Feedback Fundamentals

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Context

- Introduction
- Modeling
- Block diagrams and transfer functions
- Simple feedback systems
- Frequency response
- Feedback fundamentals
- PID Control
- Loopshaping
- State feedback and observers
- How the field developed

The Magic of Feedback

Nice properties:

- Attenuate effects of disturbances - process control, automotive
- Make good systems from bad components - feedback amplifier
- Follow command signals - robotics
- Stabilized an shape behavior - flight control

Bad properties:

- Feedback may cause instability
- Feedback feeds measurement noise into the system

Arthur C. Clarke: Any sufficiently advanced technology is indistinguishable from magic

Feedback Fundamentals

Theme: A closer look at feedback

Introduction

- A closer look at a feedback system
- Some important considerations

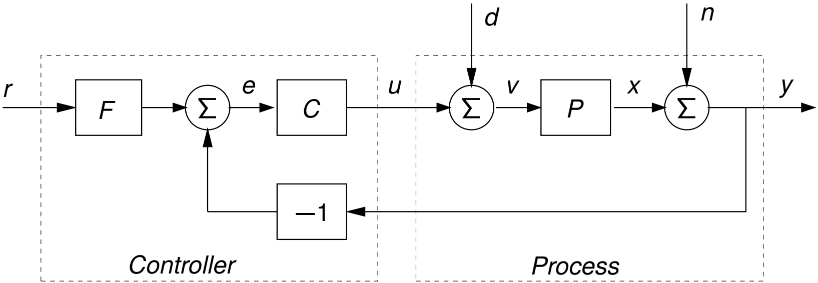
  Load disturbances

  Measurement noise

  Process variations and uncertainty

  Command signal following
- Issues related to evaluation, testing and specifications of control systems
- Insight into what can and cannot be achieved by feedback
- Quantification of performance and robustness
- Measurement and testing of performance and robustness

A Basic Feedback Loop

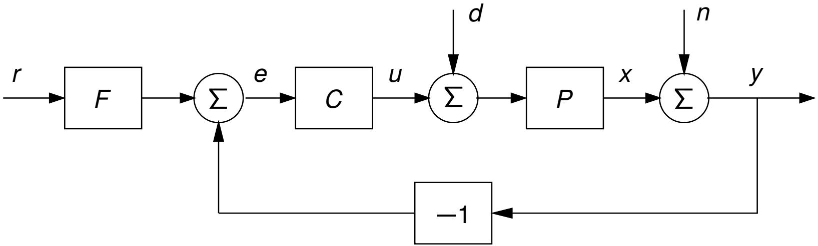


Ingredients:

- Controller: feedback C, feedforward F
- Load disturbance d : Drives the system from desired state
- Process: transfer function P
- Measurement noise n : Corrupts information about x
- Process variable x should follow reference r

Quiz

Look at the block diagram

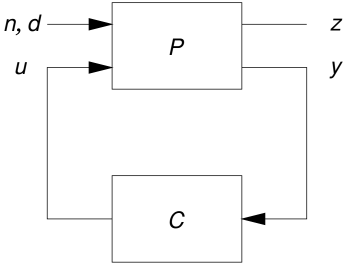


Find all relations where the signal transmissions are equal to either the sensitivity function or the complementary sensitivity function

The Audience is Thinking ...

A More General Setting

Load disturbances and measurement need not enter at the process input and measurement noise not at the output. A more general situation is.



$$w = (d, n, y_{sp}),\ z = (e, v), \text{ find } C \text{ to make } z \text{ small!}$$

These problems can be dealt with in the same way but we will stick to the simpler case. Always useful to understand disturbances, who they are and where they enter the system.

