Hierarchical Decentralized Control for Networked Dynamical Systems Towards Glocal Control

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Future Direction in Control

Realization of High Quality Products
→ Solving Social Problems such as Energy, Environments, and Medicine

Multiple Functions
High Performance
Automation
Stabilization
Modern Control
Robust Control
Hybrid Control
Classical Control

Linear motor car
Engine control
Robotics
Aerospace
Mechatronics
Steel process
Chemical process

Glocal Control

Meteorological Phenomena
Bio-systems
Energy NWs
Transportation

Watt
Glocal Control: to achieve desired global behavior by only local actions of measurement and control.

Idea of “Glocal Control”?

Real World

Locally Control

Global Behaviors By Locally

Prediction

Locally Measurement

Large-scale & Complex

Meteorological Phenomena

Energy NWs

Transportation NWs

Bio-system
Urban Heat Island Problem

Local Actions of Measurement & Control

Realization of Global Desired Environment of a Whole City

Scale of buildings and roads

Glocal Control

Scale of residential and business areas

Scale of districts/towns
Possibility by Glocal Control

Fix VS Shift actively

Temperature Difference: 15m above the surface after 15min

Much cooling affect

Control

Prediction

Measurement

Global Prediction by model

Distributed local actuators

Distributed local sensors

5km
Hierarchical Bio-Network Systems

Proteins Gene Network of Cells

HGMIS(www.ornl.gov)
Integrated Energy Networks

Integrated Energy Network

Multi-resolved Hierarchical Model

Multi-resolved Prediction

Hierarchical Decentralized Control

Regional Energy Network System

Electric Power Network + Gas Energy Network + Heat Energy Network

Information

Elec. Power

Heat Energy

Gas Energy

Regional Energy Network System
Framework for Glocal Control

Realization of Global Functions by Local Measurement and Control

Glocal Control System

Local Control

Global Prediction through hierarchical model with multiple-resolution

Local Measurement

LR model
MR model
HR model

Real World

Power NWs
Bio-Systems
Transportation

Meteorological Phenomena
Framework for Glocal Control

Realization of Global Functions by Local Measurement and Control

Hierarchical Dynamical Systems with Multi-resolution

Glocal Control System

Local Control

LR model

MR model

HR model

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Global Prediction through hierarchical model with multiple-resolution

Real World

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Meteorological Phenomena
1. Glocal Control
2. A Unified Framework for Hierarchical Networked Dynamical Systems
3. Hierarchical Decentralized Control
4. Hierarchical Control for Energy NWs
5. Conclusion
1. Glocal Control

2. A Unified Framework for Hierarchical Networked Dynamical Systems

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5. Conclusion


(Hara et al.: CDC2010)

(Shimizu, Hara: SICE2008, Hara et al.: ACC2009)
Framework for Glocal Control

Realization of Global Functions by Local Measurement and Control

Hierarchical Dynamical Systems with Multi-resolution

Q1: What is the most fundamental system representation?
LTI System with Generalized Frequency Variable

A unified representation for multi-agent dynamical systems

\[ C (sI - A)^{-1} B + D \]

\[ \frac{1}{s} \rightarrow h(s) \]

\[ \Phi(s) = \frac{1}{h(s)} \]

Group Robot  
Gene Reg. Networks

Dynamics + Information Structure
Define: Domains \( \Omega_+ := \phi(\mathbb{C}_+) \), \( \Omega^c_+ := \mathbb{C} \setminus \Omega_+ \)

Q2A: How to characterize the region?
Q2B: How to check the condition?
## Stability Tests for LTISwGFV

(Tanaka et al., ASCC, 2009)

<table>
<thead>
<tr>
<th>Graphical</th>
<th>Algebraic</th>
<th>Numeric (LMI)</th>
</tr>
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<tr>
<td><strong>Nyquist – type</strong></td>
<td><strong>Hurwitz – type</strong></td>
<td><strong>Lyapunov – type</strong></td>
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<td>Hara et al. (2007)</td>
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<tr>
<td>$h(s)$ and $\sigma(A)$</td>
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<td>$h(s)$ and $A$</td>
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**Characteristic Polynomial**

