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Parametric Local Stability of a Multi-Converter System

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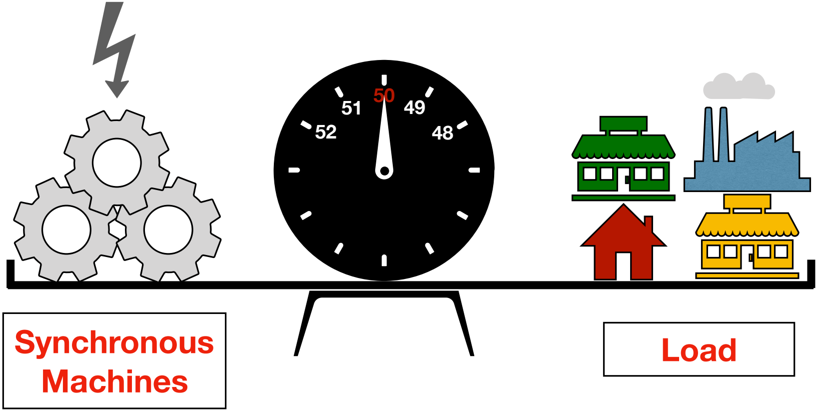
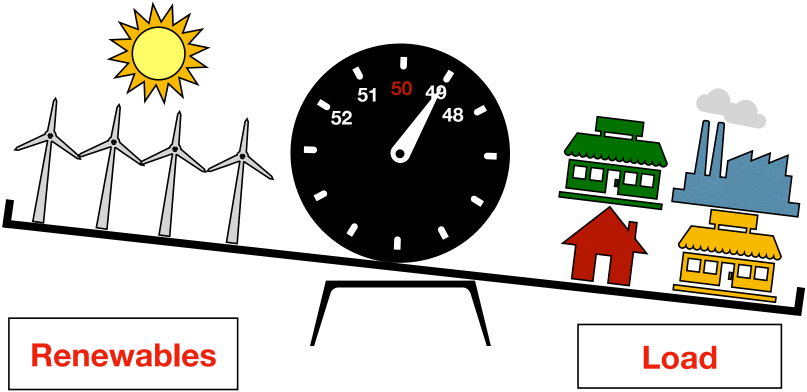
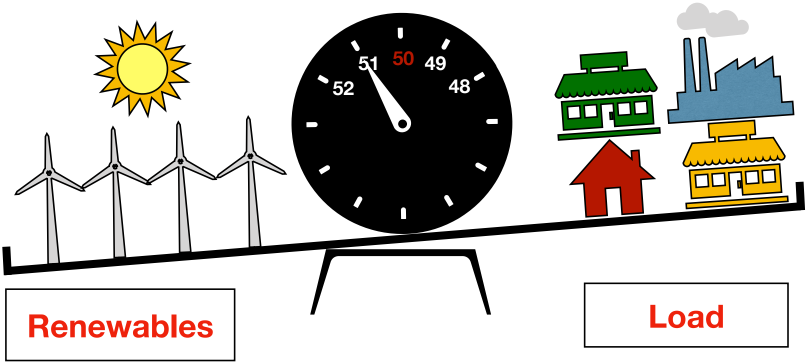
Motivation: classical operation of power system



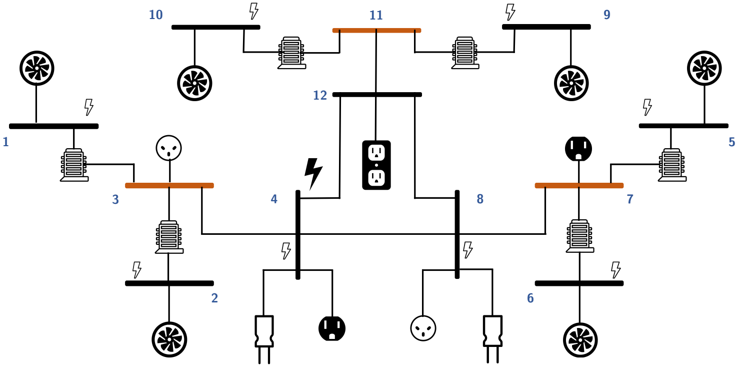
Motivation: changes in grid operation



Motivation: changes in grid operation



Generic analysis challenges



C1. assumptions on ***quasi-stationary*** steady state

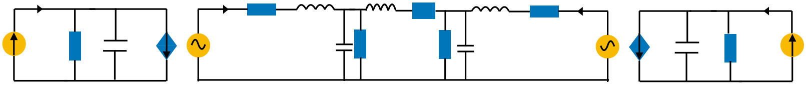
C2. transmission lines with ***non-zero*** transfer conductance

