patterson Airforce Base, 1959
- Emergence of adaptive theory 1970-1980
  Model reference adaptive control emerged from flight control stability theory
  The self tuning regulator emerged from process control and stochastic control theory
- Microprocessor based products 1980
- Robust adaptive control 1990
- L1-adaptive control - Flight control 2006
- Learning and Adaptation 2020

Publications in Scopus



Blue control red adaptive control

Flight Control – Servo Problem

P. C. Gregory March 1959. Proceedings of the Self Adaptive Flight Control Systems Symposium. Wright Air Development Center, Wright-Patterson Air Force Base, Ohio.

Most of you know that with the advent a few years ago of hypersonic and supersonic aircraft, the Air Force was faced with a control problem. This problem was two-fold; one, it was taking a great deal of time to develop a flight control system; and two, the system in existence were not capable of fulfilling future Air Force requirements. These systems lacked the ability to control the aircraft satisfactorily under all operating conditions.

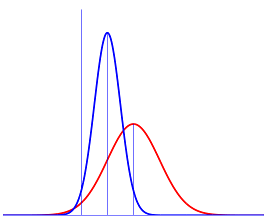
Test flights start summer 1961

- Honeywell self oscillating adaptive system X-15
- MIT model reference adaptive system on F-101A

Mishkin, E. and Brown, L Adaptive Control Systems. Mc-Graw-Hill New York, 1961

Process Control – Regulation Problem

- What can be achieved?
- What are the benefits?
- Small improvements 1% can have larege economic consequences



Some contributions

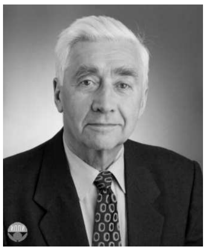
- Early pneumatic systems
- Kalman's self-optimizing controller
- The self-tuning regulator - moving average controller
- Relay auto-tuning

Some Landmarks

- Early flight control systems 1955
- Dynamic programming Bellman 1957
- Dual control Feldbaum 1960
- System identification 1965
- Learning control Tsypkin 1971
- Algorithms MRAS STR 1970
- Stability analysis 1980
  Lyapunov - Tsypkin
  Passivity - Popov Landau
  Augmented Error - Monopoli
- Industrial products 1982
- PID auto-tuning
- Robustness 1985
- Automatic tuning 1985
- Autonomous Control 1995
- Adaptation and Learning - a renaisance

Yakov Z Tsypkin 1919 - 1997

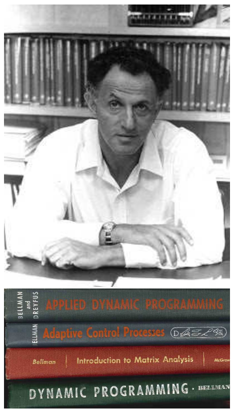
- BS, Moscow Electrical Engineering Institute 1941
- BS, Moscow State University 1943
- MS Engineering, Moscow State University 1945
- PhD, Moscow State University 1948
- Engineer, senior engineer, Chief of Department, Research Institute Aircraft Equipment 1941-1949
- Senior researcher, Institute Control Sciences, Moscow 1950-1957
- Head of laboratory, Institute Control Sciences, Moscow, since 1957
- Yakov Z. Tsypkin and C. Constanda Relay Control Systems.
- Sampling Systems Theory and Its Application Volume 1 and 2 (NY 1964 Macmillan. Translated from Russian by A. Allen an...)
- Foundations of the Theory of Learning Systems by Tsypkin, Ya. Z. (1973) Paperback



- Lenin Prize 1960
- Quazza Medal IFAC 1984
- Hartley Medal IMC 1985
- Rufus Oldenburger Medal ASME 1989

Richard Bellman 1920 - 1984

- BA math Brooklyn College 1941
- MA University of Wisconsin
- Los Alamos Theoretical Physics
- PhD Princeton Lefschetz 1946
- RAND Corporation
- Founding editor Math Biosciences
- Brain tumor 1973
- 619 papers 39 Books
- Dynamic Programming
- Bellman Equation HJB
- Curse of dimensionality
- Bellman-Ford algorithm
- John von Neumann Theory Prize (1976)
- IEEE Medal of Honor (1979)

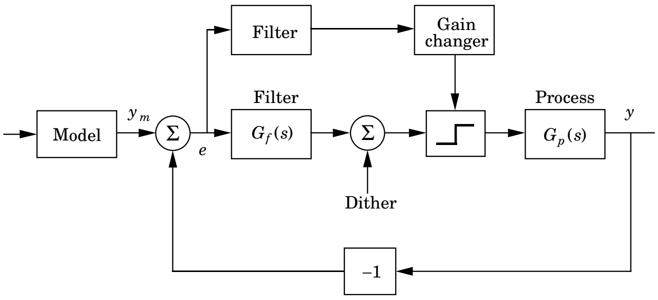


Adaptive Control

The Self-Oscillating Adaptive System H. Schuck Honeywell 1959

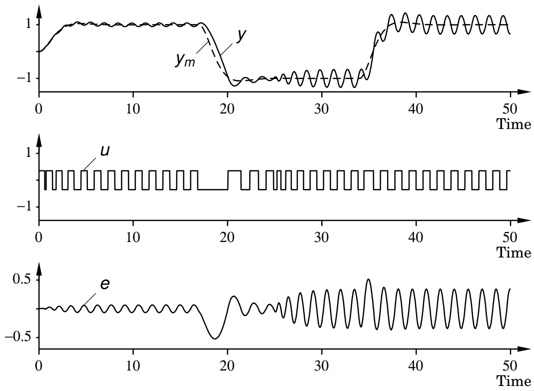
It's is rather hard to tell when we at Honeywell first became interested in adaptive control. ... In retorspect, it seems that we first conciously articulated the need in connection with our early work in automatic approach and landing. ...
Let us look at the adaptive flight control system that was proven in the flight tests on the F-94C. Conceptually it is simple, deceptively so. The input is applied to a model whose dynamic performance is what we whish the dynamic performance of the aircraft to be. The actual response is compared with the reponse of the model and the difference is used as an input to the servo. If the gain of the servo is sufficiently high, the response of the aircraft will be identical to that of the modle, no matter what the elevator effectiveness, so long as it is finite and has the right direction.
Design of the model is fairly simple. ... The big problem comes inconnection with the need to make the gain of the servo sufficiently high. An ordinary linear servo loop will not do. It simply cannot be given a sufficiently high gain and still be stable. So we go in for non-linearity, the most extreme form of non-linearity, in fact - the bang-bang type. Full available power is applied one way, or the other, depending on the the direction of the swithching order.
Now it is well known that a simple bang-bang sytem is oscillatorye. And we don't want an oscillatory aircraft. .. So we look for ways to tame it down, keeping its high-gain characteristic while reducing it oscillatory activity. ...

The Self-Oscillating Adaptive System



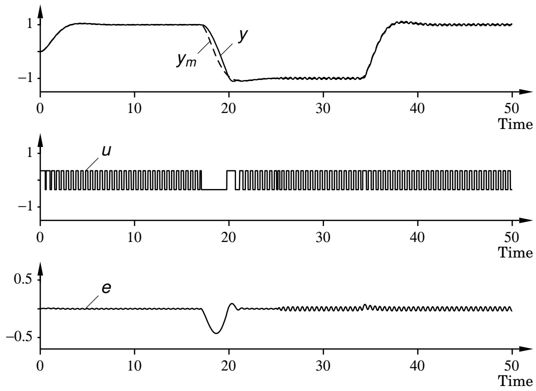
- Shape behavior by the block called Model
- Make the inner loop as fast as possible
- Relay feedback automatically adjusts to gain margin for low frequency signals to $g_m = 2$! Dual input describing functions!!!
- Relay amplitude adjusted by logic

SOAS $P(s) = \dfrac{k}{s(s+1)^2}$



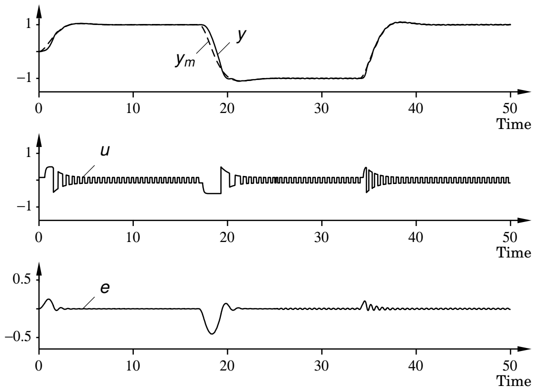
Gain k nominally 1 increased to $k = 5$ at time $t = 25$
System with linear controller unstable for $k > 2$

SOAS Simulation - Adding Lead Network



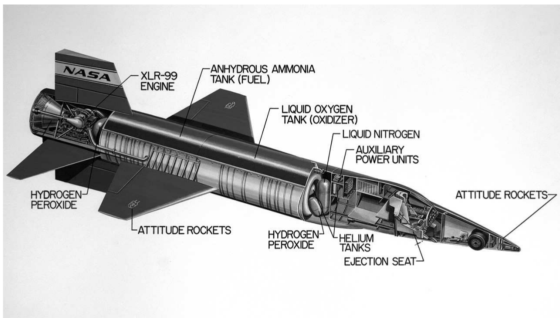
Gain increases by a factor of 5 at time $t = 25$

SOAS Simulation - Adding Gain Changer



Gain increases by a factor of 5 at time $t = 25$

The X-15 with MH-96 Autopilot Crash Nov 11 1967



Logic for gain change did not increase gain fast enough

Dydek, Zachary, Anuradha Annaswamy, and Eugene Lavretsky. "Adaptive Control and the NASA X-15-3 Flight Revisited." IEEE Control Systems Magazine 30.3 (2010): 32–48.

