Lecture 7 - Vehicle/Bicycle Dynamics

Goal: To review modeling of vehicles, particularly bicycles To discuss codesign of process and control system. To discuss factors that make system easy or difficult to control.

1. Introduction
2. Bicycle Dynamics
3. Stabilization
4. Rear Wheel Steering
5. Path Dynamics
6. More Elaborate Models
7. Summary

Thank you Jonas for a super job with the computer exercises!!

Introduction

- So far we have discussed single domain modeling
- Vehicles represent a wide range of physics
- Vehicles represent a large application area for control
- Flight control and Space have been strong driving forces
- Control often mission critical

  Satellites

  Flight control of unstable aircrafts
- Codesign of process and controller often a key issue
- Automotive applications are increasing rapidly
- Lots of examples we will only scratch the surface

A Rich Variety

- Bicycles
- Motorcycles
- Trains and subways

  Drive systems

  Start stop breaking

  Magnetic levitation

  Tilt

  Electric systems
- Ships
- Surface effects
- Helicopters
- Space Ships
- Cars

  Engine control

  Traction control ABS

  Idle speed control

  Cruise control

  Distance keeping

  Platooning
- Aircrafts

  Flight control

  Air traffic control

  Automatic landing

  Autonomous flight

Vehicle Control Problems

- Many similarities

  Control the velocity

  Generation of control forces. Forces forward or aft?

  Loss of controllability; slip and stall

  Stabilization

  Interaction, RHP poles and zeros
- Many special features

  Helicopters

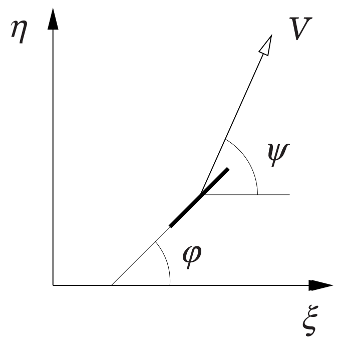
  Engine control

  Chassis control (active damping)

Kinematics

A common core for all vehicle control problems!

$$\frac{d\xi}{dt} = V\cos\phi$$
$$\frac{d\eta}{dt} = V\sin\phi$$
$$\frac{d\psi}{dt} = u$$



Vehicles differ in mechanisms for generating the control u.

The angle $\alpha = \psi - \varphi$ plays an essential role: angle of attack, side-slip angle etc.

Wilbur Wright 1901

"Men know how to construct airplanes.
Men also know how to build engines.
Inability to *balance and steer* still confronts
students of the flying problem.
When this one feature has been worked out,
the age of flying will have arrived, for
all other difficulties are of minor importance."

A beautiful example of process and controller codesign.
Control was mission critical!

Stability and Controllability

- Controllability in a broad sense
- Wright Brothers rejected the dogma that the aircraft should be inherently stable.
- Minorsky 1922: It is an old adage that a stable ship is difficult to steer.
- Integrated process and control design.

  Potentially a very BIG value add!

  Sometimes systems cannot be build without control.
- The cardinal sin of control.

The Cardinal Sin of Control

Most control theory starts like this. Assume that we have a process described by ... .This may easily lead you to believe that the process is fixed. In reality it is not and it never should be because it leads to sub-optimization of systems.

- Never ever believe that the process is given!
- When faced with a difficult control problem always try to change the process to make it easy
- An educational need

  To recognize the easy problems

  To understand what causes difficulties

Lecture 7 - Vehicle/Bicycle Dynamics

Bicycle Dynamics

- A nice prototype example
- You all have practical experience of the system
- Many complicated phenomena

  Gyroscopic forces
  Tire friction
  The front fork
- Goal

  The bicycle as a controlled object
  What factors are important?
  How do we balance?
- Much literature and much confusion!
- Thanks to Richard Klein

Questions to Think About

Use your own experience to think about the following questions.

