Session 1

Linear Control Systems. Examples. Linearization. Transition Matrix.

Reading Assignment

Rugh (1996 edition) chapters 1-4 and scan Chapter 20 until Example 20.7. The main new thing is to do linearization along a trajectory rather than at an equilibrium, and the definition and properties of the transition matrix $\Phi(t, \tau)$.

Exercise 1.1 = Rugh 1.9

Exercise 1.2 = Rugh 1.20 (spectral norm)

Exercise 1.3 = Consider the communications satellite example in Lecture 1. Show that the linearization of the nonlinear model along the circular equatorial orbit (nominal solution) yields the linearized system given in the lecture slides.

Exercise 1.4 = Rugh 3.1

Exercise 1.5 = Rugh 3.10

Exercise 1.6 = Rugh 3.12 (hint: use Lemma 3.2)

Exercise 1.7 = Rugh 20.6

Hand in problems - to be handed in at exercise session

Handin 1.1 Given

$$A(t) = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ 0 & A_{22}(t) \end{bmatrix},$$

show that the following holds

$$\Phi(t, t_0) = \begin{bmatrix} \Phi_{11}(t, t_0) & \Phi_{12}(t, t_0) \\ 0 & \Phi_{22}(t, t_0) \end{bmatrix},$$

where $\frac{d}{dt}\Phi_{ii}(t, t_0) = A_{ii}\Phi_{ii}(t, t_0)$, for $i \in \{1, 2\}$. Check that

$$A(t) = \begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix}$$

yields the following state transition matrix

$$\Phi(t, 0) = \begin{bmatrix} e^{-t} & (e^{t} - e^{-t})/2 \\ 0 & e^{-t} \end{bmatrix}.$$

Handin 1.2 Rugh 3.5

Handin 1.3 Rugh 3.6. Use a symbolic computation software such as Maple or the symbolic toolbox in MATLAB.