$p(\lambda, s) := d(s) - \lambda \cdot n(s) \quad \lambda \in \sigma(A) \quad \text{(complex)}$
Numerical Example: 4th order

\[ h(s) = \frac{100(s + 2)\left(\frac{19}{10}s^2 - \frac{1}{500000}s + \frac{21}{10}\right)}{(s - 1)^2(s + 1)(s + 100)} \]

\( \Delta_1 > 0 \)

\( \Delta_2 > 0 \)

\( \Delta_3 > 0 \)

\( \Delta_4 > 0 \)

\( \Delta_k := l_k(\lambda)^*\Phi_k l_k(\lambda) > 0 \quad l_\ell(\lambda) := \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda_\ell \end{bmatrix} \)
Algorithm \texttt{h2Phi}(h(s)) :

Input : \( h(s) = \frac{b_1 s^{\nu-1} + \cdots + b_\nu}{s^\nu + a_1 s^{\nu-1} + \cdots + a_\nu} \)

Output : \( \ell_k \) and \( \Phi_k \)

1. \( p_0 \leftarrow 1, q_0 \leftarrow 0 \)
   for \( i \leftarrow 1 \) until \( 2\nu - 1 \) do
     if \( i \leq \nu \) then
       \( p_i \leftarrow a_i - b_i x, q_i \leftarrow -b_i y \)
     else
       \( p_i \leftarrow 0, q_i \leftarrow 0 \)

2. \( \Delta_1 \leftarrow p_1 \)
   for \( k \leftarrow 2 \) until \( 2\nu \) do
     \( M \leftarrow O^{(2k-1) \times (2k-1)} \)
     for \( i \leftarrow 1 \) until \( \nu \) do
       \( M_{i,2k-i} \leftarrow \Delta_k ((\lambda + \bar{\lambda})/2, (\lambda - \bar{\lambda})/2j) \)
     \end{for}
   \endfor
   for \( i \leftarrow 1 \) until \( \nu \) do
     \( \Phi_k (m+1, l+1) \leftarrow 0 \) until \( \ell_k - 1 \) do
       \( \Phi_k (m+1, l+1) \leftarrow \) the coefficient of \( \lambda^m \bar{\lambda}^l \) in \( \Delta_k (\lambda, \bar{\lambda}) \)
     \endfor
   \endfor

Result:

\( \Phi_k \) \((k = 1, 2, \cdots, \nu)\)

Systematic methods for stability analysis

Hurwitz-type & LMI
Messages: A New Framework

① LTI system with generalized freq. variable
   a proper class of homogeneous multi-agent
dynamical systems

② Three types of stability tests, namely
   graphical, algebraic, and numeric (LMI)
   powerful tools for analysis

Q3: from Homogeneous
    to Heterogeneous ?

Q4: from Flat Structure
    to Hierarchical Structure ?
From Homogeneous to Heterogeneous

\[ \tilde{H}(s) = (I + \Delta(s)) \cdot h(s) \]

Nominal system: homogeneous

\[ h_i(s) = (1 + \delta_i(s))h(s) \]

Independent perturbations

\[ \Delta_{d\gamma} := \{ \Delta(s) | \Delta(s) = \text{diag}\{\delta_i(s)\}, \|\Delta(s)\|_{\infty} \leq 1/\gamma \} \]
Robust Stability Condition for Heterogeneous Perturbations

**Assumption**

\[ \exists D : \text{diagonal s.t. } DAD^{-1} \text{ is normal} \]

**Theorem:** The following conditions are equivalent.

(i) The system is robustly stable for \( \Delta_d \gamma \).

(ii) \[ \left\| \frac{\lambda h}{1 - \lambda h} \right\|_\infty < \gamma, \ \forall \lambda \in \sigma(A) \]

(iii) \[ \left| \frac{\lambda}{\phi - \lambda} \right| < \gamma, \ \forall \lambda \in \sigma(A), \]
\[ \forall \phi \in \Phi := \{1/h(j\omega) | \omega \in \mathbb{R} \} \]

**Symmetric Circulant**

Same results for MIMO general classes of perturbations
Hierarchical NW Dynamical Systems

\[ \dot{\mathbf{x}}(t) = A\mathbf{x}(t) \]

\[ \exists \xi, \lim_{t \to \infty} \mathbf{x}(t) = \xi \cdot 1 \]

# total agents : \( n_1 \times n_2 \times n_3 \)
Hierarchical Structure

\[ A_l = \text{diag}(A_{l-1} - I) + P \otimes \Delta \]