1. Do you stabilize the bike by steering or leaning?
2. Do you stabilize all the time?
3. Is it harder to stabilize at low or high speeds?

Think about this when you go home tonight!

The Audience is Thinking!!!

Introduction

Goals:

- Develop a simple bicycle models that catches the essentials: stabilization, path control, process and control design.

Key issues:

- How complicated models are needed?
- The choices of simplification are nontrivial!

  Develop many different models

  Experiments and analysis by Richard Klein at Illinois

An approximation:

- Control through the handle bar

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Kinematics

Angular velocity of coordinate system

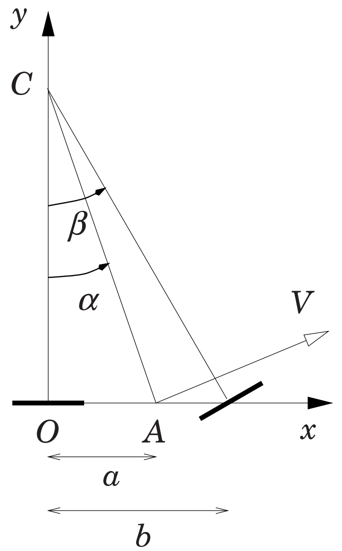
$$\frac{d\varphi}{dt} = \frac{V_0}{r_0} = \frac{V}{r} = \frac{V_0}{b}\tan\beta$$

Velocity of center of mass

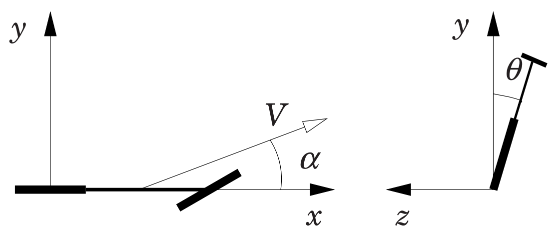
$$V = r\frac{d\varphi}{dt} = \frac{aV_0\tan\beta}{b\sin\alpha}$$

$y-$ component of velocity

$$V_y = V\sin\alpha = \frac{aV_0}{b}\tan\beta$$



Kinematics and Tilt



$$J_p\frac{d^2\theta}{dt^2} = mg\ell\sin\theta + m\ell\Big(\frac{V^2}{r}\cos\alpha + \frac{dV_y}{dt}\Big)\cos\theta$$

$$V_y = V\sin\alpha = \frac{aV_0}{b}\tan\beta$$

$$\frac{dV_y}{dt} = \frac{aV_0}{b}\frac{1}{\cos^2\beta}\frac{d\beta}{dt}$$

$$V = \frac{aV_0\tan\beta}{b\sin\alpha}$$

$$r_0 = \frac{b}{\tan\beta}$$

$$r = \frac{a}{\sin\alpha}$$

Bike 1 - A Simple Bicycle Model

$$J_p \frac{d^2\theta}{dt^2} = mg\ell \sin\theta + m\ell\Big(\frac{V^2}{r}\cos\alpha + \frac{dV_y}{dt}\Big)\cos\theta$$

$$V = \frac{aV_0 \tan\beta}{b\sin\alpha} = \frac{V_0}{\cos\alpha}$$

$$\frac{dV_y}{dt} = \frac{aV_0}{b}\frac{1}{\cos^2\beta}\frac{d\beta}{dt}$$

But

$$\frac{V^2\cos\alpha}{r} = \frac{V}{r}\frac{aV_0\tan\beta}{\tan\alpha} = \frac{V_0}{r_0}\frac{aV_0\tan\beta}{b\tan\alpha} = \frac{V_0^2}{b}\tan\beta$$

Hence

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\sin\theta + \frac{m\ell V_0^2\cos\theta}{bJ_p}\Big(\tan\beta + \frac{a}{V_0\cos^2\beta}\frac{d\beta}{dt}\Big)$$