C3. interaction of grid units with ***line dynamics***

C4. ***dq-frame*** in multi-machine case study

C5. need for ***fully decentralized*** stability conditions

Modelling from first-order principle



Balanced, three-phase, averaged DC/AC converter in ***dq-frame*** $\int_0^\tau \omega^* d\tau$

$$C_{dc}\dot{v}_{dc} = -G_{dc}\, v_{dc} - \frac{1}{2}\mathrm{diag}(m^\top)\, i_{dq} + i_{dc}$$

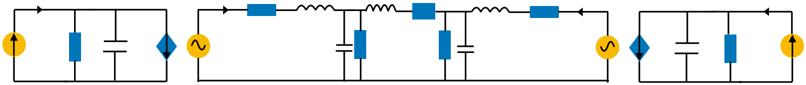
$$L\dot{i}_{dq} = -\mathbf{Z}_R\, i_{dq} + \frac{1}{2}\,\mathrm{diag}(m)\, v_{dc} - v_{dq}$$

$$C\dot{v}_{dq} = -\mathbf{Z}_V\, v_{dq} + i_{dq} - \mathbf{B}\, i_{net,dq}$$

$$L_{net}\dot{i}_{net,dq} = -\mathbf{Z}_{net}\, i_{net,dq} + \mathbf{B}^\top v_{dq}$$

$\mathbf{B}$: incidence matrix,
$\mathbf{Z}_R = R\,\mathbf{I} + \mathbf{J}\,\omega^* L, \mathbf{Z}_V = G\,\mathbf{I} + \mathbf{J}\,\omega^* C, \mathbf{Z}_{net} = R_{net}\,\mathbf{I} + \mathbf{J}\,\omega^* L_{net}$: impedance matrices, with $\mathbf{J} = \mathbf{I} \otimes \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$

Synchronous machine matching control



- synchronous machine ***matching control***

$$\dot{\gamma} = \eta\,(v_{dc} - v_{dc}^*), \eta > 0\,,$$

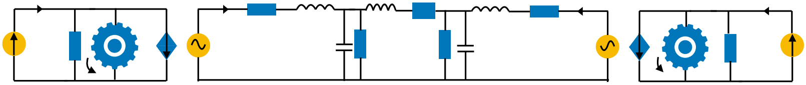
$$m = \mu \begin{bmatrix} -\sin(\gamma) \\ \cos(\gamma) \end{bmatrix},\ 1 > \mu > 0$$

- ***controllable*** current source

$$i_{dc} = i_{dc}^* - K_p(v_{dc} - v_{dc}^*) - G_{dc}v_{dc}^*,\ K_p > 0$$

[Jouini et al.'16, Arghir et al.'18]

dq trafo with respect to desired frequency



Closed-loop DC/AC converter in ***dq-frame***

$$\dot{\gamma} = \eta \left(v_{dc} - v_{dc}^{*} \mathbf{1}_n \right)$$

$$C_{dc} \dot{v}_{dc} = -\hat{K}_p \left(v_{dc} - v_{dc}^{*} \mathbf{1}_n \right) - \frac{1}{2} \text{diag}(\mu) \mathbf{R}(\gamma)^{\top} i_{dq} + i_{dc}^{*}$$

$$L \dot{i}_{dq} = -\mathbf{Z}_R \, i_{dq} + \frac{1}{2} \text{diag}(\mu) \mathbf{R}(\gamma) \, v_{dc} - v_{dq}$$

$$C \dot{v}_{dq} = -\mathbf{Z}_V \, v_{dq} + i_{dq} - \mathbf{B} \, i_{net,dq}$$

$$L_{net} \dot{i}_{net,dq} = -\mathbf{Z}_{net} \, i_{net,dq} + \mathbf{B}^{\top} v_{dq}$$

where $\hat{K}_p = K_p + G_{dc}, \; K_p > 0$

Steady state manifold $\mathcal{M}$: Invariance under rotation angle θ_0

- steady-state ***angles***

$$[\gamma] = \{\gamma \in \mathbf{T}_n \,|\, \omega^* t + \gamma(0) + \theta_0 \,\text{span}\{\mathbf{1}_n\}\}$$