Typical Requirements

A controller should

A: Reduce effects of load disturbances

B: Do not inject too much measurement noise into the system

C: Make the closed loop insensitive to variations in the process

D: Make output follow command signals

Performance is expressed by

- Response to command signals
- Attenuation of load disturbances

Robustness is expressed by sensitivity to

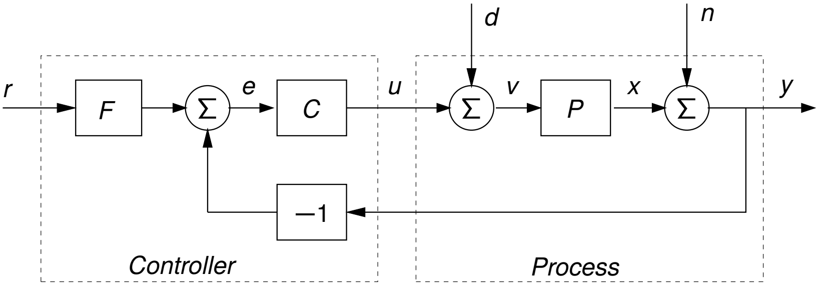
- Load disturbances
- Model uncertainty

Feedback Fundamentals

1. Introduction
2. Controllers with Two Degrees of Freedom
3. The Gangs of Four and Seven
4. The Sensitivity Functions
5. Summary

Theme: A closer look at feedback

System with Two Degrees of Freedom



- Load disturbance d : Drives the system from desired state
- Measurement noise n : Corrupts information about x

The controller has two degrees of freedom 2DOF because the signal transmissions from reference r to control u and from measurement y to control u are different. Horowitz 1963.

A Separation Principle for 2DOF Systems

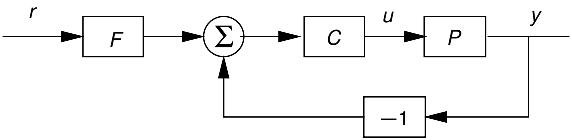
Design the feedback C to achieve

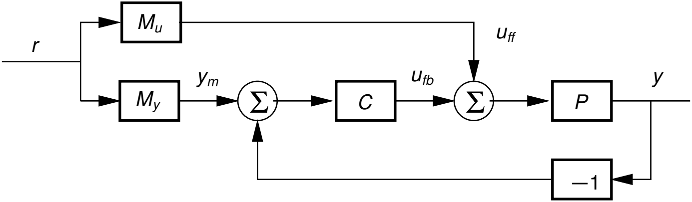
- Low sensitivity to load disturbances d
- Low injection of measurement noise n
- High robustness to process uncertainty and process variations

Design the feedforward F to achieve

- Desired response to command signals r

Many Versions of 2DOF



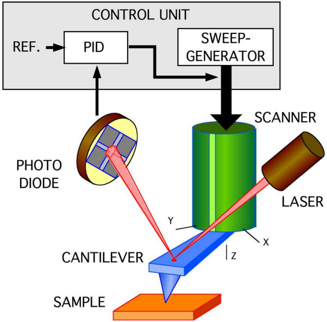


For linear systems all 2DOF configurations have the same properties. For the systems above we have $CF = M_u + CM_y$

Some Systems only Allow Error Feedback

Disk drive

Atomic Force Microscope



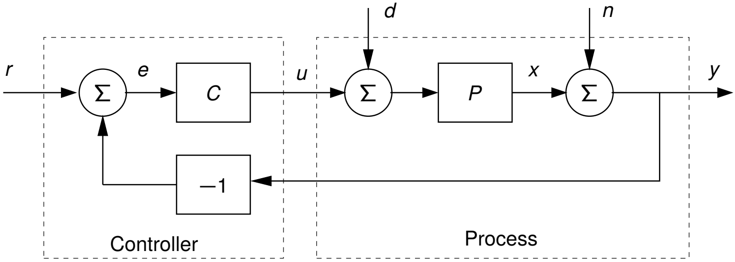
Only error can be measured

Design for command disturbance attenuation and command signal response can not be separated!