Homogeneous structure

Upper-layer structure

Property on Interactions

Low Rank Interaction:
\[ \Delta = 1 \cdot \zeta^T \]

weak interaction:
Sparse
Small gain

Share an aggregated information
Control uniformly

(Shimizu, Hara: SICE2008)
\( \angle: \) Rank 1

\[
eigs(A_1) = \bigcup_{r=1}^{n_1} \exp\left(2\pi j(r - 1)/n_1\right) - 1
\]

\[
eigs(A_2) = \begin{cases} 
\bigcup_{r=1}^{n_2} \exp\left(2\pi j(r - 1)/n_2\right) - 1 \\
\bigcup_{r=2}^{n_1} \exp\left(2\pi j(r - 1)/n_1\right) - 2
\end{cases}
\]

\[ n_1 = 25 \]
\[ > n_2 = 4 \]

\[ \Delta = 1 \cdot \zeta^T \]
\[ \Delta = I \]

\( \circ: \) rank 1
\( \times: \) Identity
Time Responses \((n_1=25, n_2=4)\)

**Rapid Consensus**

\[\Delta: \text{Rank 1} \]

\[\Delta = 1\]

\[n_1 > n_2\]

- Distribution
- Aggregation
- Low-rank Interlayer Interactions
- Multiple resolution
General Case with $h(s)$

\[ h(s) = \frac{s + 1}{s(0.1s^2 + 0.5s + 1)} \]

$n_1 = 8, \ n_2 = 5$

Stability Boundary

○: $\Delta = 1$

* : $\Delta = \text{Rank 1}$
Quorum-Sensing Networks

(Nakamura et al.: SICE2011)

\[
\hat{S}_i(t) = -k_{s0}S_i(t) + k_{s1}p_{i,1}(t) - \eta(S_i(t) - W\bar{S}(t))
\]

\[
\dot{p}_{i,j}(t) = \frac{R^2}{2T}f(p_{i,j-1}) - \frac{p_{i,j}(t)}{2T} + \frac{\kappa R^2}{2T\alpha}g_j(S_i)
\]
Messages: A New Framework

① LTI system with generalized freq. variable
a proper class of homogeneous multi-agent systems

② Three types of stability tests, namely
graphical, algebraic, and numeric (LMI)
powerful tools for analysis

③ From Homogeneous to Heterogeneous
robust stability analysis (Hinf norm condition)

④ From Flat to Hierarchical Structure
low-rank interlayer connection (aggregation & distribution)

Q5: How to Design Decentralized Control Systems?
1. Glocal Control
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(Fujimori et al.: CDC2011)
(Tsubakino et al.: ASCC2013)
A General Hierarchical Structure

(Fujimori et al.: CDC2011)

\[ \mathcal{A} = \begin{bmatrix} A_0 & \cdots & k_1 \Gamma_0 \\ \vdots & \ddots & \vdots \\ k_M \Gamma_0 & \cdots & k_M \Gamma_0 \end{bmatrix} = I_M \otimes A_0 + K \otimes \Gamma_0 ; \quad \Gamma_0 = V_0 W_0^T \]

\[ \mathcal{A} \text{ can be written using Kronecker product } \otimes. \]

\[ \mathcal{A} = \begin{bmatrix} A_1 & \cdots & k_1 \Gamma_{11} \\ \vdots & \ddots & \vdots \\ k_M \Gamma_{MM} \end{bmatrix} = \text{diag} \{ A_k \} + K \odot \Gamma \]

\[ \Gamma_{ij} := V_i W_j^T \]

We need Khatori-Rao product \( \odot \).