Physical Interpretation

$$J_p \frac{d^2\theta}{dt^2} = mg\ell \sin\theta + m\ell\Big(\frac{V^2}{r}\cos\alpha + \frac{dV_y}{dt}\Big)\cos\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\sin\theta + \frac{m\ell V_0^2\cos\theta}{bJ_p}\Big(\tan\beta + \frac{a}{V_0\cos^2\beta}\frac{d\beta}{dt}\Big)$$

Essentially a pendulum equation. Two types of forces controlled through β, the handle bar angle

- Centrifugal force proportional to $V_0^2\tan\beta$
- Acceleration force proportional to $V_0\dfrac{d\beta}{dt}$
- Which term is most important at low speed?

Normalization

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\sin\theta + \frac{m\ell V_0^2\cos\theta}{bJ_p}\Big(\tan\beta + \frac{a}{V_0\cos^2\beta}\frac{d\beta}{dt}\Big)$$

Introduce

$$\omega_0 = \frac{mg\ell}{J_p}, \qquad \gamma = \frac{V_0^2}{bg}, \qquad \kappa = aV_0$$

Physical interpretations! Introduce $1/\omega_0$ as time unit

$$\frac{d^2\theta}{dt^2} = \sin\theta + \gamma\Big(\tan\beta + \frac{\kappa}{\cos^2\beta}\frac{d\beta}{dt}\Big)\cos\theta$$

Two parameters gain κ and time constant κ

Linearization

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\sin\theta + \frac{m\ell V_0^2\cos\theta}{bJ_p}\Big(\tan\beta + \frac{a}{V_0\cos^2\beta}\frac{d\beta}{dt}\Big)$$

Linearize

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\theta + \frac{m\ell V_0^2}{bJ_p}\Big(\beta + \frac{a}{V_0}\frac{d\beta}{dt}\Big)$$

Transfer function

$$P(s) = \frac{m\ell V_0^2}{bJ_p}\frac{1 + \frac{a}{V_0}s}{s^2 - \frac{mg\ell}{J_p}} = \gamma\omega_0^2\frac{a + \kappa s}{s^2 - \omega_0^2}$$

Notice gain proportional to V_0^2, unstable pole at $\sqrt{mg\ell/J_p}$ and zero at $-V_0/a$. Discuss the difficulties of control

Numerical Values

ℓ=1.3

a=0.4

b=1.20

m=75

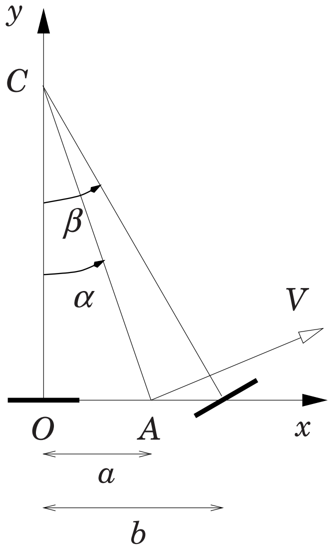
$J \approx m\ell^2$

$V_0/a = 2.5V_0$

ω_0=2.4

γ=0.64 V_0^2

κ=0.4/V_0



When is Control Easy or Difficult?

$$P(s) = \frac{m\ell V_0^2}{bJ_p} \frac{1 + \frac{a}{V_0}s}{s^2 - \frac{mg\ell}{J_p}} = \gamma\omega_0^2 \frac{a + \kappa s}{s^2 - \omega_0^2}$$

Notice gain proportional to V_0^2, unstable pole at $\sqrt{mg\ell/J_p}$ and zero at $-V_0/a$.

- Better control authority at high speeds
- Unstable pole $p = \sqrt{mg\ell/J_p} \approx \sqrt{g/\ell}$ slower when ℓ is large! A full size bike is easier to ride than a children's bike!
- The zero $z = -V_0/a$ is slower at low speeds. Tendency of peaking. May require cancellation.