Adaptive Control

K. J. Åström

1. Introduction
2. Self-oscillating Adaptive Control
3. Model Reference Adaptive Control
   - The MIT rule -sensitivity derivatives
   - Direct MARS - update parameters of a process model
   - Indirect MRAS - update controller parameters directly
   - L1 adaptive control - avoid dividing with estimates
4. Estimation and Excitation
5. Self-Tuning Regulators
6. Learning and Dual Control
7. Applications
8. Related Fields
9. Summary

Global Stability

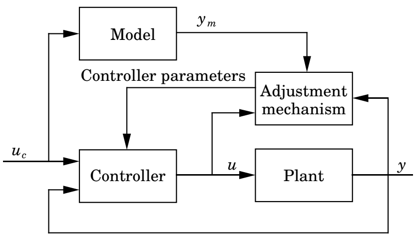
A long story

- MRAS and the MIT rule 1959
- Empirical evidence of instability
- Analysis
- Butchart and Shackloth 1965
- SPR rules
- Parks 1966
- Landau 1969
- The augmented error Monopoli 1974
- Counterexamples Feuer and Morse 1978
- Stability proofs
  - Egardt 1979
  - Goodwin Ramage Caines 1980
  - Narendra 1980
  - Morse 1980
  - Many others

Model Reference Adaptive Control – P. Whitaker MIT 1959

We have further suggested the name model-reference adaptive system for the type of system under consideration. A model-reference system is characterized by the fact that the dynamic specification for a desired system output are embodied in a unit which is called the model-reference for the system, and which forms part of the equipment installation. The commandsignal input to the control system is also fed to the model. The difference between the output signal of the model and the corresponding output quantity of the system is then the response error. The design objectie of the adaptive portion of this type of system is to minimze this response error under all operational conditions of the system. Specifically the adjustment is done by the MIT Rule.

Model Reference Adaptive Control – MRAS



Linear feedback from $e = y - y_m$ is not adequate for parameter adjustment!
The MIT rule

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta}$$

Many other versions

A First Order System

Process

$$\frac{dy}{dt} = -ay + bu$$

Model

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

Controller

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t)$$

Ideal controller parameters

$$\theta_1 = \theta_1^0 = \frac{b_m}{b}$$
$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b}$$

Find a feedback that changes the controller parameters so that the closed loop response is equal to the desired model

MIT Rule - Sensitivity Derivatives

The error

$$e = y - y_m$$
$$y = \frac{b\theta_1}{p + a + b\theta_2} u_c$$
$$\frac{\partial e}{\partial \theta_1} = \frac{b}{p + a + b\theta_2} u_c$$
$$\frac{\partial e}{\partial \theta_2} = -\frac{b^2\theta_1}{(p + a + b\theta_2)^2} u_c = -\frac{b}{p + a + b\theta_2} y$$

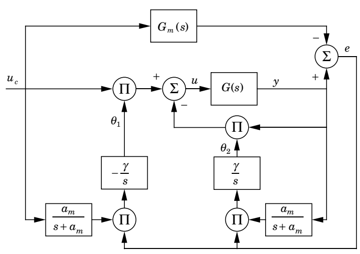
Approximate

$$p + a + b\theta_2 \approx p + a_m$$

Hence

$$\frac{d\theta_1}{dt} = -\gamma\left(\frac{a_m}{p + a_m} u_c\right) e$$
$$\frac{d\theta_2}{dt} = \gamma\left(\frac{a_m}{p + a_m} y\right) e$$

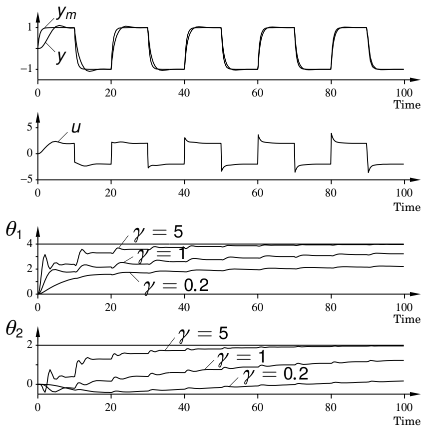
Block Diagram



$$\frac{d\theta_1}{dt} = -\gamma\left(\frac{a_m}{p + a_m} u_c\right) e$$
$$\frac{d\theta_2}{dt} = \gamma\left(\frac{a_m}{p + a_m} y\right) e$$

Example $a = 1$, $b = 0.5$, $a_m = b_m = 2$.

Simulation $a = 1$, $b = 0.5$, $a_m = b_m = 2$.



MRAS - The MIT Rule

The error

$$e = y - y_m, \qquad y = \frac{b\theta_1}{p + a + b\theta_2} u_c \qquad p = \frac{dx}{dt}$$

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{p + a + b\theta_2} u_c$$

$$\frac{\partial e}{\partial \theta_2} = -\frac{b^2\theta_1}{(p + a + b\theta_2)^2} u_c = -\frac{b}{p + a + b\theta_2} y$$

Approximate

$$p + a + b\theta_2 \approx p + a_m$$

The MIT rule: Minimize $e^2(t)$

$$\frac{d\theta_1}{dt} = -\gamma \left(\frac{a_m}{p + a_m} u_c \right) e, \qquad \frac{d\theta_2}{dt} = \gamma \left(\frac{a_m}{p + a_m} y \right) e$$

Adaptation Laws from Lyapunov Theory

Replace ad hoc with desings that give guaranteed stability

- Lyapunov function $V(x) > 0$ positive definite

$$\frac{dx}{dt} = f(x),$$

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = \frac{DV}{dx} f(x) < 0$$

- Determine a controller structure
- Derive the Error Equation
- Find a Lyapunov function
- $\frac{dV}{dt} \le 0$ Barbalat's lemma
- Determine an adaptation law

First Order System

Process model and desired behavior

$$\frac{dy}{dt} = -ay + bu, \qquad \frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

Controller and error

$$u = \theta_1 u_c - \theta_2 y, \qquad e = y - y_m$$

Ideal parameters

$$\theta_1 = \frac{b}{b_m}, \qquad \theta_2 = \frac{a_m - a}{b}$$

The derivative of the error

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)\,u_c$$

Candidate for Lyapunov function

$$V(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2 \right)$$

Derivative of Lyapunov Function

$$V(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2 \right)$$

Derivative of error and Lyapunov function

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)\,u_c$$

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$

$$= -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma y e\right)$$

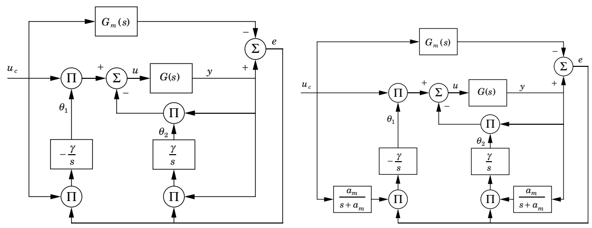
$$+ \frac{1}{\gamma}(b\theta_1 - b_m)\left(\frac{d\theta_1}{dt} + \gamma u_c e\right)$$

Adaptation law

$$\frac{d\theta_1}{dt} = -\gamma u_c e, \qquad \frac{d\theta_2}{dt} = \gamma y e \Rightarrow \frac{de}{dt} = -e^2$$

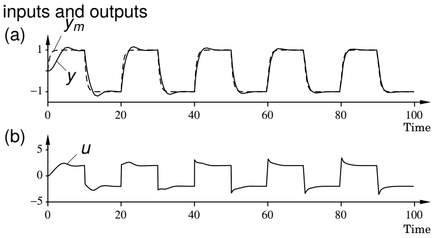
Error will always go to zero, what about parameters, Barbara's lemma!