Homogeneous

Heterogeneous
Numerical Examples (1/2)

\[ h(s) = \frac{b}{s(s + a)} e^{-\tau s}; \quad a = \pi, \quad b = \frac{\pi^2}{2}; \quad n_1 = 4, n_2 = 3 \]

\[ \tau = 0 \]

Without Control

Rank 1

\( \Omega^c \)

Real part

Imaginary part

\( \Omega^c \)

Real part

Imaginary part

\( \O \) : eigenvalue of \( A \)
Numerical Examples (2/2)

\[ \tau = 0.25 \]

Without Control

Rank 1

Rank 2
Theorem: Rank 2 Case

Assumption

\[ \forall k = 1, \ldots, M \]

- \( A_k \): has at least two simple eigenvalues \( \lambda_{k1}, \lambda_{k2} \)
- \( \Gamma \) is a right eigen-connection matrix of \( \{ A_k \} \)
  associated with eigenvalue \( \{ \lambda_{k1} \}, \{ \lambda_{k2} \} \)

Theorem: Rank 2

For any \( K \), the set of all the eigenvalues of \( \mathcal{A} \) is given by

\[
\sigma(\mathcal{A}) = \bigcup_{k=1}^{M} \left( \sigma(A_k) \setminus \{ \lambda_{k1}, \lambda_{k2} \} \right) \cup \sigma \left( S(K \otimes I_2) \Phi + \Lambda \right)
\]

\[
S = \text{diag} \{ S_k \} \quad \Lambda = \text{diag} \left\{ \begin{bmatrix} \lambda_{k1} & 0 \\ 0 & \lambda_{k2} \end{bmatrix} \right\} \quad \Phi = \text{diag} \left\{ \begin{bmatrix} w_{k1} & w_{k2} \end{bmatrix}^\top \begin{bmatrix} v_{k1} & v_{k2} \end{bmatrix} \right\}
\]

An analogous result is obtained for left eigen-connection matrices.
Hierarchical Optimal Control Problem

Optimal Control Problem

\[ \dot{x} = A_L x + B_L u \quad Q_L \geq 0, \quad R_L = I \]
\[ J(x_o, u) = \int_0^\infty (x(t)^T Q_L x(t) + u(t)^T R_L u(t)) \, dt \]

Optimal Control Law

\[ u = Kx \quad K = -R_L^{-1} B_L^T P \]
\[ A_L^T P + P A_L - P B_L R_L^{-1} B_L^T P + Q_L = 0 \]

Q6: Under what condition, the optimal control gain \( K \)
- preserves the hierarchical structure
- belongs to a desired decentralized structure?
An Example with 3 Subgroups

\[ J = W_l J_l + W_g J_g, \]

Local

\[ J_l = \int_0^{\infty} \left( 30 \sum_{i=1}^{5} (N_i^2 + P_i^2 + Z_i^2) + \sum_{i}^{10} (u_i^2) \right) dt, \]

Global

\[ J_g = \int_0^{\infty} \left( 30(N^2 + P^2 + Z^2) + \sum_{i}^{10} (u_i^2) \right) dt \]

Structure of feedback control in \( N \)

\[ u_i = k_l N_i + k_g \bar{N} \]

\( \bar{N} : N_i \) (Average)

How is the general case?
Theorem: class of desired structure

\[ A_L, B_L, Q_L \in \mathcal{H}_L \]

\[ \mathcal{G}_i = \{G_{ij}\} : \text{inter-layer interactions} \]

\[ \mathcal{H}_1 = \left\{ H_1 \in \mathbb{R}^{n_1 \times n_1} | H_1 = \sum_{j}^{N_1} a_{1j} G_{1j}, \ a_{1j} \in \mathbb{C} \right\}, \]

\[ \mathcal{H}_L = \left\{ H_L \in \mathbb{R}^{n_L \cdots n_L} | H_L = \sum_{m}^{N_L} a_{Lj} G_{Lj} \otimes H_{L-1,j}, \ a_{Lj} \in \mathbb{R}, \ H_{L-1,j} \in \mathcal{H}_{L-1} \right\} \]

Theorem \( \mathcal{G}_i \) : a semi-group

\[ K \in \mathcal{H}_L, \]

\[ \mathcal{G}_i(i = 1, 2, \cdots, L) \]

\[ = \left\{ G_{ij} \in \mathbb{C}^{n_i \times n_i} | \forall j, k, \ \exists l, \ G_{ij} G_{ik} = G_{il} \right\} \]