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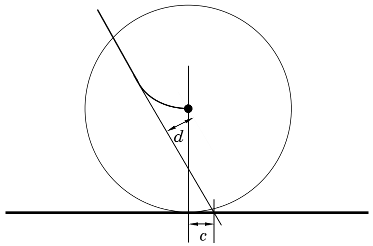
The Front Fork is the Key

Effects of front fork

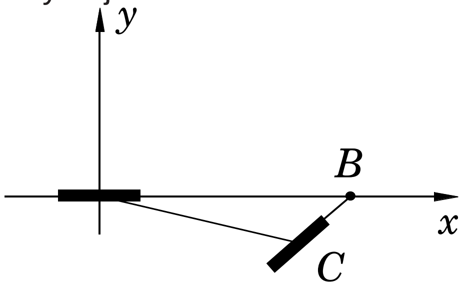


The distance c, typically 4-8 cm, is called the trail. It has a major effect on stabilization. There are two mechanisms

- It makes the front wheel line up automatically due to the caster effect.
- It turns the front wheel when the bicycle is tilted

Caster

Caster - A small wheel on a swivel attached under a piece of furniture or a heavy object to make it easier to move.



Introducing caster action on the front fork has a major impact. It makes the front wheel line up automatically and it makes it possible to bike with no hands on the handle bar.

A Simple Front Fork Model

The design of the front fork thus introduces a *feedback* in the bicycle. This feedback has dramatic effects on the behavior of the bicycle.

A simple model for the combination of trail and the caster effect is

$$\beta = -k\theta$$

Critical Velocity

Linearized bicycle dynamics (kinematics and tilt)

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\theta + \frac{m\ell V_0^2}{bJ_p}\left(\beta + \frac{a}{V_0}\frac{d\beta}{dt}\right)$$

Front fork feedback $\beta = -k\theta$. Closed loop system

$$\frac{d^2\theta}{dt^2} + \frac{am\ell kV_0}{bJ_p}\frac{d\theta}{dt} + \frac{m\ell}{J_p}\Big(\frac{kV_0^2}{b} - g\Big)\theta = 0$$

Stable if the velocity is greater than the critical velocity

$$V_c > \sqrt{\frac{bg}{k}}$$

Practical consequence

Summary

- The design of the front fork is essential
- Properly done it introduces a feedback that stabilizes the bicycle if the speed is sufficiently high $V \geq V_c$
- This is one reason why it is hard for children to learn how to bike. At slow speed they must stabilize manually. If they go fast enough the system is stable automatically.
- Gain increases with the square of the velocity
- Also teach them to balance with the handle bar

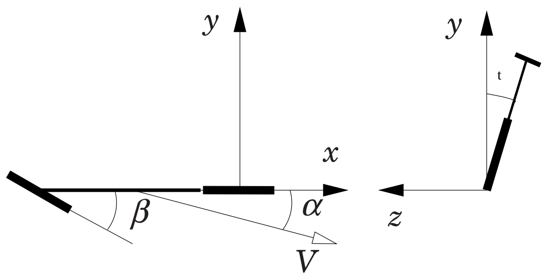
Bicycle Dynamics

- Introduction
- A simple Bicycle Model
- Properties
- Stabilization
- Rear wheel steering
- Adding complexity

Rear Wheel Steering

- To steer in front or in the rear is a classic problem in vehicle control
  - Wright Brothers
  - Ships
- To illustrate some consequences we will analyze a bicycle with rear wheel steering
- Results will also be a good illustration of things to look at when judging the difficulty of a control problem

Kinematics and Tilt



$$J_p \frac{d^2\theta}{dt^2} = mg\ell \sin\theta + m\ell\Big(\frac{V^2}{r}\cos\alpha - \frac{dV_y}{dt}\Big)\cos\theta \qquad V = \frac{aV_0 \tan\beta}{b\sin\alpha}$$

$$V_y = -V\sin\alpha = -\frac{aV_0}{b}\tan\beta \qquad r_0 = \frac{b}{\tan\beta}$$

$$\frac{dV_y}{dt} = \frac{aV_0}{b}\frac{1}{\cos^2\beta}\frac{d\beta}{dt} \qquad r = \frac{a}{\sin\alpha}$$