- steady-state ***frequency*** and ***DC voltages***

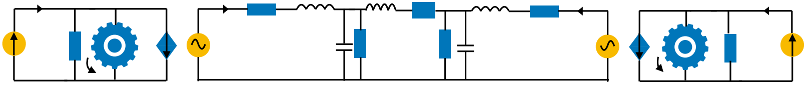
$$[\omega] = \{\omega \in \mathbf{R}^n_{\geq 0} | \omega = \omega^*\}$$
$$[v_{dc}] = \{v_{dc} \in \mathbf{R}^n_{\geq 0} | v_{dc} = v^*_{dc}\}$$

- steady-state ***AC quantities***

$$[i_{dq}] = \{i_{dq} \in \mathbf{R}^{2n} | i^*_{dq} + \text{span}\{\mathcal{R}(\theta_0) i^*_{dq}\}\}$$
$$[v_{dq}] = \{v_{dq} \in \mathbf{R}^{2n} | v^*_{dq} + \text{span}\{\mathcal{R}(\theta_0) v^*_{dq}\}\}\}$$
$$[i_{net,dq}] = \{i_{net,dq} \in \mathbf{R}^{2m} | i^*_{net,dq} + \text{span}\{\mathcal{R}(\theta_0) i^*_{net,dq}\}\}\}$$

$\mathcal{R}(\theta_0)$: rotating matrix with angle θ_0

Linearisation around a desired steady state x^*



Linearised converter model at $x^* = \left[\, \gamma^* \;\; v_{dc}^* \mathbf{1}_n \;\; i_{dq}^* \;\; v_{dq}^* \;\; i_{net,dq}^* \,\right]^\top$

$$\dot{x} = \left[\begin{array}{cc|ccc} 0 & \eta \mathbf{I} & 0 & 0 & 0 \\ -\mathbf{A}_{21} & -C_{dc}^{-1}\hat{K}_p \mathbf{I} & -\mathbf{A}_{32}^\top & 0 & 0 \\ \hline \mathbf{A}_{31} & \mathbf{A}_{32} & -L^{-1}\mathbf{Z}_R & -L^{-1}\mathbf{I} & 0 \\ 0 & 0 & C^{-1}\mathbf{I} & -C^{-1}\mathbf{Z}_v & -C^{-1}\mathbf{B} \\ 0 & 0 & 0 & L_{net}^{-1}\mathbf{B}^\top & -L_{net}^{-1}\mathbf{Z}_{net} \end{array}\right] x$$

$v(x^*) = \text{span}\left\{\left[\mathbf{1}_n \; \mathbf{0} \; \mathbf{J}\, i_{dq}^* \; \mathbf{J}\, v_{dq}^* \; \mathbf{J}\, i_{net,dq}^*\right]\right\} \in \ker(A(x^*)),\; v(x^*) = \left[\, v_1(x^*) v_2(x^*) \,\right]$

Definition of a steady state tangent space $T_x^* \mathcal{M}$

- tangent ***angles*** subspace

$$[\gamma] = \{\gamma \in \mathbf{T}_n | \, \omega^* t + \gamma(0) + \text{span}\{\mathbf{1}_n\}\}$$

- tangent ***frequency*** and ***DC voltages*** subspace

$$[\omega] = \{\omega \in \mathbf{R}^n_{\geq 0} | \omega = \omega^*\}$$
$$[v_{dc}] = \{v_{dc} \in \mathbf{R}^n_{\geq 0} | v_{dc} = v^*_{dc}\}$$

- tangent ***AC quantities*** subspace

$$[i_{dq}] = \{i_{dq} \in \mathbf{R}^{2n} | i^*_{dq} + \text{span}\{\mathbf{J}\, i^*_{dq}\}\}$$
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$$[i_{net,dq}] = \{i_{net,dq} \in \mathbf{R}^{2m} | i^*_{net,dq} + \text{span}\{\mathbf{J}\, i^*_{net,dq}\}\}\}$$

Separable Lyapunov analysis for systems with symmetries

A class of ***partitioned*** linear systems

$$\dot{x} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] x \tag{5}$$

Three assumptions to go:

- the sub-block A_{11} is ***Hurwitz***
- there ***exists*** a vector $v = \begin{bmatrix} v_1 \; v_2 \end{bmatrix}^\top : A \cdot \text{span}\{v\} = 0$
- . . . (added in a later step)

