This paper presents the solution to a general decentralized state-feedback problem, in which the plant and controller must satisfy the same combination of delay constraints and sparsity constraints. The control problem is decomposed into independent subproblems, which are solved by dynamic programming. In special cases with only sparsity or only delay constraints, the controller reduces to existing solutions.

Find a policy that minimizes a quadratic cost

\[
E \left\{ \sum_{t=0}^{\infty} x^T Q x + z^T R z \right\}
\]

subject to linear dynamics and a partially nested information constraint. Plant sparsity and information constraints are encoded by a network graph:

\[
d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

For the network graph above, dynamics and constraints are

\[
\begin{aligned}
\begin{bmatrix} x_{1,t+1} \\ x_{2,t+1} \\ x_{3,t+1} \\ x_{4,t+1} 
\end{bmatrix} &= \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} \\ 0 & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \\ x_{4,t} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} u_t + \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \\ w_{4,t} \end{bmatrix} \\
\end{aligned}
\]

The states \( x_i \) are influenced.

Each \( \zeta_i \) corresponds to a node in the information graph. The \( \{\zeta_i\} \) are independent because they are functions of different noise terms.

Define matrices \( X^*_t \) recursively by \( X^*_0 = Q^*_0 \) and

\[
X^*_t = Q^*_t + A^* T X^*_t \begin{bmatrix} A^* & B^* \end{bmatrix} - (S^* + A^* T X^*_t \begin{bmatrix} A^* & B^* \end{bmatrix})^T (R^* + B^* T X^*_t \begin{bmatrix} A^* & B^* \end{bmatrix}) \]

where \( a \) is the unique node with \( r \rightarrow a \) in the information graph.

The optimal control policy is given by \( u_t = \sum_{r \in U} \gamma^r K^*_t \zeta^r_t \).

The states \( \zeta^r_t \) evolve according to \( \zeta^r_t = \sum_{s \in \mathcal{G}} I_{s \rightarrow r} \zeta^s_{t-1} \) and

\[
\zeta^r_{t+1} = \sum_{s \in \mathcal{G}} (A^s + B^s K^*_t \zeta^s_t + \sum_{s \in \mathcal{G}} I_{s \rightarrow r} \zeta^s_t)
\]

Notation: \( M^r \) is the block submatrix \( (M_{ij})_{i \in U, j \in E} \).

Sparsity only [1, 2]: Acyclic graph with delay 0 on all edges.

Delay only [3]: Strongly connected graph with delay 1 on all edges.

References


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Further information is available on our websites (see below).

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