Predictive Control - Homework 1

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In this homework exercise we recapitulate theory for discrete time signals and systems in assignments 1-2. Recursive Least square estimation (RLS) is treated in assignment 3. The exercise also gives the opportunity to practice Julia/Matlab/Simulink. E-mail your solutions in pdf-format to `marcus.greiff@control.lth.se`. Attach any code you might have used. Include figures in the pdf report, and verify that any included code is formatted such that copy-paste works (not always the case when using mcode.sty).

1. Provide a discrete-time state space representation of the system

$$G(s) = \frac{s+1}{s^2}. \quad (1)$$

a. Do the calculations by hand, with a parametric sample period h.

b. Do the calculations using Julia/Matlab, with sample period $h = 0.1$.

c. How does the sampling period affect the result, and what would be a reasonable sampling time to choose in this case?

2. This problem concerns the notion of stability for discrete-time systems.

a. Give the transfer function $H(z)$ from r to y of the positive feedback interconnection in Figure 1, where $H_1(z) = z+2$, $H_2(z) = (z^2+2z+1)^{-1}$ and $H_3(z) = 3z(z+2)^{-1}$.

b. Where are the poles of the closed loop pulse transfer function $H(z)$ located? Given the location of the poles, would you say that the system is unstable, marginally stable or asymptotically stable? Plot the response y when r is a step function to support your stability claim.

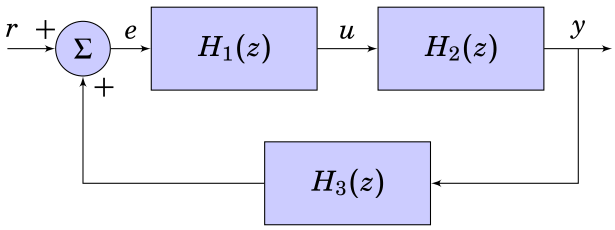

Figure 1 Interconnection in assignment 2.

3. In this problem we will implement and investigate an RLS scheme.

a. Use Julia/Matlab/Simulink to demonstrate RLS identification of $\Theta = [a\ b]^T$ for $y_k = ay_{k-1} + bu_{k-1} + w_k$, where w_k is a Gaussian white noise.

b. Comment on the choice of input signal and how it affects the result.

c. Give an interpretation of what the covariance matrix $\mathbf{P}_k$ in the RLS scheme is. How does the initial value of $\mathbf{P}_0$ affect the estimate of Θ?

d. Assume a is time varying, how can the method be modified to identify a? How do you use information about variations in a to tune our algorithm?