Feedback Fundamentals

Theme: A closer look at feedback

System with Error Feedback



$$
\begin{aligned}
G_{xr} &= \frac{PC}{1+PC}, & G_{xd} &= \frac{P}{1+PC}, & G_{xn} &= -\frac{PC}{1+PC}, \\
G_{yr} &= \frac{PC}{1+PC}, & G_{yd} &= \frac{P}{1+PC}, & G_{yn} &= \frac{1}{1+PC}, \\
G_{ur} &= \frac{C}{1+PC}, & G_{ud} &= -\frac{PC}{1+PC}, & G_{un} &= -\frac{C}{1+PC}, \\
G_{yr} &= \frac{1}{1+PC}, & G_{ed} &= -\frac{P}{1+PC}, & G_{en} &= -\frac{1}{1+PC},
\end{aligned}
$$

The Gang of Four - GOF

$$G_{xr} = \frac{PC}{1+PC}, \quad G_{xd} = \frac{P}{1+PC} \quad G_{xn} = -\frac{PC}{1+PC},$$

$$G_{yr} = \frac{PC}{1+PC}, \quad G_{yd} = \frac{P}{1+PC}, \quad G_{yn} = \frac{1}{1+PC},$$

$$G_{ur} = \frac{C}{1+PC}, \quad G_{ud} = -\frac{PC}{1+PC}, \quad G_{un} = -\frac{C}{1+PC},$$

$$G_{er} = \frac{1}{1+PC}, \quad G_{ed} = -\frac{P}{1+PC}, \quad G_{en} = -\frac{1}{1+PC},$$

Only four transfer functions!!! (Sensitivity functions - the Gang of Four)!

$$S = \frac{1}{1+PC}, \quad T = \frac{PC}{1+PC} = 1 - S, \quad PS = \frac{P}{1+PC}, \quad CS = \frac{C}{1+PC}$$

Interpretation of The Gang of Four

Robustness to process variations is characterized by

$$S = \frac{1}{1+PC}, \quad T = \frac{PC}{1+PC}$$

Response of output y to load disturbance d is characterized by

$$G_{yd} = \frac{P}{1+PC} = PS = \frac{T}{C}$$

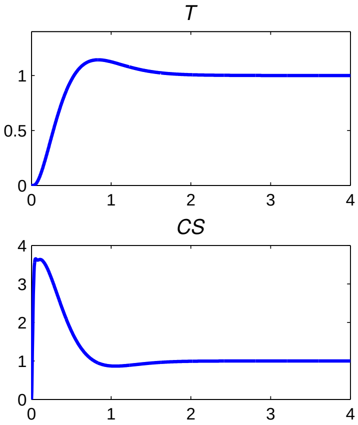
Response of control signal u to measurement noise n is characterized by

$$G_{un} = -\frac{C}{1+PC} = -CS = -\frac{T}{P}$$

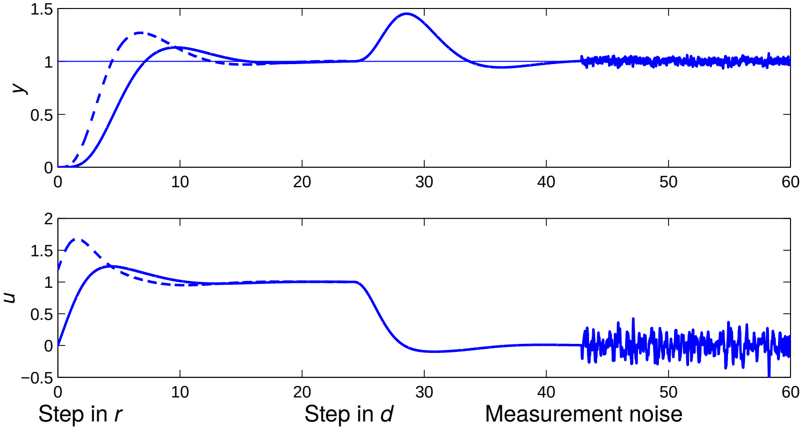
Responses of y and u to reference signal r are characterized by