Comparison with MIT rule

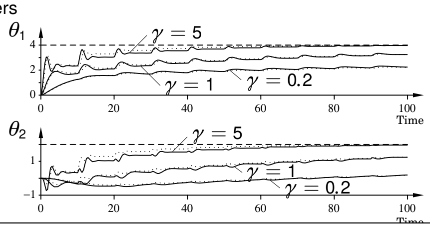


Simulation - Dotted Parameters MIT rule

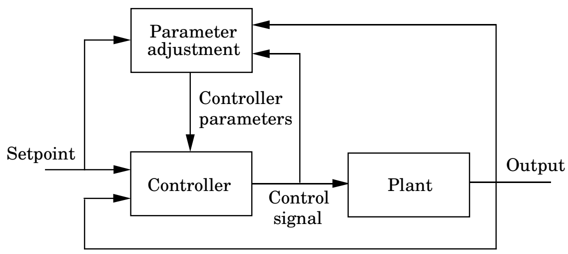
Process inputs and outputs



Parameters



Indirect MRAS



Two loops

- Regular feedback loop
- Parameter adjustment loop

Schemes

- Model Reference Adaptive Control MRAS
- Self-tuning Regulator STR
- Learning and Dual Control

Indirect MRAS - Estimate Process Model

Process and estimator

$$\frac{dx}{dt} = ax + bu, \qquad \frac{d\hat{x}}{dt} = \hat{a}\hat{x} + \hat{b}u$$

Nominal controller gains: $k_x = k_x^0 = (a - a_m)/b, \quad k_r = k_r^0 = b_m/b$.
Estimation error $e = \hat{x} - x$ has the derivative

$$\frac{de}{dt} = \hat{a}x + \hat{b}u - ax - bu = ae + (\hat{a} - a)\hat{x} + (\hat{b} - b)u = ae + \tilde{a}\hat{x} + \tilde{b}u,$$

where $\tilde{a} = \hat{a} - a$ and $\tilde{b} = \hat{b} - a$. Lyapunov function

$$2V = e^2 + \frac{1}{\gamma}\Big(\tilde{a}^2 + \tilde{b}^2\Big).$$

Its derivative becomes

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}\Big(\tilde{a}\frac{d\hat{a}}{dt} + \tilde{b}\frac{d\hat{b}}{dt}\Big) = ae^2 + \Big(e\hat{x} + \frac{1}{\gamma}\frac{d\tilde{a}}{dt}\Big)\tilde{a} + \Big(eu + \frac{1}{\gamma}\frac{d\tilde{b}}{dt}\Big)\tilde{b}$$

Indirect MRAS

Process and estimator

$$\frac{dx}{dt} = ax + bu, \qquad \frac{d\hat{x}}{dt} = \hat{a}\hat{x} + \hat{b}u$$

Control law

$$u = -\frac{\hat{a} - a_m}{\hat{b}}x + \frac{b_m}{\hat{b}}r$$

Very bad to divide by $\hat{b}$!

L1 Adaptive Control - Hovkimian and Cao 2006

Replace

$$u = -\frac{\hat{a} - a_m}{\hat{b}}x + \frac{b_m}{\hat{b}}r$$
$$\hat{b}u + (\hat{a} - a_m)x - b_m r = 0$$

with the differential equation

$$\frac{du}{dt} = K\big(b_m r - (\hat{a} - a_m)x - \hat{b}u\big)$$

Avoid division by $\hat{b}$, can be interpreted as sending the signal $\hat{b}_m r + (a_m - \hat{a})x$ through a first order filter with the pole $s = -K\hat{b}$.

L1 has been tested on several airplanes
I. M. Gregory, C. Cao, E. Xargay, N. Hovakimyan and X. Zou L1 Adaptive Control Design for NASA AirSTAR Flight Test Vehicle. AIAA Guidance, Navigation, and Control Conference Portland Oregon August 2011

Adaptive Control

1. Introduction
2. Self-oscillating Adaptive Control
3. Model Reference Adaptive Control
4. Estimation and Excitation
5. Minimum Variance Control
6. Self-Tuning Regulators
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9. Related Fields
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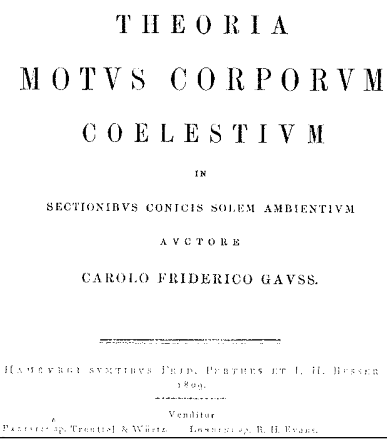
The Least Squares Method

The problem: The Orbit of Ceres
The problem solver: Karl Friedrich Gauss
The principle: *Therefore, that will be the most probable system of values of the unknown quantities, in which the sum of the squares of the differences between the observed and computed values, multiplied by numbers that measure the degree of precision, is a minimum.*
In conclusion, the principle that the sum of the squares of the differences between the observed and computed quantities must be a minimum, may be considered independently of the calculus of probabilities.
An observation: Other criteria could be used. *But of all these principles ours is the most simple; by the others we should be led into the most complicated calculations.*

The Book


THEORIA
MOTVS CORPORVM
COELESTIVM
IN
SECTIONIBVS CONICIS SOLEM AMBIENTIVM
AVCTORE
CAROLO FRIDERICO GAVSS.

1809.
Venditur

Recursive Least Squares

$$\begin{aligned} y_{t+1} &= -a_1 y_t - a_2 y_{t-1} + \cdots + b_1 u_t + \cdots + e_{t+1} = \varphi_t^T \theta + e_{t+1} \\ \varphi_t &= [-y_t - y_{t-1} \cdots u_t u_{t-1} \cdots] \\ \theta &= [a_1 a_2 \cdots b_1 b_2 \cdots], \end{aligned}$$

the parameter estimates are given by

$$\begin{aligned} \hat{\theta}_t &= \hat{\theta}_{t-1} + K_t(y_t - \varphi_t \hat{\theta}_{t-1}) \\ K_t &= P_t \varphi_t \\ P_t &= P_{t-1}\varphi_t(\lambda + \varphi_t^T P_{t-1}\varphi_t)^{-1} \end{aligned}$$

The parameter λ controls how quickly old dat is discounted, many versions: directional forgetting, square root filtering etc.

Persistent Excitation PE

Introduce

$$c(k) = \lim_{t\to\infty} \frac{1}{t}\sum_{i=1}^{t} u(i)u(i-k)$$

A signal u is called *persistently exciting* (PE) of order n if the matrix C_n is positive definite.

$$C_n = \begin{pmatrix} c(0) & c(1) & \ldots & c(n-1) \\ c(1) & c(0) & \ldots & c(n-2) \\ \vdots & & & \\ c(n-1) & c(n-2) & \ldots & c(0) \end{pmatrix}$$

A signal u is persistently exciting of order n if and only if

$$U = \lim_{t\to\infty} \frac{1}{t}\sum_{k=1}^{t} (A(q)u(k))^2 > 0$$

for all nonzero polynomials A of degree $n-1$ or less.

Persistent Excitation - Examples

- A step is PE of order 1

$$(q-1)u(t) = 0$$

- A sinusoid is PE of order 2

$$(q^2 - 2q\cos\omega h + 1)u(t) = 0$$

- White noise
- PRBS
- Physical meaning
- Mathematical meaning

Lack of Identifiability due to Feedback

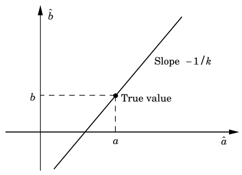
$$y(t) = ay(t-1) + bu(t-1) + e(t), \qquad u(t) = -ky(t)$$

Multiply by α and add, hence

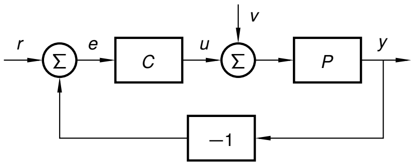
$$y(t) = (a+\alpha k)y(t-1) + (b+\alpha)u(t-1) + e(t)$$

Same I/O relation for all $\hat{a}$ and $\hat{b}$ such that

$$\hat{a} = a + \alpha k, \qquad \hat{b} = b + \alpha$$



Lack of Identifiability due to Feedback



$$Y(s) = \frac{P(s)C(s)}{1+P(s)C(s)}R(s) + \frac{P(s)}{1+P(s)C(s)}V(s)$$

$$U(s) = \frac{C(s)}{1+P(s)C(s)}R(s) - \frac{C(s)P(s)}{1+P(s)C(s)}V(s)$$

$Y(s) = P(s)U(s)$ if $v = 0$, and $Y(s) = -\frac{1}{C(s)}U(s)$ if $r = 0$.

Identification will then give the negative inverse of controller transfer function! Any signal entering between u and v will influence closed loop identification severely A good model can only be obtained if $v = 0$, or if v is much smaller than r!

Example

Model

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$

Parameters

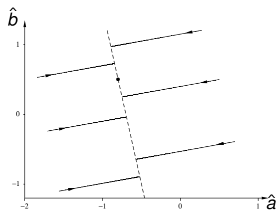
$$a = -0.9,\ b = 0.5,\ \sigma = 0.5,\ \hat{\theta}(0) = 0,\ P(0) = 100I$$

Excitation

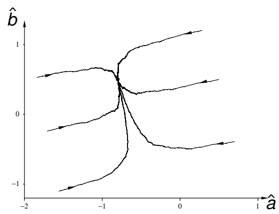
- Unit pulse at $t = 50$
- Square wave of unit amplitude and period 100
- Two cases
  a) $u(t) = -0.2y(t)$
  b) $u(t) = -0.32y(t-1)$

Example ...

a) $u(t) = -0.2y(t)$



b) $u(t) = -0.32y(t-1)$



No convergence with constant feedback with compatible structure, slow convergence to low dimensional subspace with irregular feedback!