Averaging, Circulant

Same results for

Output Feedback & Hinf Control
Generalization of Literatures

<table>
<thead>
<tr>
<th></th>
<th>Lth layer</th>
<th>⋯</th>
<th>2nd layer</th>
<th>1st Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brockett et al. (1974)</td>
<td>none</td>
<td>⋯</td>
<td>any</td>
<td>circulant</td>
</tr>
<tr>
<td>Sandereshan et al. (1991)</td>
<td>none</td>
<td>⋯</td>
<td>{I, \frac{1}{n}11^\top}</td>
<td>any</td>
</tr>
<tr>
<td>Bollerri et al. (2008)</td>
<td>none</td>
<td>⋯</td>
<td>{I, \frac{1}{n}11^\top}</td>
<td>any</td>
</tr>
</tbody>
</table>

\[ \mathcal{B} = I \otimes B_l, \mathcal{R} = I \otimes R_l, \]

**Bollerri**

\[ \mathcal{A} = I \otimes A_l \]

\[ Q = I \otimes (Q_1 - Q_2) + 11^\top \otimes Q_2 \]

\[ \implies \mathcal{K} = I \otimes (K_1 - K_2) + 11^\top \otimes K_2 \]

**Sandereshan**

\[ Q = I \otimes Q_l \]

\[ \mathcal{A} = I \otimes (A_1 - A_2) + 11^\top \otimes A_2 \]

\[ \implies \mathcal{K} = I \otimes (K_1 - K_2) + 11^\top \otimes K_2 \]
Desired Hierarchical Structures

\{I, \hat{I}\} \quad \{I, \frac{1}{n}11^\top\} \quad : \text{averaging} \quad \hat{i} = \begin{bmatrix} 0 & \cdots & 1 \\ 1 & \cdots & 0 \end{bmatrix}

\{I, L, L^2, \ldots, L^{n-1}\} \quad : \text{Circulant} \quad \begin{bmatrix} 0 & I \\ 1 & 0 \end{bmatrix}

\mathcal{T} = \{A|T(g_i)A = AT(g_i), \forall g_i \in G\}

Spatially Decay Operator

\mathcal{S}_\tau = \{\mathbb{R}^{n \times n}| \exists C, \exists \alpha \in \mathbb{R}, 0 < \alpha < \tau, \quad A = [A_{ki}], \|A_{ki}\| \leq C \exp(-\alpha |k - i|)\}
Cooling in Iron Plate Production

\[ \frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial y^2} + u, \quad x(0, t) = 0, \quad x(1, t) = 0, \]

Discretization in space

\[ x_i(t) = x(ih, t) \]

Optimal Control with SD Structure

\[
\begin{bmatrix}
  -2 & 1 & 1 & 1 \\
  1 & -2 & 1 & 1 \\
  & & \ddots & \ddots \\
  1 & -2 & 1 & 1 \\
  1 & -2 & 1 & -2
\end{bmatrix} \quad \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + u(t)
\]

Performance Index

\[
Q_c = 10 \begin{bmatrix}
  2 & -1 & -1 & -1 & -1 \\
  -1 & 3 & -1 & -1 & -1 \\
  -1 & -1 & 3 & -1 & -1 \\
  -1 & -1 & -1 & 3 & -1 \\
  -1 & -1 & -1 & -1 & 2
\end{bmatrix}
\]
Example: optimal feedback gain

$$(W_s, W_c) = \left( \frac{1}{2}, \frac{1}{2} \right)$$

$K = \begin{bmatrix}
1.7061 & -0.1245 & -0.0222 & -0.0045 & -0.0010 & -0.0002 & -0.0001 & -0.0000 & -0.0000 & -0.0000 \\
-0.1245 & 2.3181 & -0.0437 & -0.0107 & -0.0027 & 0.0007 & -0.0002 & -0.0000 & -0.0000 & -0.0000 \\
-0.0222 & -0.0437 & 2.3393 & -0.0394 & -0.0098 & -0.0025 & -0.0007 & -0.0000 & -0.0000 & -0.0000 \\
-0.0045 & -0.0107 & -0.0394 & 2.3405 & -0.0391 & -0.0098 & -0.0025 & -0.0007 & -0.0000 & -0.0000 \\
-0.0010 & -0.0027 & -0.0098 & -0.0391 & 2.3405 & -0.0391 & -0.0098 & -0.0025 & -0.0007 & -0.0000 \\
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-0.0000 & -0.0000 & -0.0025 & -0.0098 & -0.0394 & 2.3393 & -0.0437 & -0.0222 & -0.0049 & -0.0010 \\
\end{bmatrix}$