$$G_{yr} = \frac{PC}{1+PC} = T, \quad G_{ur} = \frac{C}{1+PC} = CS$$

Visualizing the GOF - Time Responses

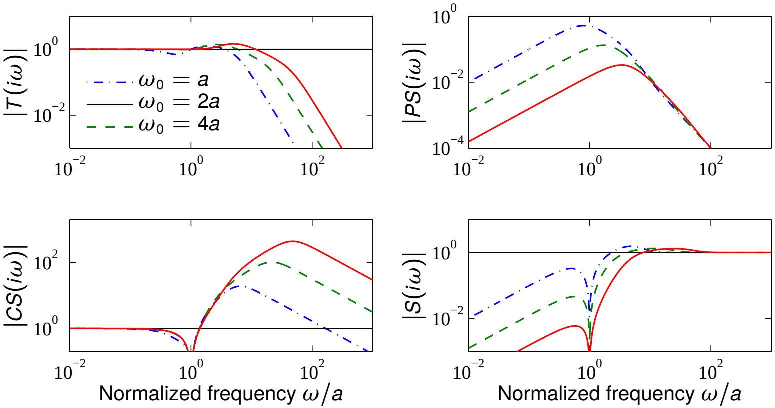


Another Way to Show Time Responses



Interactive Learning Module

Visualizing the GOF - Frequency Responses



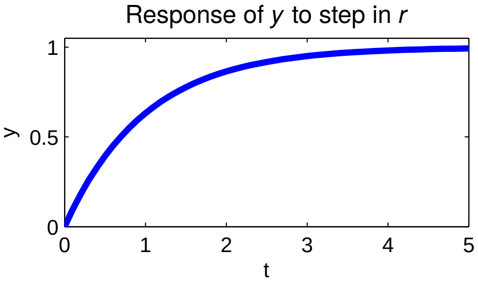
Gain curves for the GOF is a good way to get a quick overview of a feedback system. Curves represent three different controller are designed for a nano-positioner

Discuss!

A Warning!

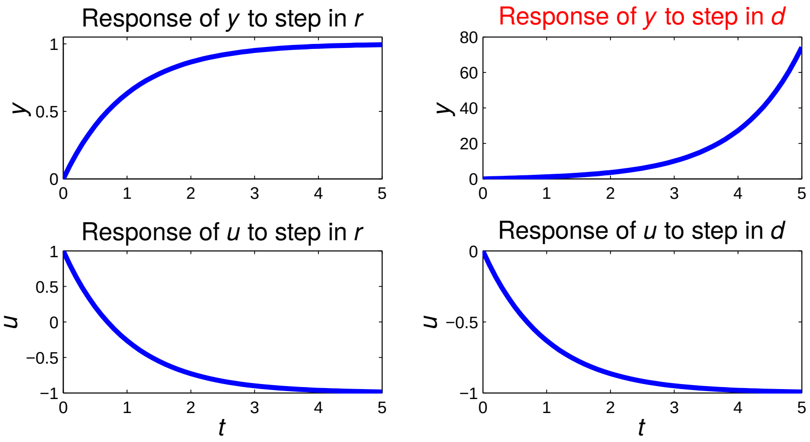
Remember to always look at all responses!

The step response below looks fine



BUT ...

All Four Responses



The system is unstable!

What is going on?

