IQC toolbox

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IQC - Integral Quadratic Constraints

- A unifying framework for systems analysis
- Generalizes stability theorems such as small gain theorem and passivity theorem
- Generalizes many concepts from robust control analysis
- (Fairly) easy to build computer tools (convex optimization)

Outline

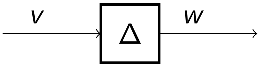
- Some theory on IQC
- IQCβ toolbox
- Live demo

ICQ - Theory

For a comprehensive review of the theory, see:

Megretski, Alexandre, and Anders Rantzer. "System analysis via integral quadratic constraints." *Automatic Control, IEEE Transactions on* 42.6 (1997): 819-830.

IQC - Theory

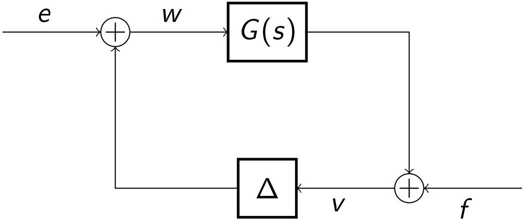


Definition (IQC)

An operator Δ is said to satisfied the IQC defined by Π, if for all input-output pairs $w = \Delta(v)$, where $v \in L_2[0, \infty)$, the integral inequality

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix} d\omega \geq 0.$$

IQC - Theory



Definition (Well-posedness and stability)

A feedback interconnection of G and Δ is well-posed if the map $(v, w) \to (e, f)$ has a causal inverse. The interconnection is stable, in addition, the inverse is bounded, i.e., if there exists a constant $C > 0$ such that

$$\int_0^T (|v|^2 + |w|^2)dt \leq C \int_0^T (|f|^2 + |c|^2)dt,$$

for any $T > 0$.

ICQ - Theory

Theorem

Let $G(s) \in RH_\infty$, and let Δ be a bounded causal operator. Assume that

1. *for every $\tau \in [0, 1]$, the interconnection of G and $\tau\Delta$ is well-posted;*
2. *for every $\tau \in [0, 1]$, the IQC defined by Π is satisified by $\tau\Delta$;*
3. *there exists $\epsilon > 0$, such that*

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} \leq -\epsilon I, \quad \forall \omega \in \mathbb{R}.$$

Then, the feedback interconnection of G and Δ is stable.

Proof.

See [Megretski and Rantzer 1997]. □

IQC - Small gain theorem

Let

$$\Pi(j\omega) = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

Then the small-gain theorem is recovered.

IQC - Passivity theorem

Let

$$\Pi(j\omega) = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Then the passivity theorem is recovered.

Some other ICQs

Δ	Π	Constraint
$\lVert\Delta(j\omega)\rVert \leq 1$	$\begin{bmatrix} x(j\omega)I & 0 \\ 0 & -x(j\omega)I \end{bmatrix}$	$x(j\omega) \geq 0$
$\Delta \in [-1,1]$	$\begin{bmatrix} X(j\omega) & Y(j\omega) \\ Y(j\omega)^* & -X(j\omega) \end{bmatrix}$	$X = X^* \geq 0,\ Y = -Y^*$
$\Delta(t) \in [-1,1]$	$\begin{bmatrix} X & Y \\ Y^T & -X \end{bmatrix}$	$x(j\omega) \geq 0$
$\Delta(s) = e^{-\theta s} - 1$	$\begin{bmatrix} x(j\omega)\rho(\omega)^2 & 0 \\ 0 & -x(j\omega)I \end{bmatrix}$	$\rho(\omega) = 2 \max_{\lvert\theta\rvert \leq \theta_0} \sin(\theta\omega/2)$

The IQCβ Toolbox

- Can be downloaded from: `http://actrol.ee.nsysu.edu.tw/`
- Mainly maintained by Chung-Yao (Isaac) Kao
- For a paper about it, see:
  Kao, Chung-Yao, Alexandre Megretski, Ulf T. Jönsson, and Anders Rantzer. *A MATLAB toolbox for robustness analysis.* In Computer Aided Control Systems Design, 2004 IEEE International Symposium on, pp. 297-302.

IQCβ Toolbox - Idea

For a given parametrized IQC $\Pi_{k,\lambda}$, replace the operator Δ with the set-values function

$$\mathcal{D}_{k,\lambda}(v_k) = \left\{ w_k \in L_2[0,\infty) \mid \left\langle \begin{bmatrix} v_k \\ w_k \end{bmatrix}, \Pi_{k,\lambda} \begin{bmatrix} v_k \\ w_k \end{bmatrix} \right\rangle \right\}$$

The goal is not just to check stability, stability comes from solving a robust control problem, e.g., does there exists a $\gamma > 0$ such that

$$\int_0^\infty (|z|^2 - \gamma |e_0|^2) dt \leq 0.$$

The toolbox, fixes a γ, searches for a feasible λ.

DEMO TIME!

(hopefully it will work...)