Normalization

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\sin\theta + \frac{m\ell V_0^2\cos\theta}{bJ_p}\Big(\tan\beta - \frac{a}{V_0\cos^2\beta}\frac{d\beta}{dt}\Big)$$

Introduce

$$\omega_0 = \frac{mg\ell}{J_p}, \qquad \gamma = \frac{V_0^2}{bg}, \qquad \kappa = aV_0$$

Physical interpretations! Introduce $1/\omega_0$ as time unit

$$\frac{d^2\theta}{dt^2} = \sin\theta + \gamma\Big(\tan\beta - \frac{\kappa}{\cos^2\beta}\frac{d\beta}{dt}\Big)\cos\theta$$

Two parameters gain κ and time constant κ

Linearization

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\sin\theta + \frac{m\ell V_0^2\cos\theta}{bJ_p}\Big(\tan\beta - \frac{a}{V_0\cos^2\beta}\frac{d\beta}{dt}\Big)$$

Linearize

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\theta + \frac{m\ell V_0^2}{bJ_p}\Big(\beta - \frac{a}{V_0}\frac{d\beta}{dt}\Big)$$

Transfer function

$$P(s) = \frac{m\ell V_0^2}{bJ_p}\frac{1-\frac{a}{V_0}s}{s^2-\frac{mg\ell}{J_p}} = \gamma\omega_0^2\frac{a+\kappa s}{s^2-\omega_0^2}$$

Poles and zeros in the RHP. Potential trouble!!!

Stabilization

Linearized dynamics (kinematics and tilt)

$$\frac{d^2\theta}{dt^2} = \frac{mg\ell}{J_p}\theta + \frac{m\ell V_0^2}{bJ_p}(\beta - \frac{a}{V_0}\frac{d\beta}{dt})$$

Front fork feedback $\beta = -k\theta$. Closed loop system

$$\frac{d^2\theta}{dt^2} - \frac{am\ell kV_0}{bJ_p}\frac{d\theta}{dt} + \frac{m\ell}{J_p}\Big(\frac{kV_0^2}{b} - g\Big)\theta = 0$$

Always unstable!

Does it help to have a more complex controller?

Fundamental Limitations

Ref KJÅ European Journal on Control Jan 2000.

Achievable phase margin for $n_{gc} = -1/2$ and different zero-pole ratios z/p

z/p	2	2.24	3.86	5	5.83	8.68	10	20
φ_m	-6.0	0	30	38.6	45	60	64.8	84.6

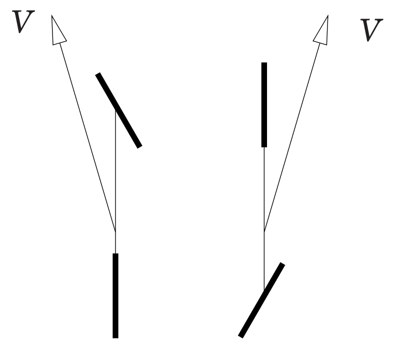
For the bicycle with rear wheel steering we have

$$\frac{z}{p} = \frac{a}{V_0}\sqrt{\frac{ml}{bJ_p}} \approx \frac{a}{\sqrt{b\ell}} = 0.32$$

Hopeless to get any controller to work!