The Gang of Four

Process: $P(s) = \dfrac{1}{s-1}$, Controller: $C(s) = \dfrac{s-1}{s}$

Loop transfer function: $L = PC = \dfrac{1}{s-1} \times \dfrac{s-1}{s} = \dfrac{1}{s}$

Notice cancellation of the factor $s-1$! The Gang of Four

$$\frac{PC}{1+PC} = \frac{1}{s+1} \qquad \frac{P}{1+PC} = \frac{s}{(s+1)(s-1)}$$

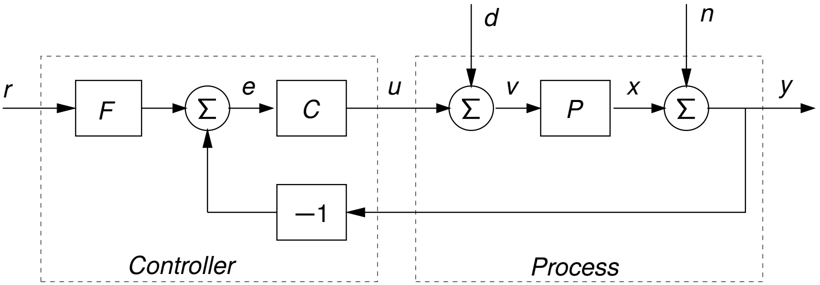
$$\frac{C}{1+PC} = \frac{s-1}{s+1} \qquad \frac{1}{1+PC} = \frac{1}{s+1}$$

Response of y to step in load disturbance d

$$G_{yd}(s) = \frac{P}{1+PC} = \frac{s}{(s+1)(s-1)}$$

This transfer function represents an unstable system

2DOF System - The Gang of Seven



The Gang of Four and transfer functions from reference r to e, x, y and u

$$G_{er} = \frac{F}{1+PC}, \quad G_{xr} = \frac{PCF}{1+PC}, \quad G_{yr} = \frac{PCF}{1+PC}, \quad G_{ur} = \frac{CF}{1+PC},$$

Some Observations

- To fully understand a system it is necessary to look at all transfer functions
- A system based on error feedback is characterized by *four* transfer functions *The Gang of Four*
- The system with a controller having two degrees of freedom is characterized by *seven* transfer function *The Gang of Seven*
- It may be strongly misleading to only show properties of a few systems for example the response of the output to command signals. A common omission in many papers and books.
- The properties of the different transfer functions can be illustrated by their transient or frequency responses.

Feedback Fundamentals

Theme: Exploiting representations of $G(i\omega)$

The Sensitivity Functions

- Sensitivity function $S = \dfrac{1}{1+PC} = \dfrac{1}{1+L}$
- Complementary sensitivity function $T = 1 - S = \dfrac{PC}{1+PC} = \dfrac{L}{1+L}$
- Input sensitivity function $G_{xl} = \dfrac{P}{1+PC} = PS$
- Output sensitivity function $G_{un} = \dfrac{C}{1+PC} = CS$

are called sensitivity functions. They have interesting properties and useful physical interpretations. We have

- The functions S and T only depend on the loop transfer function L
- $S + T = 1$
- Typically $S(0)$ small and $S(\infty) = 1$ and consequently $T(0) = 1$ and $T(\infty)$ small

Poles, Zeros and Sensitivity Functions

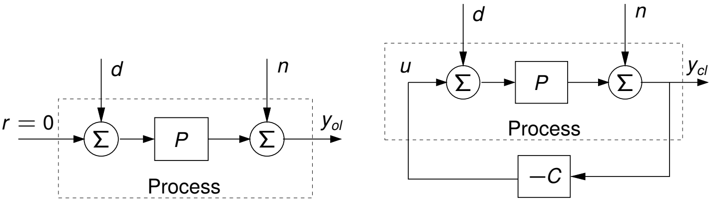
The sensitivity functions depend only on the loop transfer function

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}$$

Notice that

- The sensitivity function S is zero and the complementary sensitivity function is one at the poles of L
- The sensitivity function S is one and complementary sensitivity function T is zero at the zeros of L