T. Hägglund and KJÅ, Supervision of adaptive control algorithms. Automatica 36 (2000) 1171-1180

Adaptive Control

1. Introduction
2. Self-oscillating Adaptive Control
3. Model Reference Adaptive Control
4. Estimation and Excitation
5. Minimum Variance Control
   - Motivation
   - Stochastic control theory
   - A model structure
   - System identification
   - The control algorithm
6. Self-Tuning Regulators
7. Learning and Dual Control
8. Applications
9. Related Fields
10. Summary

The Billerud-IBM Project

- IBM and Computer Control
  - IBM dominated computer market totally in late 1950
  - Saw big market in the process industry
  - Started research group in math department of IBM Research, hired Kalman, Bertram and Koepcke 1958
  - Bad experience with installation in US paper industry
  - IBM Nordic Laboratory 1959 hired KJ Jan1960
- Billerud
  - Visionary manager Tryggve Bergek
  - Had approached Datasaab earlier for computer control
- Project Goals
  - Billerud: Exploit computer control to improve quality and profit!
  - IBM: Gain experience in computer control, recover prestige and find a suitable computer architecture!
- Schedule
  - Start April 1963, computer Installed December 1964
  - System identification and on-line control March 1965
  - Full operation September 1966
  - 40 many-ears effort in about 3 years

The Billerud Plant

Steady State Regulation of Basis Weight and Moisture Content

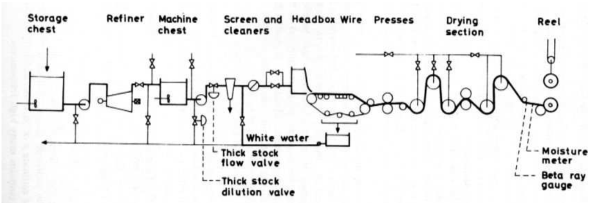
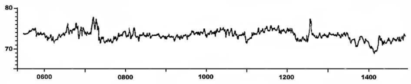
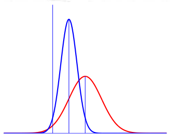


Fig. 6.9. Simplified diagram of a kraft paper machine.

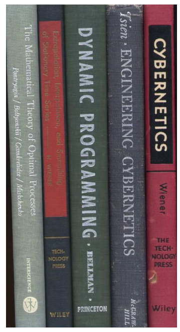




Small improvements 1% are very valuable

The Scene of 1960

- Servomechanism theory 1945
- IFAC 1956 (50 year jubilee in 2006)
- Widespread education and industrial use of control
- The First IFAC World Congress Moscow 1960
- Exciting new ideas
  - Dynamic Programming Bellman 1957
  - Maximum Principle Pontryagin 1961
  - Kalman Filtering ASME 1960
- Exciting new development
  - The space race (Sputnik 1957)
  - Computer Control Port Arthur 1959
- IBM and IBM Nordic Laboratory 1960
  - Computerized Process Control



The RAND Corporation

Set up as an independent non-profit research organization (Think Tank) for the US Airforce by Douglas Aircraft Corporation in 1945.
Saab R System

- Richard Bellman
- Georg Danzig LP
- Henry Kissinger
- John von Neumann
- Condolezza Rice
- Donald Rumsfeld
- Paul Samuelson

Stochastic Control Theory

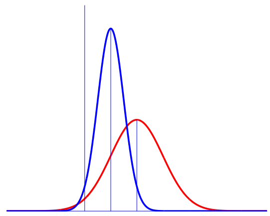
Kalman filtering, quadratic control, separation theorem

Process model

$$dx = Axdt + Budt + dv$$
$$dy = Cxdt + de$$

Controller

$$d\hat{x} = A\hat{x} + Bu + K(dy - C\hat{x}dt)$$
$$u = L(x_m - \hat{x}) + u_{ff}$$



A natural approach for regulation of industrial processes.

Model Structures

Process model

$$dx = Axdt + Budt + dv$$
$$dy = Cxdt + de$$

Much redundancy $z = Tx$ + noise model. Start by transforming to innovations representation, ϵ is Wiener process

$$d\hat{x} = A\hat{x}dt + Budt + K(dy - C\hat{x}dt)$$
$$= (A - KC)\hat{x}dt + Budt + K\,d\epsilon$$
$$dy = C\hat{x}dt + d\epsilon$$

Transfor to observable canonical form

$$d\hat{x} = \begin{pmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & & 0 \\ \vdots & & & & \\ -a_{n-1} & 0 & 0 & & 1 \\ -a_n & 0 & 0 & & 0 \end{pmatrix} \hat{x}dt + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} udt + \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} d\epsilon$$

$$dy = \begin{pmatrix} 1 & 0 & 0 \dots & 0 \end{pmatrix} \hat{x} + d\epsilon$$

Model Structures

...

$$d\hat{x} = \begin{pmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & & 0 \\ \vdots & & & & \\ -a_{n-1} & 0 & 0 & & 1 \\ -a_n & 0 & 0 & & 0 \end{pmatrix} \hat{x}\,dt + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} u\,dt + \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} d\epsilon$$

$$dy = \begin{pmatrix} 1 & 0 & 0 \dots & 0 \end{pmatrix} + \hat{x} + d\,\epsilon$$

Input output representation

$$Y = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}{s^n + a_1 s^{n-1} + \dots + a_n} U + \Big(1 + \frac{k_1 s^{n-1} + k_2 s^{n-2} + \dots + k_n}{s^n + a_1 s^{n-1} + \dots + a_n}\Big) E$$

- Filter gains k_i appear explicitely
- Dynamics of system a_i is characteristic polynomial of estimator eigenvalues of $A - KC$

Corresponding sampled system

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t)$$

The Sampled Model

The basic sampled model for stochastic SISO system is

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t)$$

Notice symmetries

- y can be computed from e, dynamics A
- e can be computed from y, dynamics $A - KC$ (observer dynamics)
- Kalman filter gains explicitly in model $A(s) - C(s)$

Modeling from Data (Identification)

The Likelihood function (Bayes rule)

$$p(\mathcal{Y}_t, \theta) = p(y(t)|\mathcal{Y}_{t-1}, \theta) = \dots = -\frac{1}{2}\sum_{1}^{N} \frac{\epsilon^2(t)}{\sigma^2} - \frac{N}{2}\log 2\pi\sigma^2$$

$$\theta = (a_1, \dots, a_n, b_1, \dots, b_n, c_1, \dots, c_n, \epsilon(1), ..,)$$
$$Ay(t) = Bu(t) + Ce(t) \qquad C\epsilon(t) = Ay(t) - Bu(t)$$
$$\epsilon = \text{one step ahead prediction error}$$

Efficient computations

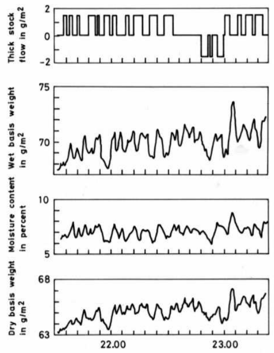
$$\frac{\partial J}{\partial a_k} = \sum_{1}^{N} \epsilon(t)\frac{\partial \epsilon(t)}{\partial a_k} \qquad C\frac{\partial \epsilon(t)}{\partial a_k} = q_k y(t)$$

- Good match identification and control. Prediction error is minimized in both cases!

KJÅ and T. Bohlin, Numerical Identification of Linear Dynamic Systems from Normal Operating Records. In Hammond, *Theory of Self-Adaptive Control Systems, Plenum Press, January 1966.*

Practical Issues

- Sampling period
- To perturb or not to perturb
- Open or closed loop experiments
- Model validation
- 20 min for two-pass compilation of Fortran program!
- Control design
- Skills and experiences



Minimum Variance Control - Example

Consider the first order system

$$y(t+1) + ay(t) = bu(t) + e(t+1) + ce(t)$$

Consider the situation at time t, we have

$$y(t+1) = -a\underline{y(t)} + bu(t) + e(t+1) + c\underline{e(t)}$$

The control signal $u(t)$ can be chosen, the underlined terms are known at time t and $e(t+1)$ is independent of all data available at time t. The controller that minimizes Ey^2 is thus given by

$$bu(t) = ay(t) - ce(t)$$

and the control error is $y(t+1) = e(t+1)$, i.e. the one step prediction error. The control law becomes $u(t) = \frac{a-c}{b} y(t)$.

Minimum Variance (Moving Average Control)

Process model

$$Ay(t) = Bu(t) + Ce(t)$$

Factor $B = B^+B^-$, solve (minimum G-degree solution)

$$AF + B^-G = C$$

$$Cy = AFy + B^-Gy = F(Bu + Ce) + B^-Gy = CFe + B^-(B^+Fu + Gy)$$

Control law and output are given by

$$B^+Fu(t) = -Gy(t), \qquad y(t) = Fe(t)$$

where $\deg F \geq$ pole excess of B/A

True minimum variance control $V = E\frac{1}{T}\int_0^T y^2(t)dt$

Properties of Minimum Variance Control

- The output is a moving average

$$y = Fe, \qquad \deg F \leq \deg A - \deg B^+.$$

  Easy to validate!
- Interpretation for $B^- = 1$ (all process zeros canceled), y is a moving average of degree $n_{pz} = \deg A - \deg B$. It is equal to the error in predicting the output n_{pz} step ahead.
- Closed loop characteristic polynomial is

$$B^+Cz^{\deg A - \deg B^+} = B^+Cz^{\deg A - \deg B + \deg B^-}.$$

- The sampling period an important design variable!
- Sampled zeros depend on sampling period. For a stable system all zeros are stable for sufficiently long sampling periods.