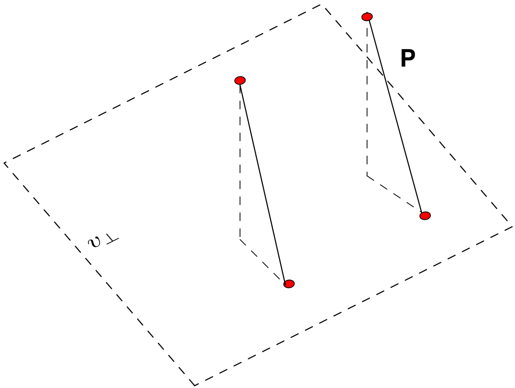
Derive ***fully decentralized*** conditions so that span$\{v\}$ is ***asymptotically stable*** $\Leftrightarrow$ the system matrix has ***all its eigenvalues*** in the ***left-half*** plane except for ***one*** at ***zero***

Construction of the Lyapunov function

- define LF as an ***oblique projector*** into $v_\perp$ in **P**-columns direction

$$\mathcal{V}(x) = x^\top \left(\mathbf{P} - \frac{\mathbf{P}vv^\top\mathbf{P}}{v^\top\mathbf{P}v} \right) x : \; A \cdot \mathrm{span}\{v\} = 0,$$

P is a ***positive definite*** matrix.



Separable Lyapunov function

- fix the ***structure*** of the matrix $\mathcal{Q}(\mathbf{P})$ so that $\mathbf{P}A + A^\top \mathbf{P} = -\mathcal{Q}(\mathbf{P})$

$$\mathcal{Q}(\mathbf{P}) = \left[\begin{array}{c|c} \mathbf{Q}_1 & H(\mathbf{P})^\top \\ \hline H(\mathbf{P}) & H(\mathbf{P})\mathbf{Q}_1^{-1}H^\top(\mathbf{P}) + \mathbf{Q}_2 \end{array}\right], H(\mathbf{P}) = A_{12}^\top \mathbf{P}_1 + \mathbf{P}_2 A_{21}$$

  $\mathbf{Q}_1$ is ***positive definite***, $\mathbf{Q}_2$ is ***positive semi-definite*** with respect to v_2

- choose block diagonal matrix $\mathbf{P}$

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{P}_1 & 0 \\ \hline 0 & \mathbf{P}_2 \end{array}\right], \mathbf{P}_1 = \mathbf{P}_1^\top, \mathbf{P}_2 = \mathbf{P}_2^\top$$

- $\mathbf{P}A + A^\top \mathbf{P} = -\mathcal{Q}(\mathbf{P})$ Algebraic Ricatti Equation
- ***Third*** assumption: define $\mathbf{F} = A_{22} + A_{21}\mathbf{Q}_1^{-1}P_1 A_{12}$
  - ***necessary*** and ***sufficient*** condition: $\left(\mathbf{F}, A_{21}\mathbf{Q}_1^{-1/2}\right)$ is stabilizable and $(\mathbf{F}, D)$ is detectable, where $DD^\top = A_{12}^\top P_1 \mathbf{Q}_1^{-1} P_1 A_{12} + \mathbf{Q}_2$
  - $\mathbf{F}$ is a ***Hurwitz*** matrix (stricter condition)

Main result and contextualization

If ***three*** assumptions are satisfied, then span$\{v\}$ is ***asymptotically stable***

The matrix **F** is Hurwitz:

- ***small-gain*** theorem for A_{22} Hurwitz
- ***passive*** system: LE, KYP Lemma

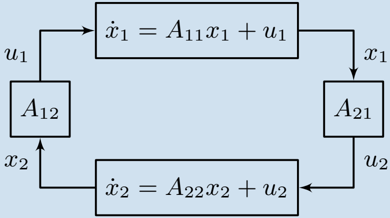
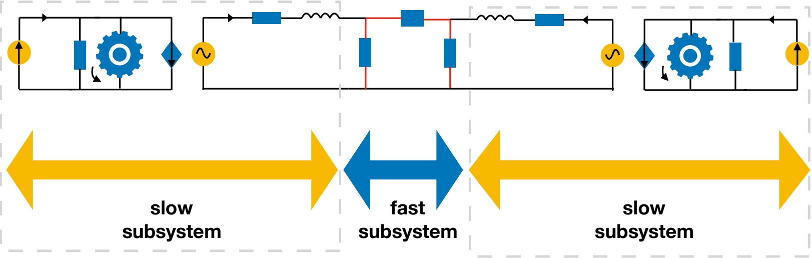