Disturbance Attenuation



Output without control $Y = Y_{ol}(s) = N(s) + P(s)D(s)$

Output with feedback control

$$Y_{cl} = \frac{1}{1+PC}\big(N + PL\big) = \frac{1}{1+PC} Y_{ol} = SY_{ol}$$

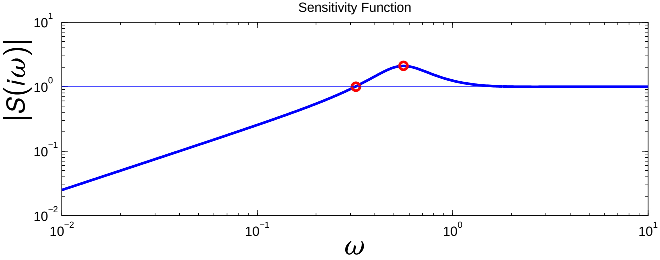
The effect of feedback is thus like sending the open loop output through a system with the transfer function $S = 1/(1+PC)$. Disturbances with frequencies such that $|S(i\omega)| < 1$ are reduced by feedback, disturbances with frequencies such that $|S(i\omega)| > 1$ are amplified by feedback.

Assessment of Disturbance Reduction - Bode

We have

$$\frac{Y_{cl}(s)}{Y_{cl}(s)} = S(s) = \frac{1}{1 + P(s)C(s)}$$

Feedback attenuates disturbances of frequencies ω such that $|S(i\omega)| <$.
It amplifies disturbances of frequencies such that $|S(i\omega)| > 1$

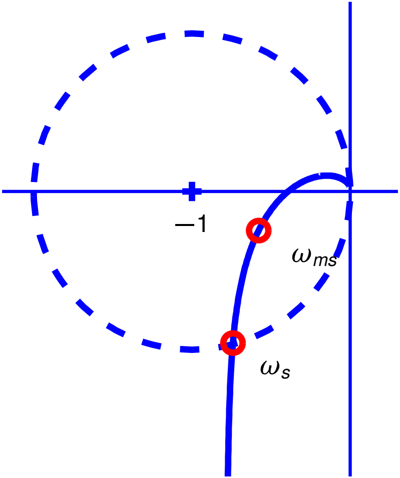


Assessment of Disturbance Reduction - Nyquist

$$\frac{Y_{cl}}{Y_{ol}} = \frac{1}{1 + PC} = S$$

Geometric interpretation: Disturbances with frequencies inside the circle are amplified by feedback. Disturbances with frequencies outside are reduced.

Disturbances with frequencies less than ω_s are reduced by feedback.



Properties of the Sensitivity Function

- Can the sensitivity be small for all frequencies?

  No we have $S(\infty) = 1$!
- Can we have $|S(i\omega)| \leq 1$?

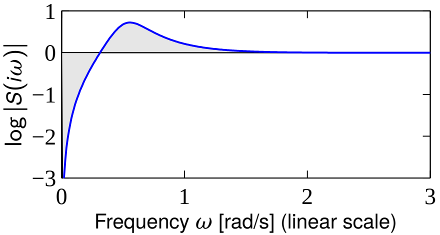
  If the Nyquist curve of $L = PC$ is in the first and third quadrant!

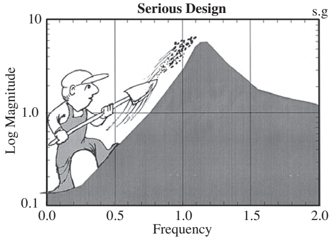
  Passive systems!
- Bode's integral, p_k RHP poles of $L(s)$, z_k RHP zeros of $L(s)$

$$\int_0^\infty \log |S(i\omega)| d\omega = \pi \sum \mathrm{Re}\, p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

- Fast poles (and slow zeros) in the RHP are bad!
- Useful to let the loop transfer function go to zero rapidly for high frequencies (*high-frequency roll-off*) because the last term vanishes
- The "water-bed effect". Push the curve down at one frequency and it pops up at another! Design is a compromise!

The Water Bed Effect - Bode's Integral





$$\int_0^\infty \log|S(i\omega)| d\omega = \pi \sum \mathrm{Re}\, p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

The sensitivity can be decreased at one frequency at the cost of increasing it at another frequency.