$K_t = \begin{bmatrix}
1.7061 & -0.1245 & -0.0222 & -0.0045 \\
-0.1245 & 2.3181 & -0.0437 & -0.0107 & -0.0027 \\
-0.0222 & -0.0437 & 2.3393 & -0.0394 & -0.0098 & -0.0025 \\
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-0.0025 & -0.0107 & -0.0437 & 2.3181 & -0.01245 & -0.0045 & -0.0222 & -0.01245 \\
\end{bmatrix}$

approximation

Error: $9.4081 \times 10^{-7}$

Centralized

Decentralized
\[ J = W_s J_s + W_c J_c, \]
\[ J_s = \int_0^\infty \left( 10 \sum_{i=1}^{10} (x_i^2) + \sum_{i}^{10} (u_i^2) \right) dt \]

**Local**

\[ J_c = \int_0^\infty \left( 10 \sum_{i=1}^{9} (x_i - x_{i+1})^2 + \sum_{i}^{10} (u_i^2) \right) dt \]

**Cooperation**

**Control Structure**

\[ u_i = k_s x_i \]
\[ + \sum_{j=1}^{3} k_{c,j} (x_{i-j} + x_{i+j}) \]
\[ J = W_s J_s + W_c J_c, \]
\[ J_s = \int_0^\infty \left( 10 \sum_{i=1}^{10} (x_i^2) + \sum_{i} (u_i^2) \right) dt \]

**Local**

\[ J_c = \int_0^\infty \left( 10 \sum_{i=1}^{9} (x_i - x_{i+1})^2 + \sum_{i} (u_i^2) \right) dt \]

**Cooperation**

Control Structure

\[ u_i = k_s u_i \]
\[ + \sum_{j=1}^{3} k_{cj} (x_{i-j} + x_{i+j}) \]
① Proper ways of aggregation and distribution are important to achieve rapid consensus.
② Low rankness of interlayer connection captures them properly.
③ Heterogeneous agents: *Khatri-Rao Product*
   hierarchical network synthesis based on left eigenvectors
④ LQR optimal control with desired hierarchical structure
   certain semi-group property
1. Glocal Control
2. A Unified Framework for Hierarchical Networked Dynamical Systems
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4. Hierarchical Control for Energy NWs
5. Conclusion

Workshop @ CDC2013, Florence, Italy
Features of Energy Networks

★ Energy
  - not uniformly distributed in time/space
  - unbalance between demand & supply
    → **Control = balancing energy in time/space**
  - but, transfer is very costly
    → **only local actions with exchanges in neighbors are available**
    → shifting elements in time/space are important

★ To reduce total energy
  → **Utilizing Nature & Control Strategy**

★ Key Points
  - Hierarchical with Multi-resolution
  - Aggregation & Distribution
  - **Passivity**
Integrated Energy Networks

Integrated Energy Network

- Electric power network
- Gas energy network
- Heat energy network

Multi-resolved Hierarchical Modeling

Regional Energy Network System

Electric power network + Gas energy network + Heat energy network

Information
Elec. Power
Heat Energy
Gas Energy

Regional Energy Network System
OUTLINE

1. with Fujitsu
2. with Azbil
3. with Tokyo-Gas

- Different Target Systems
- Different Shift Elements
- Different Focuses
Integrated BEMS by Heat Transfer

Purpose
Energy Management Control by Heat Transfer with Thermal Energy Storages

On Going Work
1) Hierarchical Modeling & Decentralized Control
2) Design Guideline for NWs (TESs, GEs)
Features of Decentralized Control

Advantages

- Reduction of computation load in each control device
- Localization of confidential information (e.g., facility information, energy consumption)
- Adaptation capability for facility replacement and performance degradation with updating of subsystems

Parameters
- Heat loss ratio of all pipes
- Performance of chillers
- Capacity of chillers
- Capacity of TES
- Heat loss ratio
- Cooling demand

Variables
- Interchanged heat of all pipes
- Produced heat
- State-of-charge of TES
- Interchanged heat of pipes
Decentralized Control for Int. BEMS

Electricity → Chilled Water → Chilled Water → (with Azbil) Air conditioning

- Global
- Local

Chiller → Thermal Energy Storage → Air conditioner

Minimizing Total Energy

Maximizing Each Utility
Modeling of Elements and Setting of Objective Functions

Modelling of Elements

Thermal Storage

\[ Q(t) = x(t) - v(t) \]

heat loss

Conduit

\[ u(t) \]

\[ \frac{1 - \lambda}{Ts + 1} \]

first-order lag

Setting of Objective Functions

To minimize Cost Function In Chillers

\[ \text{Convex} \]

