Exercise Session 2

1. Describe your results on Handin 1.

2. Sketch the Nichols curves for the following systems

$$\frac{1}{s(s+1)(s+10)}, \quad \frac{1}{1-s}, \quad \frac{\exp(-s)}{1+s}, \quad \frac{1-s}{s(1+s)}, \quad \frac{1}{s^2+2\zeta s+1}, \quad (\zeta \text{ small})$$

For what feedback gains is the closed loop system stable?

3. Plot the root-loci for the following systems

$$\frac{s}{s^2-1}, \quad \frac{(s+1)^2}{s^3}, \quad \frac{1}{s(s^2+2\zeta s+1)}, \quad (\zeta \text{ small})$$

4. Transform the systems in the previous two exercises to discrete time using e.g. zero-order hold sampling with $h = 0.02$ (`c2d(sys,h)`) and redo the exercises.

5. The following frequency domain based code can be used (why?) to simulate the step response of the system $1/(s+1)$.

```
N=2^12; dt=0.01; T=N*dt; dw=2*pi/T;
t = dt*(0:N-1);
omega = -pi/dt:dw:(pi/dt-dw);
u = [ones(1,N/2) zeros(1,N/2)];
U = fft(u);
P = 1./(i*omega+1);
y = ifft(fftshift(P).*U);
plot(t+dt/2,real(y),'-bx');
hold on;grid on
plot(t,1-exp(-t),'-ro')
```

Simulate the step response of the open loop system $P(s) = \exp(-\sqrt{s})$ and of the closed loop system under PI-control with $k_p = k_i = 1$ (you might need to tune N and dt).

Compare the rise time to 50% and the settling times to 99% of the final value for open loop vs closed loop control (Answer: Rise time around 1 second in both cases. Settling time 3200 seconds (open) vs 14 seconds (closed)!).

Also suggest some improved PI parameters giving a faster step response. How small rise time is reasonable to achieve with PI control?

6. Consider the transfer function $P(s) = \frac{s+p}{s-p}e^{-sL}$ show that the phase lag is always more than π if $pL > 2$.

7. Derive the limitations for the sensitivity and the complementary sensitivity for systems with a complex pole pair in the RHP

8. Define the *average residence time* T_{ar} as the first moment of the impulse response, i.e. $T_{ar} = \int_0^\infty th(t)dt / \int_0^\infty h(t)dt$. A possible definition of *rise time* T_r that is sometimes used is given by

$$T_r := \frac{\int_0^\infty (t - T_{ar})^2 h(t)dt}{\int_0^\infty h(t)dt}.$$

a) Show that $T_{ar} = -\dfrac{P'(0)}{P(0)}$ and $T_r = \dfrac{P''(0)}{P(0)} - T_{ar}^2$

b) Calculate T_{ar} and T_r for e^{-sT}, $1/(1+sT)^n$ and $(1-e^{-sh})/(sh)$.

c) Consider a system composed of n cascaded systems, each with $h_k(t) \geq 0$ (monotone step responses) and residence time $T_{ar,k}$ and rise time of $T_{r,k}$ respectively. Show that the delay time and rise time of the cascaded system equals

$$T_{ar,tot} = T_{ar,1} + \ldots + T_{ar,n},$$
$$T_{r,tot} = (T_{r,1}^2 + \ldots + T_{r,n}^2)^{\frac{1}{2}}.$$

9. Show that the system $G(s) = \exp(-\sqrt{s})$ satisfies Bode's relations between amplitude and phase for stable minimum phase systems

$$v(i\omega_0) = \frac{2\omega_0}{\pi} \int_0^\infty \frac{u(i\omega) - u(\omega_0)}{\omega^2 - \omega_0^2} d\omega$$

with $u + iv = \log |G| + i \arg G$.