Feedback design is a trade-off

Robustness

Effect of small process changes dP on closed loop response $T = PC/(1+PC)$

$$\frac{dT}{dP} = \frac{C}{(1+PC)^2} = \frac{ST}{P}, \qquad \frac{dT}{T} = S\frac{dP}{P}$$

Effect of large process canges: how much ΔP can the process change without making the closed loop unstable?

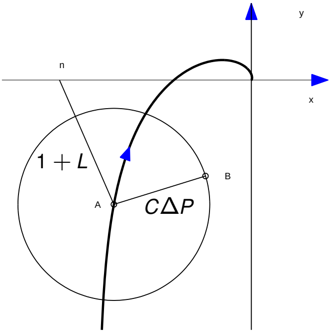
$$|C\Delta P| < |1+PC|$$

or

$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

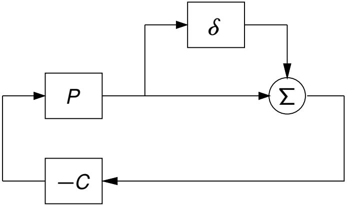
ΔP must be stable

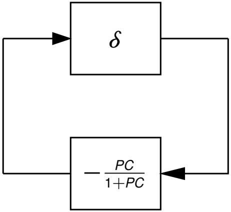
Nyquist plot



Another View of Robustness

A feedback system where the process has multiplicative uncertainty, i.e. $P + \Delta P = P(1 + \delta)$, where $\delta = \Delta P/P$ is the relative error, can be represented with the following block diagrams





The small gain theorem gives the stability condition

$$|\delta| = \frac{|\Delta P|}{|P|} < \left|\frac{1 + PC}{PC}\right| = \frac{1}{|T|}$$

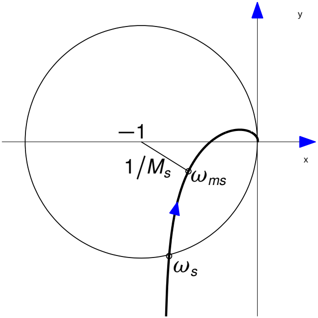
Same result as obtained before!

Robustness and Sensitivity

Gain and phase margins

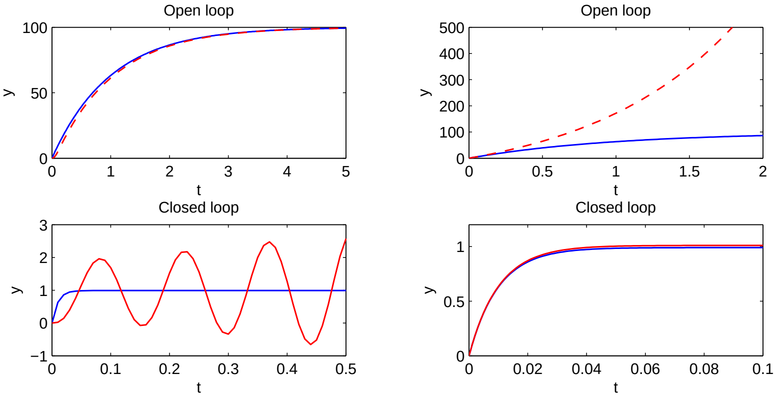
$$g_m \geq \frac{M_s}{M_s - 1}, \quad \varphi_m \geq 2 \arcsin \frac{1}{2M_s}$$

Constraints on both gain and phase margins can be replaced by constraints on maximum sensitivity M_s.



- $M_s = 2$ guarantees $g_m \geq 2$ and $\varphi_m \geq 30^\circ$
- $M_s = 1.6$ guarantees $g_m \geq 2.7$ and $\varphi_m \geq 36^\circ$
- $M_s = 1.4$ guarantees $g_m \geq 3.5$ and $\varphi_m \geq 42^\circ$
- $M_s = 1$ guarantees $g_m = \infty$ and $\varphi_m \geq 60^\circ$

When are Two Systems Close?