Summary

- Rear wheel steering introduces a RHP zero
- This zero is close to the RHP pole which implies that the system cannot be controlled!
- Close is $z/p < 5$!
- An inherently bad design
- An intuitive explanation
- Centrifugal forces same direction
- Acceleration forces different directions



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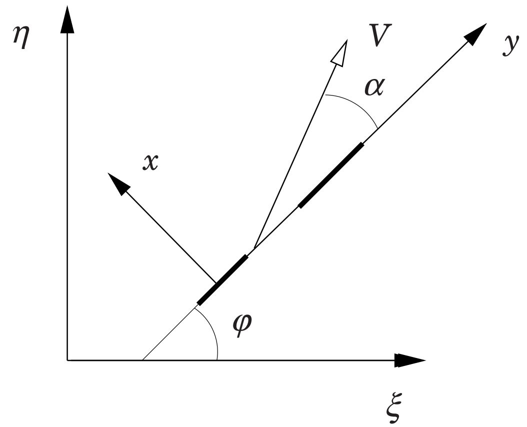
Path Dynamics

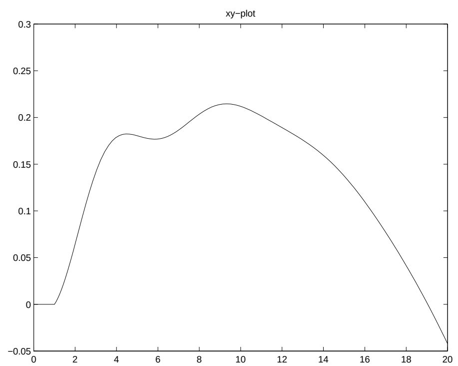
$$
\begin{aligned}
\frac{d\xi}{dt} &= V\cos\psi \\
\frac{d\eta}{dt} &= V\sin\psi \\
\psi &= \alpha + \varphi \\
\frac{d\phi}{dt} &= \frac{V_0}{b}\tan\beta \\
V &= \frac{V_0}{\cos\alpha} \\
\tan\alpha &= \frac{a}{b}\tan\beta
\end{aligned}
$$



You cannot choose β freely because you must stabilize!

The Inexperienced Biker

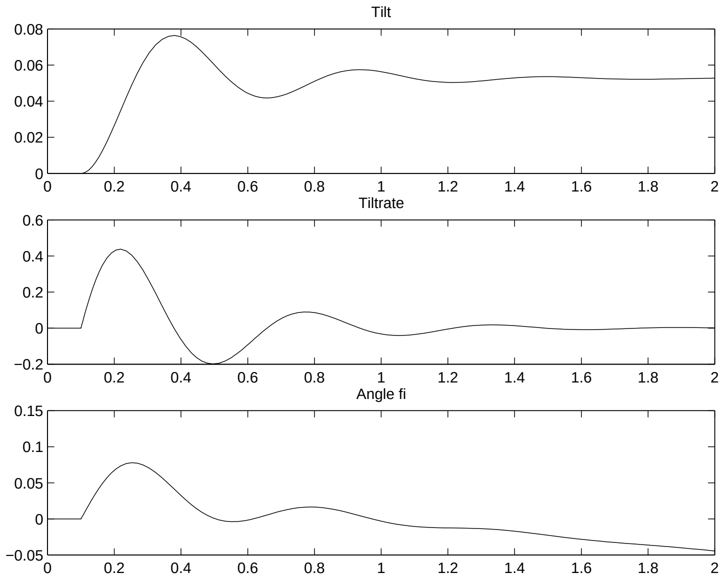
A bike moves forward at constant speed. A car starts to drive out a little ahead. The biker turns away from the car, BUT...!



Can you explain this strange behavior?

The Audience is Thinking!!!