KJÅ, P Hagander, J Sternby Zeros of sampled systems. Automatica 20 (1), 31-38, 1984

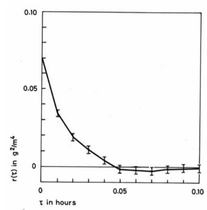
Minimum Variance Control

Process model

$$y_t + a_1y_{t-1} + ... = b_1u_{t-k} + ... + e_t + c_1e_{t-1} + ...$$
$$Ay_t = Bu_{t-k} + Ce_t$$

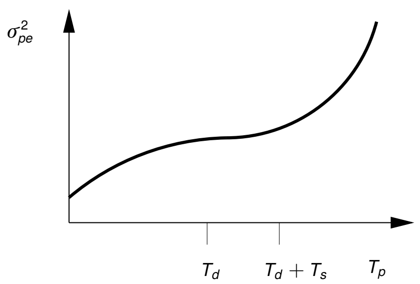
- Ordinary differential equation with time delay
- Disturbances are statinary stochastic process with rational spectra
- The predition horizon: tru delay and one sampling period
- Control law $Ru = -Sy$
- Output becomes a moving averate of white noise $y_{t+k} = Fe_t$
- Robustness and tuning

The output is a moving average $y_{t+j} = Fe_t$, which is easy to validate!



Performance ($B^- = 1$) and Sampling Period

Plot prediction error as a function of prediction horizon T_p

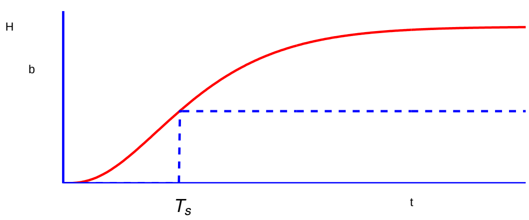


T_d is the time delay and T_s is the sampling period. Decreasing T_s reduces the variance but decreases the response time.

A Robustness Result

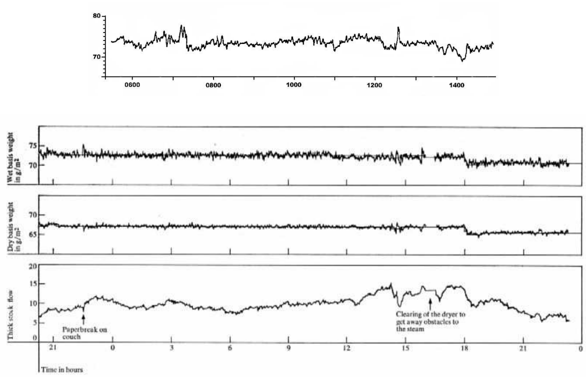
A simple digital controller for systems with **monotone** step response (design based on the model $y(k+1) = bu(k)$)

$$u_k = k(y_{sp} - y_k) + u_{k-1}, \qquad k < \frac{2}{g(\infty)}$$



Stable if $g(T_s) > \frac{g(\infty)}{2}$ kjå: Automatica **16** 1980, pp 313–315.

Back to Billerud - Performance of Minimum Variance Control

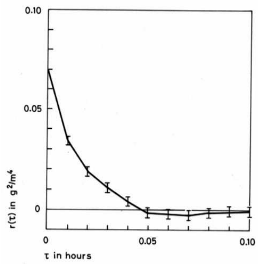


KJÅ, Computer control of a paper machine—An application of linear stochastic control theory. IBM Journal of research and development 11 (4), 389-405, 1967

IBM Scientific Symposium Control Theory and Applications 1964

Summary

- Regulation can be done effectively by minimum variance control
- Easy to validate because regulated output is a moving average of white noise!
- Robustness depends critically on the sampling period
- Sampling period is the **design variable**!
- The Harris Index and related criteria
- OK to assess but how about adaptation?



Adaptive Control

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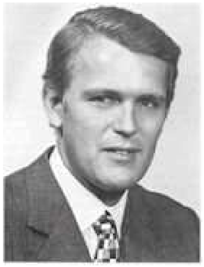
2. Self-oscillating Adaptive Control
3. Model Reference Adaptive Control
4. Estimation and Excitation
5. Minimum Variance Control
6. Self-Tuning Regulators
   - The self-tuning regulator
   - Properties
7. Learning and Dual Control
8. Applications
9. Related Fields
10. Summary

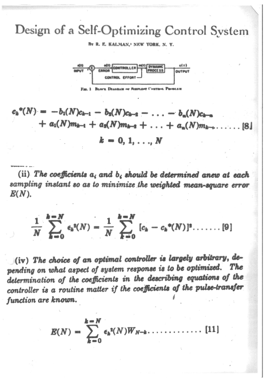
Rudolf Emile Kalman 1930-2016

- Born in Budapest 1930
- BS MIT 1953
- MS MIT 1954
- PhD Columbia University NY 1957
- IBM Research Yorktown Heights 1957-58
- RIAS Baltimore 1958-1964
- Professor Stanford 1964-1971
- Professor University of Florida 1971-1992
- Professor 1973 Professor ETH 1973-2016



Kalman's Self-Optimizing Regulator 1

R. E. Kalman, Design of a self optimizing control system. Trans. ASME 80,468– 478 (1958)



Design of a Self-Optimizing Control System

By R. E. Kalman, New York, N. Y.

$$c_k^*(N) = -b_1(N)c_{k-1} - b_2(N)c_{k-2} - \dots - b_n(N)c_{k-n} + a_1(N)m_{k-1} + a_2(N)m_{k-2} + \dots + a_n(N)m_{k-n} \quad [8]$$

$$k = 0, 1, \dots, N$$

(ii) *The coefficients a_i and b_i should be determined anew at each sampling instant so as to minimize the weighted mean-square error $E(N)$.*

$$\frac{1}{N}\sum_{k=0}^{k=N} \epsilon_k^2(N) = \frac{1}{N}\sum_{k=0}^{k=N} [c_k - c_k^*(N)]^2 \quad [9]$$

(iv) *The choice of an optimal controller is largely arbitrary, depending on what aspect of system response is to be optimized. The determination of the coefficients in the describing equations of the controller is a routine matter if the coefficients of the pulse-transfer function are known.*

$$E(N) = \sum_{k=0}^{k=N} \epsilon_k^2(N) W_{N-k} \quad [11]$$

Inspired by work at IBM Research and DuPont

Repeat the following two steps at each sampling instant

Step 1: Estimate the parameters

$$a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n$$

in the model (8)

Step 2: Use a control law that gives the shortest settling time for a step change in the reference signal

Remark: Many other methods can be used for parameter estimation and control design

Kalman's Self-Optimizing Regulator 2

R. E. Kalman, Design of a self optimizing control system. Trans. ASME 80,468– 478 (1958)

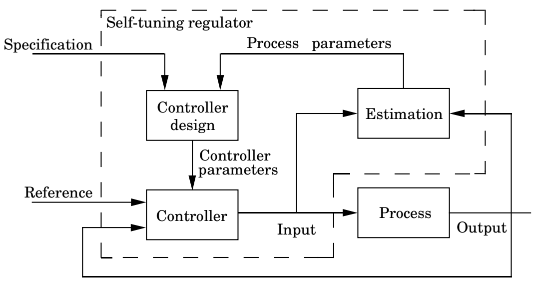
Remark on computations

In practical applications, however, a general-purpose digital computer is an expensive, bulky, extremely complex, and sometimes awkward pieze of equipment. Moreover, the computational capabilities (speed, storage capacity, accuracy) of even smaller commercially available general-purpose digital computers are considerably in access of what is demanded in performing the computations listed in the Appendix. For these reasons, a small special-purpose computer was constructed which could be externally digital and internally analog

Columbia University had a computer of this type

Unfortunately Kalman's regulator never worked!

The Self-Tuning Regulator



- Certainty Equivalence - Design as if the estimates were correct (Simon)
- Many control and estimation schemes

The Self-Tuning Regulator STR

Process model, estimation model and control law

$$y_t + a_1 y_{t-1} + \dots + a_n y_{t-n} = b_0 u_{t-k} + \dots + b_m u_{t-m} + e_t + c_1 e_{t-1} + \dots + c_n e_{t-n}$$

$$y_{t+k} = s_0 y_t + s_1 y_{t-1} + \dots + s_m y_{t-m} + r_0 (u_t + r_1 u_{t-1} + \dots r_n u_{t-\ell})$$

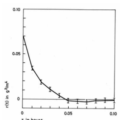
$$u_t + \hat{r}_1 u_{t-1} + \dots \hat{r}_n u_{t-\ell} = -(\hat{s}_0 y_t + \hat{s}_1 y_{t-1} + \dots + \hat{s}_m y_{t-m})/r_0$$

If estimate converge and $0.5 < r_0/b_0 < \infty$

$$r_y(\tau) = 0, \ \tau = k, k+1, \dots k+m+1$$

$$r_{yu}(\tau) = 0, \ \tau = k, k+1, \dots k+\ell$$

If degrees sufficiently large $r_y(\tau) = 0, \ \forall \tau \geq k$



- Converges to minimum variance control even if $c_i \neq 0$, Surprising!
- Automates identification and minimum variance control in about 35 lines of code.
- The controller that drives covariances to zero

KJÅ and B. Wittenmark. On Self-Tuning Regulators, Automatica **9** (1973),185-199
Björn Wittenmark, Self-Tuning Regulators. PhD Thesis April 1973

The Self-Tuning Regulator STR ...