To maximize Utility Function In Amenity

\[ \text{Concave} \]
Decentralized Control: Optimization

\[
\max \sum \text{Objective function} \quad \text{State variables}
\]

s.t. \( \text{Equality constraints (linear)} \)
**Primal-Dual Algorithm**

\[ x \in \mathbb{R}^n \]

\[ f : \mathbb{R}^n \rightarrow \mathbb{R} \text{, C2 class, strictly concave function} \]

\[ \max_x f(x) \quad \text{(P)} \]

\[ \text{s.t. } Rx = 0 \]

\[ \mathcal{L}(x, \lambda) = f(x) + \lambda^T Rx : \text{Lagrangian} \]

\[ \dot{x} = \frac{\partial \mathcal{L}}{\partial x}(x, \lambda) = \frac{\partial f}{\partial x}(x) + R^T \lambda \]

\[ \dot{\lambda} = -\frac{\partial \mathcal{L}}{\partial \lambda}(x, \lambda) = -Rx \]

\[(x^*, \lambda^*)\]

A simple gradient method guarantees the convergence to the unique optimal

(Arrow, Hurwicz, Uzawa, 1958)
Control Theoretic Interpretation

(Yamamoto, Tsumura: METR 2012)

Incremental passive

Storage Function

\[ S_{s_i} := \frac{1}{2}(s_i - s_i^*)^2 \]

\((c''_i(\xi_{s_i}) > 0, \text{strictly convex})\)

\(f(s)\) : any passive system
e.g. PI-type
A Numerical Example

- Reduction of Computational Cost
- Possibility of Receding Horizon Strategy

The same values
An Example: two buildings (with Azbil)

Cost

\[ E_1(u_i[h]; h) \]

Cold energy

\[ u_1[h] \xrightarrow{T_i^c, \lambda_i^c} x_1[h] \xrightarrow{\nu_{11}[h]} d_{11}[h] \xrightarrow{T_{ij}^l, \lambda_{ij}^l} d_{21}[h] \xrightarrow{A_1(b_1[h]; h)} \]

Cold energy

Amenity

\[ v_{12}[h] \]

Better Efficiency

\[ \max \sum_{h} \sum_{i} (A_i(b_i[h]; h) - E_i(u_i[h]; h)) \]

\[ H = 24 \quad \Delta t = 1.0 \]

\[ \begin{bmatrix} 0.13 \\ 0.30 \end{bmatrix} \leq u[h] \leq \begin{bmatrix} 1.30 \\ 3.00 \end{bmatrix} \]

\[ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq v[h] \]

\[ \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \leq Q[h] \leq \begin{bmatrix} 15.0 \\ 15.0 \end{bmatrix} \]

\[ Q[0] = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \]

\[ A(b[h]; h) = -100(b - \text{demand})^2 \]
$1 \leq Q_2 \leq 5$

Enough Capacity

- $u_1$
- $v_{11}$
- $v_{12}$
- $Q_1$
- $b_1$
- $u_2$
- $v_{22}$
- $v_{21}$
- $Q_2$
- $b_2$
Design Guideline of Integrated Energy Networks from the View Point of Hierarchical Decentralized Control
1. Glocal Control
2. A Unified Framework for Hierarchical Networked Dynamical Systems
3. Hierarchical Decentralized Control
4. Hierarchical Control for Energy NWs
5. Conclusion
A Unified Model for Energy NWs
A Unified Model for Energy NWs

Ongoing Work

- Hierarchical Algorithm with Multi-time scales (multi-resolution)
- Adaptive Algorithm by Combination with supply/demand prediction

\[ h(s)I \]

\[ A \quad B \]
\[ C \quad D \]

\[ y \quad u \]

\[ \text{diag}\left\{ \frac{1}{1+T_{is}} \right\} \]

\[ \text{diag}\{\text{Sat}(\sigma_i)\} \]
Key Notion for “Future”

Harmony with Nature and Social Systems

Physical

Integrated Control NW
(Measurement, Prediction & Control)

Human NW

Economic NW

Thank you very much!