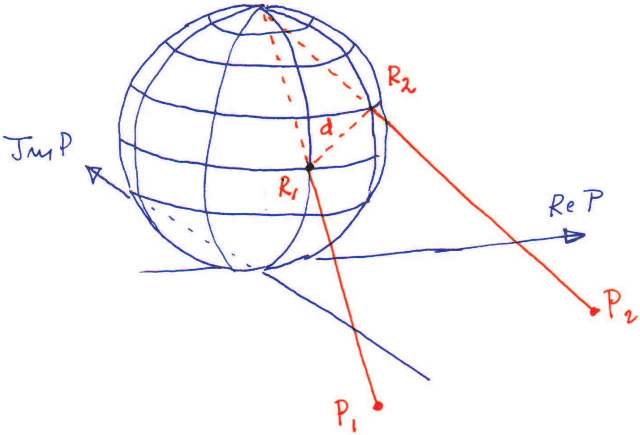


Comparing step responses can be misleading!

Frequency responses are better

Better to compare closed loop responses

Vinnicombe's Metric



Summary of the Sensitivity Functions

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}, \quad M_s = \max|S(i\omega)|, \quad M_t = \max|T(i\omega)|$$

The value $1/M_s$ is the shortest distance from the Nyquist curve of the loop transfer function $L(i\omega)$ to the critical point -1.

$$S = \frac{\partial \log T}{\partial \log P} = \frac{Y_{cl}(s)}{Y_{ol}(s)}$$

How much can the process be changed with stable ΔP without making the closed loop system unstable?

$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

Bode's integral the water bed effect.

$$\int_0^\infty \log|S(i\omega)|d\omega = \pi \sum \mathrm{Re}\, p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

Requirements and Sensitivity Functions

Disturbances

- Effect of feedback: $y_{cl} = Sy_{ol}$
- Load disturbances: $G_{yd} = PS$
- Measurement noise: $G_{un} = -CS$

Process uncertainty

- Small variations: $\delta T/T = S\delta P/P$
- Large variations: $|\Delta P|/|P| \leq 1/|T|$, stable ΔP
- Gain and phase and sensitivity margins: g_m, φ_m, $s_m = \frac{1}{M_s}$

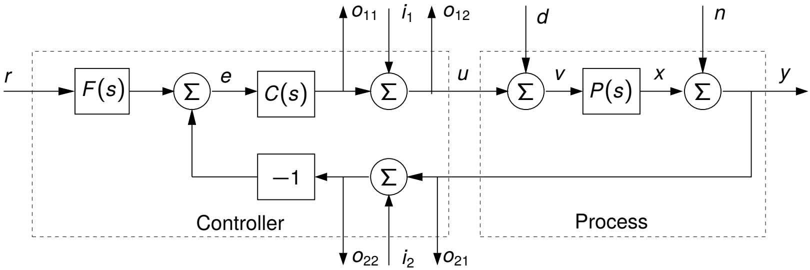
Command signal following

- Error feedback: $G_{yr} = T$, $G_{ur} = CS$
- 2DOF: $G_{yr} = TF$, $G_{ur} = CSF$

Testing Requirements

Introduce test points in the control system!

Use test signals in design phase and on the real system!



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Theme: Exploiting representations of $G(i\omega)$

Summary

- Systems with error feedback and systems with two degrees of freedom 2DOF

  A system with two degrees of freedom allows separation of command signal response from the other requirements
- A system with error feedback is characterized by four transfer functions (Gang of Four GOF: S, T, PS, CS)
- A system with two degrees of freedom is characterized by seven transfer functions (Gang of Seven = GOF, FS, FT and FCS)
- Several transfer functions are required to understand a feedback system. Analysis and specifications should cover all transfer functions!
- Performance and robustness can conveniently be characterized in terms of 4 transfer functions or 7 transfer functions
- Introduce test points in the control system to measure all relevant transfer functions