Estimates of parameters in

$$y_t + a_1 y_{t-1} + \cdots + a_n y_{t-n} = b_0 u_{t-k} + \cdots + b_m u_{t-m} + e_t + c_1 e_{t-1} + \cdots + c_n e_{t-n}$$

gives unbiased estimates if $c_i = 0$. Very surprising that the self-tuner works with $c_i \neq 0$. Discovered empirically in simulations by Wieslander, Wittenmark and independently by Peterka.

Johan Wieslander and Björn Wittenmark An approach to adaptive control using real time identification. Proc 2^{nd} IFAC Symposium on Identification and Process Parameter Estimation, Prague 1970

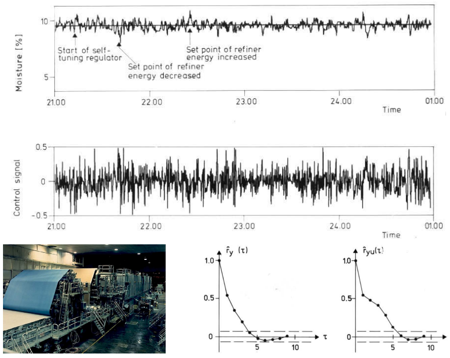
V. Peterka Adaptive digital regulation of noisy systems. Proc 2^{nd} IFAC Symposium on Identification and Process Parameter Estimation, Prague 1970

Proven in

KJÅ and Wittenmark, Björn", On Self Tuning Regulators, April 1972, Technical Report TFRT-7017, Department of Automatic Control, Lund University, Sweden. Presented at 5th IFAC World Congress Paris March 1972.

KJÅ and B. Wittenmark. On Self-Tuning Regulators, Automatica **9** (1973),185-199.

Test at Billerud 1973



U. Borisson and B. Wittenmark. An Industrial Application of a Self-Tuning Regulator, 4th IFAC/IFIP Symposium on Digital Computer Applications to Process Control 1974.
U. Borisson. Self-Tuning Regulators - Industrial Application and Multivariable Theory. PhD thesis LTH 1975.

Convergence Proof

Process model $Ay = Bu + Ce$

$$y_t + a_1 y_{t-1} + \cdots + a_n y_{t-n} = b_0 u_{t-k} + \cdots b_m u_{t-n} + e_t + c_1 e_{t-1} + \cdots + c_n e_{t-n}$$

Estimation model

$$y_{t+k} = s_0 y_t + s_1 y_{t-1} + \cdots + s_m y_{t-m} + r_0 (u_t + r_1 u_{t-1} + \cdots r_n u_{t-\ell})$$

Theorem: Assume that

- Time delay k of the sampled system is known
- Upper bounds of the degrees of A, B and C are known
- Polynomial B has all its zeros inside the unit disc
- Sign of b_0 is known

The the sequences u_t and y_t are bounded and the parameters converge to the minimum variance controller

G. C. Goodwin, P. J. Ramage, P. E. Caines, Discrete-time multivariable adaptive control. IEEE **AC-25** 1980, 449–456

Convergence Proof

Markov processes and differential equations

$$dx = f(x)dt + g(x)dw, \quad \frac{\partial p}{\partial t} = -\frac{\partial p}{\partial x}\left(\frac{\partial fp}{\partial x}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2} g^2 f = 0$$

$$\theta_{t+1} = \theta_t + \gamma_t \varphi e, \quad \frac{d\theta}{d\tau} = f(\theta) = E\varphi e$$

Method for convergence of recursive algorithms. Global stability of STR ($Ay = Bu + Ce$) if $G(z) = 1/C(z) - 0.5$ is SPR

L. Ljung, Analysis of Recursive Stochastic Algorithms IEEE Trans **AC-22** (1967) 551–575.

Converges locally if $\Re C(z_k) > 0$ for all z_k such that $B(z_k) = 0$

Jan Holst, Local Convergence of Some Recursive Stochastic Algorithms. 5th IFAC Symposium on Identification and System Parameter Estimation, 1979

General convergence conditions

Lei Gui and Han-Fu Chen, The Åström-Wittenmark Self-tuning Regulator Revisited and ELS-Based Adaptive Trackers. IEEE Trans **AC36:7** 802–812.

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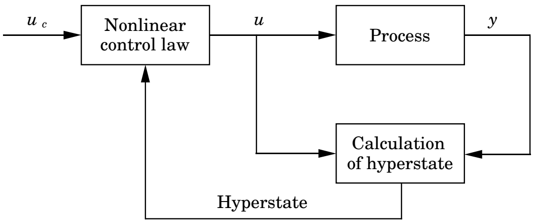
Dual Control - Alexander Aronovich Fel'dbaum

Control should be probing as well as directing

- Dual control theory I A. A. Feldbaum Avtomat. i Telemekh., 1960, 21:9, 1240–1249
- Dual control theory II A. A. Feldbaum Avtomat. i Telemekh., 1960, I21:11, 1453–1464
- R. E. Bellman Dynamic Programming Academic Press 1957
- Stochastic control theory - Adaptive control
- Decisionmaking under uncertainty - Economics
- Optimization Hamilton Jacobi Bellman

1913 – 1969

Dual Control - Feldbaum



- No certainty equivalence
- Control should be directing as well as investigating!
- Intentional perturbation to obtain better information
- Conceptually very interesting
- Unfortunately very complicated

Helmerson KJA
Is it time for a second look?

The Problem

Consider the system

$$y_{t+1} = y_t + bu_t + e_{t+1}$$

where e_t is a sequence of independent normal $(0, \sigma^2)$ random variables and b a constant but unknown parameter with a normal $\hat{b}$, $P(0)$ prior or a random wai.
Find a control llaw such that u_t based on the information available at time t

$$\mathcal{X}_t = y_t, y_{t-1}, \ldots, y_0, u_{t-1}, u_{t-2}, \ldots, u_0,$$

that minimizes the cost function

$$V = E \sum_{k=1}^{T} y^2(k).$$

KJÅ and A. Helmersson. *Dual Control of an Integrator with Unkown Gain*, Computers and Mathematics with Applications 12:6A, pp 653–662, 1986.

The Hamilton-Jacobi-Bellman Equation

The solution to the problem is given by the Bellman equation

$$V_t(\mathcal{X}_t) = E_{\mathcal{X}_t} \min_{u_t} E\Big(y_{t+1}^2 + V_{t+1}(\mathcal{X}_{t+1})\Big|\mathcal{X}_t\Big)$$

The state is $\mathcal{X}_t = y_t, y_{t-1}, y_{t-2}, \dots, y_0, u_{t-1}, u_{t-2}, \dots, u_0$. The derivation is general applies also to

$$x_{t+1} = f(x_t, u_t, e_t)$$
$$y_t = g(x_t, u_t, v_t)$$
$$\min E \sum q(x_t, u_t)$$

How to solve the optimization problem?
The curse of dimensionality: X_t has high dimension

A Sufficient Statistic - Hyperstate

It can be shown that a sufficient statistic for estimating future outputs is y_t and the conditional distribution of b given $\mathcal{X}_t$. In our setting the conditional distribution is gaussian $N\big(\hat{b}_t, P_t\big)$

$$\hat{b}_t = E(b|\mathcal{X}_t), \qquad P_t = E[(\hat{b}_t - b)^2|\mathcal{X}_t]$$
$$\hat{b}_{t+1} = \hat{b}_t + K_t[y_{t+1} - y_t - \hat{b}_t u_t] = \hat{b}_t + K_t e_{t+1}$$
$$K_t = \frac{u_t P_t}{\sigma^2 + u_t^2 P_t}$$
$$P_{t+1} = [1 - K_t u_t]P_t = \frac{\sigma^2 P_t}{\sigma^2 + u_t^2 P_t}$$

In our particular case the conditional distrubution depens only on by y, $\hat{b}$ and P - a significant reduction of dimensionality!