Time Histories



Matlab Program

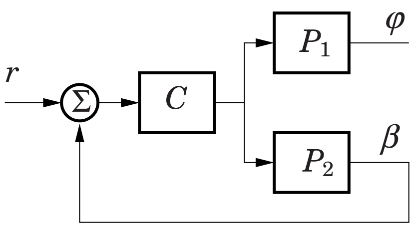
```
function dy=der(t,y)
m=75;a=0.4;b=1.2;
l=1.3;J=m*(l^2+l^2/12);
g=9.81;V0=10;k=2;
A=m*g*l/J;B=m*l/(b*J);
x1=y(1);
x2=y(2);
fi=y(3);
th=y(4);
dth=y(5);
del=0.1*(t>0.1);
be=-k*y(4)+del;
dbe=-k*y(5);
al=a*be/b;
psi=fi+al;
V=V0/cos(al);
dy(1)=V*cos(psi);
dy(2)=V*sin(psi);
dy(3)=V0/b*tan(be);
dy(4)=dth;
dy(5)=m*g*l/J*th
+m*l*V0^2/J*(be+a*dbe/V);
dy=dy’;
%end function
[t,y]=ode45(’der’,
[0 2],[0 0 0 0 0]);
plot(y(:,1),y(:,2))
```

Stabilization and Path Following

- Can we explain the nonintuitive behavior of the system?
- What is the remedy?
- Do you pay a penalty in path following because you have to stabilize?
- A fundamental problem!
- We have seen it also for the inverted pendulum! (Maybe not noticed it?)
- To understand basic problems we should look at them in the simplest setting!

Structure of the Problem

$$\frac{d\xi}{dt} = V\cos\psi$$
$$\frac{d\eta}{dt} = V\sin\psi$$
$$\psi = \alpha + \varphi$$
$$\frac{d\phi}{dt} = \frac{V_0}{b}\tan\beta$$
$$V = \frac{V_0}{\cos\alpha}$$
$$b\tan\alpha = a\tan\beta$$
$$\frac{d^2\theta}{dt^2} = \omega_0^2\Big(\theta + \gamma\big(\beta + \kappa\frac{d\beta}{dt}\big)\Big)$$



Transfer function

$$G_{\varphi r} = \frac{P_1 C}{1 + P_2 C}$$

Poles of P_2 are zeros of $G_{\varphi r}$

Stabilize P_2 imposes severe constraints on the signal transmission from r to φ. RHP poles of P_2 appear as zeros.

Insight

- We pay a penalty in path following when we have to stabilize at the same time.
- The unstable pole of the system appears as a right half plane zero in path following.
- This explains why we get the nonintuitive (inverse response, non-minimum phase) behavior
- The remedy is to use more control variables. Lean with the body!!

Lecture 7 - Vehicle/Bicycle Dynamics

Summary of Bike 1

- Very simple second order model kinematics and tilt
- Explains much of the bicycle behavior

  Mechanisms for stabilization
  Critical velocity
  Non-minimum phase in path following
  Much better to steer with the front wheel
  Shows how design parameters influence behavior
- Many things have been neglected

  It cannot be stabilized when $V_0 = 0$
  Front fork model very simplistic $\beta = -k\theta$
  Tire-road interaction, Gyroscopic effects
  Tilting of drivers body

A More Elaborate Model

Improved physics based front fork model!

More control variables.

- Five rigid bodies

  Front fork
  Two wheels
  Frame
  Drivers upper body
  10 states
  3 control variables
- Three rigid bodies

  Front fork
  Frame
  Drivers upper body
  6 states
  3 control variables

How to build a suitable library

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Summary

- The bicycle is a simple prototype problem where many things can be done easily
- Notice that simple models to not come easily. They require a lot of trial and error supported by experiments. Thanks to Richard Klein.
- Elements of the bicycle model are found in all vehicles

  Kinetics and force generation

  Control forces proportional to V^2
- Codesign of process and controller.What makes a control problem difficult.

  RHP poles and zeros are **real** difficulties
- An interesting structural problem associated with stabilization and path following. Same thing for pendulum on cart.

Hand In # 3

1. Replace the Matlab code for simulating path dynamics with Modelica code.
2. Make a structured Modelica model of Bike 1 with Kinematics. Design the model so that Bike 1 can be substituted with another model. Simulate tilt dynamics and path following.