Figure: Interconnected closed-loop system

ECC Paper: application to reduced model



- application of ***Tikhonov's*** Theorem
- ***slow*** subsystem: $\{\gamma, v_{dc}, i_{dq}\}$ and ***fast*** subsystem: $\{v_{dq}, i_{net,dq}\}$
- fast subsystem is ***exponentially stable*** relative to $\left[\, v^*_{dq} \;\; i^*_{net,dq} \,\right]^\top$
- ***sufficient*** and ***fully decentralized*** condition: reactive power support and resistive damping
- arXiv paper: ***full-order*** model analysis [Jouini et al.'19]

Application: Local stability of multi DC/AC converter

Algebraic synchronization condition at the k-th converter:

$$\boxed{16\, Q^*_{x,k} > \frac{\mu_k^2 v_{dc}^{*2}}{R}} \qquad (6)$$

where $Q^*_{x,k} = \frac{1}{2} v^*_{dc} \mu_k J r(\gamma^*_k)^\top i^*_{dq}$, $\mathbf{Q}_1 = \mathbf{I}$, $\mathbf{Q}_2 = \mathbf{I} - \frac{v_2(x^*) v_2(x^*)^\top}{v_2(x^*)^\top v_2(x^*)}$

- ***sufficient*** condition (6) is ***fully decentralized***
- if (6) is ***true***, then span$\{v(x^*)\}$ is ***locally*** asymptotically stable
- ***sufficient*** reactive power support, ***resistive damping***
- reinforced by ***virtual*** impedance control:

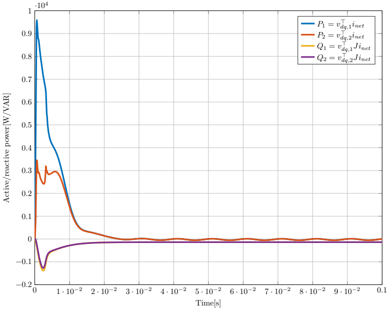
$$m' = m + 2k_m i_{dq}/v_{dc},\; i'_{dc} = i_{dc} + k_m {i_{dq}}^\top i_{dq}/v_{dc},\; k_m > 0$$

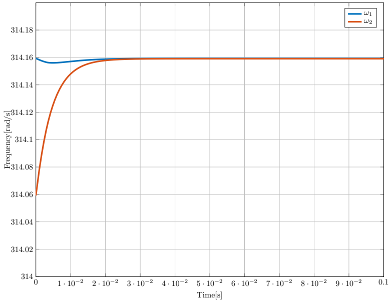
- aligned with ***practitioners*** insights [Wang et al.'15, Barbanov et al.'16]

Simulations: Eye candy

- identical DC/AC converters initialized differently $v_{dc,1}(0) \neq v_{dc,2}(0)$
- same input pair (i^*_{dc}, μ^*)
- stability condition satisfied for $k_m > 47.84$

	Converter 1	Converter 2	RL Line
i^*_{dc}	$1.9 \cdot 10^3$	$1.9 \cdot 10^3$	–
v^*_{dc}	10^3	10^3	–
C_{dc}	10^{-3}	10^{-3}	–
G_{dc}	10^{-5}	10^{-5}	–
η	10^{-4}	10^{-4}	–
R	0.1	0.1	–
L	$5 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	–
C	10^{-5}	10^{-5}	–
μ^*	0.33	0.33	–
G	0.05	0.05	–
K_p	0.2	0.2	–
R_{net}	–	–	2
L_{net}	–	–	10^{-4}





Reminder: generic analysis challenges

C1. assumptions on ***quasi-stationary*** steady state

C2. transmission lines with ***non-zero*** transfer conductance

C3. interaction of grid units with ***line dynamics***

C4. ***dq-frame*** in multi-machine case study

C5. need for ***fully decentralized*** stability conditions

Our remedy in this work

R1. modelling from ***first-order*** principles

R2. stability analysis of ***reduced-order*** DC/AC converter model

R3. extensions to ***full-order*** DC/AC converter model

R4. dq-transformation with respect to ***steady state frequency***

R5. scalable LF: ***parametrised*** Lyapunov/Ricatti equation

Future venues

- study of conservativeness of derived stability condition
- extensive simulations of multi-converter system

Thank you for your attention