The Bellman Equation

$$V_t(\mathcal{X}_t) = E_{\mathcal{X}_t} \min_{u_t} E\Big(y_{t+1}^2 + V_{t+1}(\mathcal{X}_{t+1})\Big|\mathcal{X}_t\Big)$$

Use hyperstate to replace $\mathcal{X}_t = y_t, y_{t-1}, y_{t-2}, \dots, y_0, u_{t-1}, u_{t-2}, \dots, u_0$ with $y_t, \hat{b}_t, P_t$. Introduce

$$V_t(y_t, \hat{b}_t, P_t) = \min_{u_t}\Big(E\sum_{k=t+1}^{T} y_k^2\Big|y_t, \hat{b}_t, P_t\Big)$$

$$y_{t+1} = y_t + \hat{b}_t u_t + e_{t+1}, \quad \hat{b}_{t+1} = \hat{b}_t + K_t e_{t+1}, \quad P_{t+1} = \frac{\sigma^2 P_t}{\sigma^2 + u_t^2 P_t}$$

and the Bellman equation becomes

$$V_t(y, \hat{b}, P) = \min_u E\Big(y_t^2 + V_{t+1}\big(y_{t+1}, \hat{b}_{t+1}, P_{t+1}\big)|y, \hat{b}_t, P_t\Big)$$

Short Time Horizon - 1 Step Ahead

Consider situation at time t and look one step ahead

$$V_{T-1}(y, \hat{b}, P) = \min_u E\sum_{k=T}^{T} y_k^2 = \min_u y_T^2$$
$$y_T = y_{T-1} + bu_{T-1} + e_T$$

We know y_t have an estimate $\hat{b}$ of b with covariance P

$$V_T(y, \hat{b}, P) = \min_u Ey_T^2 = \min_u\Big((y + \hat{b}u)^2 + u^2 P + \sigma^2\Big)$$
$$= \min_u\Big(y^2 + 2y\hat{b}u + u^2(\hat{b}^2 + P) + \sigma^2\Big) = \sigma^2 + \frac{Py^2}{\hat{b}^2 + P}$$

where minimum occurs for

$$u = -\frac{\hat{b}}{\hat{b}^2 + P}y \quad \Rightarrow \quad u = -\frac{1}{\hat{b}}y \quad \text{as } P \to 0$$

These control laws are called cautious control and certainty equivalence control (Herbert Simon).

The Solution and Scaling

$$V_t(y, \hat{b}, P) = \min_u\Big((y + \hat{b}u)^2 + \sigma^2 + u^2 P + V_{t+1}\big(y_{t+1}, \hat{b}_{t+1}, P_{t+1}\big)\Big)$$
$$V_T(y, \hat{b}, P) = \sigma^2 + \frac{Py^2}{\hat{b}^2 + P}$$

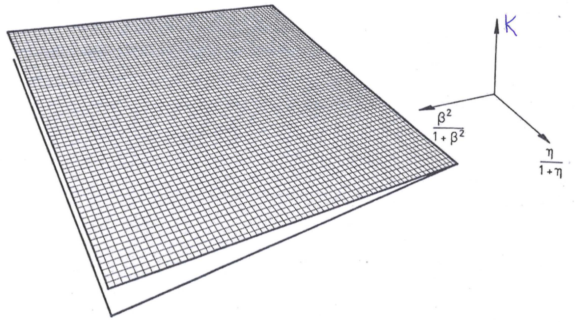
Iterate backward in time. An important observation, $V_T(y, \hat{P}, P)$ does not depend on y, state is thus two-dimensional!!
Scaling

$$\eta = \frac{y}{\sigma}. \qquad \beta = \frac{\hat{b}}{\sqrt{P}}, \qquad \mu = \frac{u\sqrt{P}}{\sigma}$$

Introduce
Two functions: the value function and the policy function

Controller Gain - Cautious Control

$$u = -\frac{\hat{b}}{\hat{b}^2 + P}y = Ky, \eta = \frac{y}{\sigma}. \qquad \beta = \frac{\hat{b}}{\sqrt{P}},$$



Solving the Bellman Equation Numerically

The scaled Bellman equation

$$W_t(\eta, \beta) = \min_\mu U_t(\eta, \beta, \mu), \qquad \varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

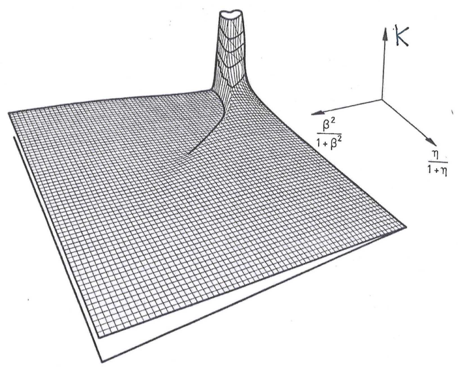
where

$$U_t(\eta, \beta, \mu) = (\eta + \beta\mu)^2 + 1 + \mu^2$$
$$+ \int_{-\infty}^{\infty}\Big(W_{t+1}(\eta + \beta\mu + \epsilon\sqrt{1 + \mu^2}, \beta\sqrt{1 + \mu^2} + \mu\epsilon\Big)\varphi(\epsilon)d\epsilon$$

Solving minimization gives control law $\mu = \Pi(\eta, \beta)$, $\mu = \frac{u\sqrt{P}}{\sigma}$, $u = \frac{\sigma}{\sqrt{P}\,\Pi(\eta,\beta)}$
Numerics:

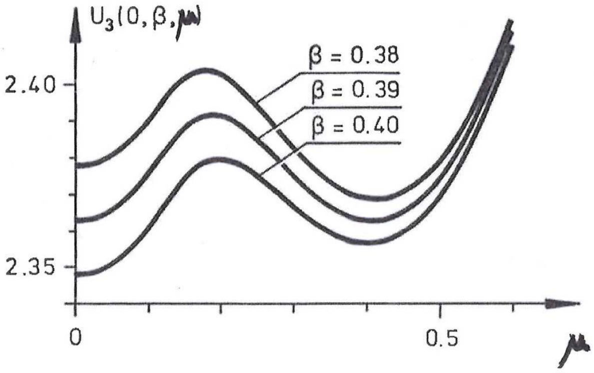
- Transform to the interval $(0\,1)$, quantize U function 128×128
- Store the a gridded version of the function $U(\eta, \beta, mu)$
- Evaluate the function $W(\eta, \beta, \mu)$ by extrapolation, and numeric integration
- Minimize $W(\eta, \beta, \mu)$ with respet to μ

Controller Gain - 3 Steps



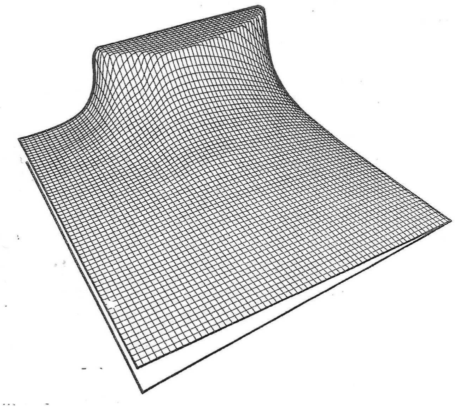
$K(\eta, \beta)$ larger than 3 not shown

Understanding Probing

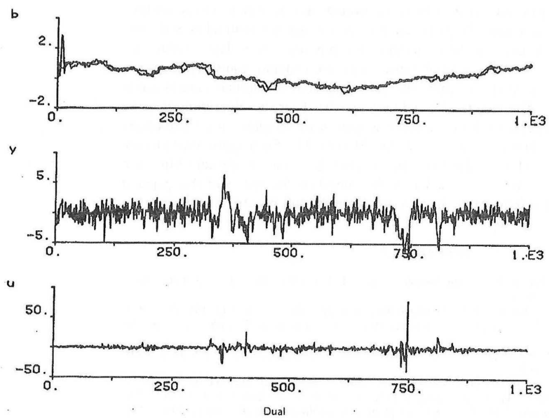


Notice jump!!

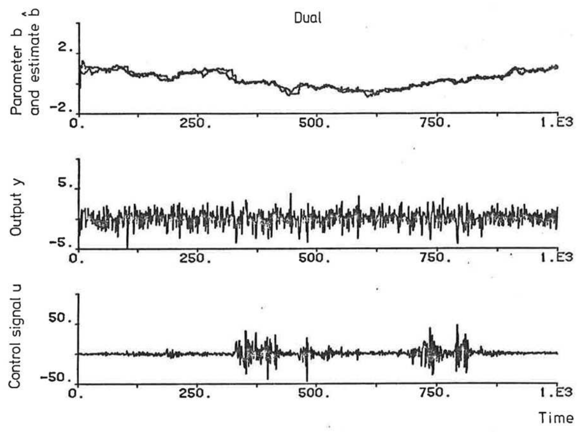
Controller gain for 30 Steps



Cautious Control - Drifting Parameters

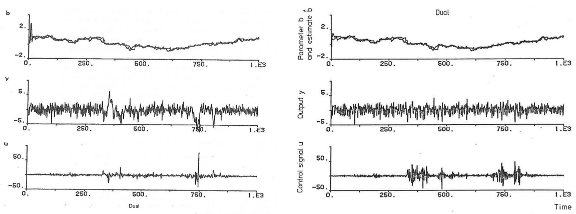


Dual Control - Drifting Parameters



Comparison

Cautious Control   Dual Control



Summary

- Use dynamic programming (move optimization inside the integral)

$$\min_{\mathcal{U}_t} E \sum_{k=t+1}^{T} y_t^2 = \min_{\mathcal{U}_t} E_{\mathcal{X}_t} E\Big(\sum_{k=t+1}^{T} y_k^2 \Big| \mathcal{X}_t\Big)$$

$$= E_{\mathcal{X}_t} \min_{\mathcal{U}_t} E\Big(\sum_{k=t+1}^{T} y_k^2 \Big| \mathcal{X}_t\Big) = E_{\mathcal{X}_t} \min_{u_t} \min_{\mathcal{U}_{t+1}} E\Big(\sum_{k=t+1}^{T} y_k^2 \Big| \mathcal{X}_t\Big)$$

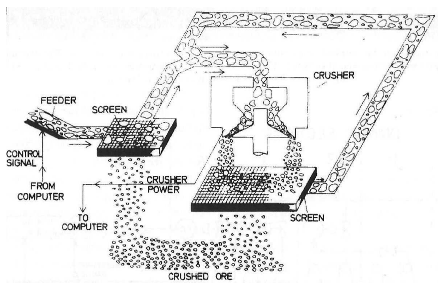
$$= E_{\mathcal{X}_t} \min_{u_t} E\Big(y_{t+1}^2 + \min_{\mathcal{U}_{t+1}} \sum_{k=t+2}^{T} y_k^2 \Big| \mathcal{X}_t\Big)$$

- State reduction $y, \hat{b}, P \rightarrow \eta = y/\sigma, \beta = \hat{b}/\sqrt{P}$
- Certainty equivalence, cautious and dual
- K. J. Åström Control of Markov Chains with Incomplete State Information JMAA, 10, pp. 174–205, 1965.
- K. J. Åström and A. Helmersson. *Dual Control of an Integrator with Unkown Gain*, Computers and Mathematics with Applications 12:6A, pp 653–662, 1986.

Adaptive Control

1. Introduction
2. Self-oscillating Adaptive Control
3. Model Reference Adaptive Control
4. Estimation and Excitation
5. Minimum Variance Control
6. Self-Tuning Regulators
7. Learning and Dual Control
8. Applications
9. Related Fields
10. Summary

Control of Orecrusher 1973



Forget Physics! - Hope an STR can work!

Power increased from 170 kW to 200 kW

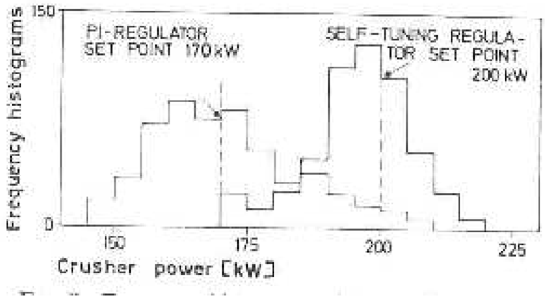
R. Syding, Undersökning av Honeywells adaptiva reglersystem. MS Thesis LTH 1975

U. Borisson, and R. Syding, Self-Tuning Control of an Ore Crusher, Automatica 1976, **12:1**, 1–7

Control over Long Distance 1973

Plant in Kiruna, computer in Lund. Distance Lund-Kiruna 1400 km, home-made modem (Leif Andersson), supervision over phone, sampling period 20s.

Results



Significant improvement of production 15 %!

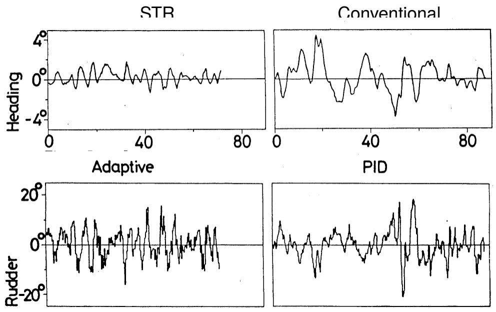
Steermaster

Kockums

- Ship dynamics
- SSPA Kockums
- Full scale tests on ships in operation

NORTHROP GRUMMAN

Ship Steering - 3% less fuel consumption



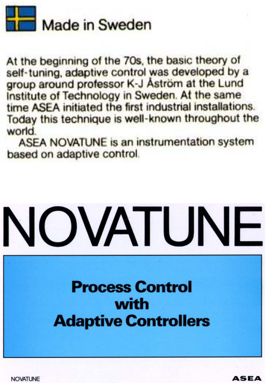
C. Källström Identification and Adaptive Control Applied to Ship Steering PhD Thesis Department of Automatic Control, Lund University, Sweden, April 1979.
C. Källström, KJÅ, N. E. Thorell, J. Eriksson, L. Sten, Adaptive Autopilots for Tankers, Automatica, **15** 1979, 241-254

ABB

- ASEA Innovation 1981
- DCS system with STR
- Grew quickly to 30 people and 50 MSEK in 1984
- Strong grpup
- Wide range of applications
- Adaptive feedforward

Incorporated in ABB Master 1984 and later in ABB 800xA

- Difficult to transfer to standard sales and commision workforce



Arthur D . Little Innovation at ASEA. 1985

First Controll

1. Winner of IEEE CSS Control Technology Award
FIRST CONTROL SYSTEMS AB
Ängsgärdsg.4, S-721 30 Västerås, Sweden Ph +46 21 417880 Fax +46 21 412810

- Gunnar Bengtsson
- Founder of ABB Novatune
- Rolling mills
- Continuous casting
- Semiconductor manufacturing
- Microcontroller XC05IX
  - Raspberry pie, linux
  - Robust adaptive control
  - Grahpical programming
  - Modelica simulation

Relay Auto-Tuner Tore

- One-button tuning
- Automatic generation of gain schedules
- Adaptation of feedback and feedforward gains
- Many versions
  - Single loop controllers
  - DCS systems
- Robust
- Excellent industrial experience
- Large numbers

Adaptive Control

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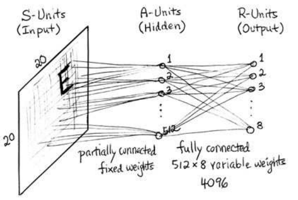
   - Adaptive Signal Processing
   - Neural Networks
   - Boxes
10. Summary

The Perceptron - Rosenblatt

- PhD Experimental Psychology, Cornell 1956: broad interests mathematics ...
- Cornell Aeronautical Lab 1957-59
- Probabilistic model for information storage in the brain
- One layer neural network classifier

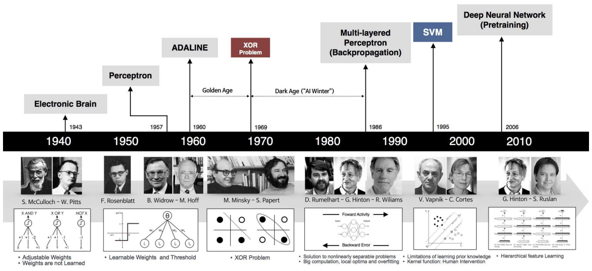


Rosenblatt, Frank (1957). "The Perceptron—a perceiving and recognizing automaton". Report 85-460-1. Cornell Aeronautical Laboratory.

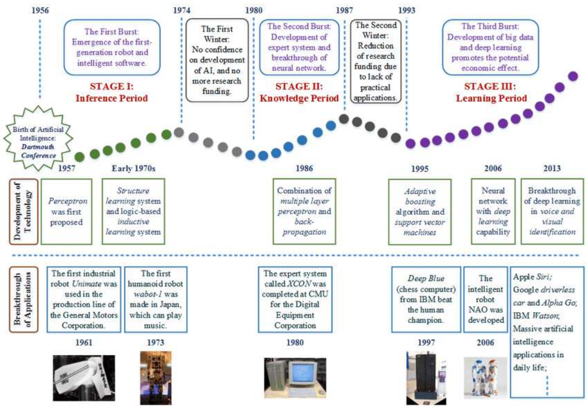
Addaline - Bernard Widrow & Ted Hoff (Intel 4004)

- Adaline - ADAptive LInear Neuron - Single layer neuron network $y = \mathrm{sign} \sum_{j=1}^{m} w_{ij} u_j$, least Mean Square (LMS)
- B. Widrow and M. E. Hoff, "Adaptive Switching Circuits," 1960 IRE WESCON Convention Record, 1960, pp. 96-104.
- Analog implementation potentiometers
- Separating hyperplane
- Madaline a multilayer version

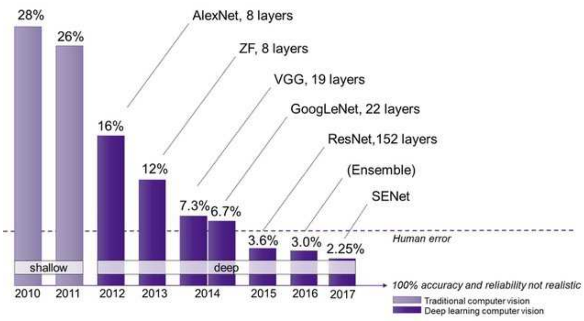
Neural Networks



Development of Learning



Development of Image Recognition



Very rapid development because of good test batch and competing algorithms

BOXES 1960 - Donald Michie

- Cryptography Bletchley Park
- Director Department of Machine Intelligence and Perception, University of Edinburgh (previously the Experimental Programming Unit 1965)
- Quantize state in boxes
- Each box stores information about control actions taken and the performance
- Local demon and global demons
- Used for playing games like Tick-Tack-Toe and broom balancing

D. Michie and R. A. Chambers, "BOXES: An experiment in adaptive control," in Machine Intelligence 2, E. Dale and D. Michie, Eds. Edinburgh: Oliver and Boyd, (1968), pp. 137–152.

Adaptive Control

Summary

- A glimpse of an interesting and useful field of control
- Nonlinear system, not trivial to analyse and design
- Several good algorithms: self-oscillating, MRAS and STR
  - Why is not STR more widely used?
  - Natural candidate for regulation in noisy environment?
  - Computer systems?
- A number of successful industrial applications
- Currently renewed interest because of connections to learning

KJÅ and B. Wittenmark. Adaptive Control. Second Edition. Dover 2008.