Example 5.2 - short recapitulation

Consider a system described by the model

$$\frac{dy}{dt} = -ay + bu \qquad (5.6a)$$

where u is the control variable and y is the measured output.
Assume that we want to obtain *a closed-loop system* described by

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \qquad (5.6b)$$

Let the controller be given by

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \qquad (5.7)$$

The controller has two parameters. If they are chosen to be

$$\begin{aligned} \theta_1 &= \theta_1^0 = \frac{b_m}{b} \\ \theta_2 &= \theta_2^0 = \frac{a_m - a}{b} \end{aligned} \qquad (5.8)$$

the input-output relations of the system and the model are the same.
This is called perfect model-following.

The problem is that **we don't know exact values for a and b**, so we can't choose θ_1 and θ_2 as in eq.(5.8). Instead we let them change according to the following scheme based on an update law called **the MIT rule**.

Introduce the error

$$e = y - y_m$$

where y denotes the output of the closed-loop system. It follows from the derivation in *Example 5.2, (page 190)* that the equations for updating the controller parameters are

$$\begin{aligned} \frac{d\theta_1}{dt} &= -\gamma \left(\frac{a_m}{p + a_m} u_c \right) e \\ \frac{d\theta_2}{dt} &= \gamma \left(\frac{a_m}{p + a_m} y \right) e \end{aligned} \qquad (5.9)$$

where γ is called **the adaptation gain**.

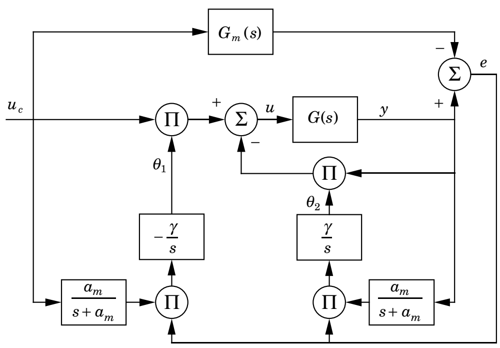

Figure 1 Block diagram for the model-reference controller in Example 5.2

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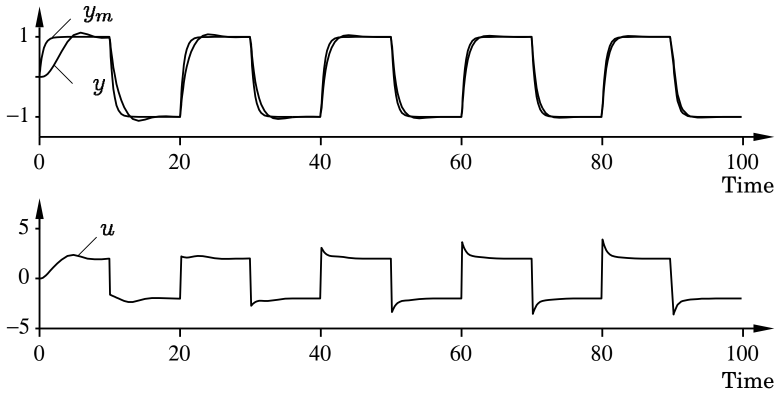

Figure 5.5 Simulation of the system in Example 5.2 using an MRAS. The parameter values are $a = 1, b = 0.5, a_m = b_m = 2$, and $\gamma = 1$.

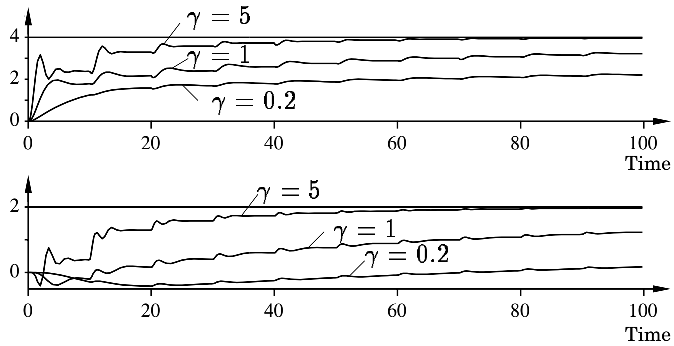

Figure 5.6 Controller parameters θ_1 and θ_2 for the system in Example 5.2 when $\gamma = 0.2, 1$ and 5.

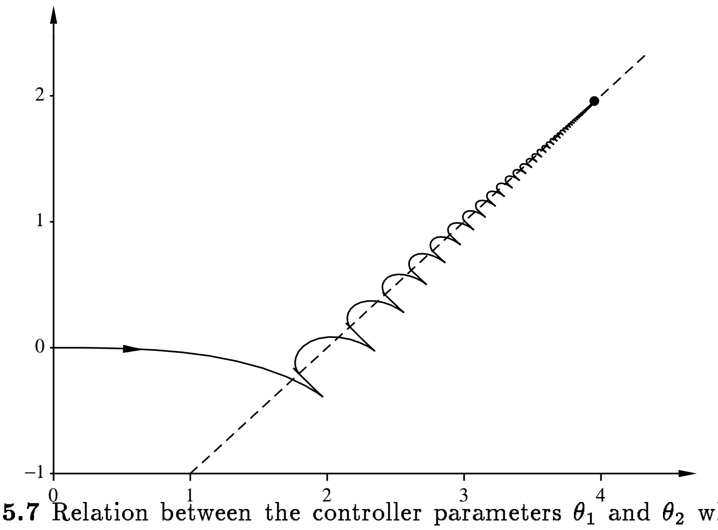

Figure 5.7 Relation between the controller parameters θ_1 and θ_2 when the system in Example 5.2 is simulated for 500 time units. The dashed line shows the the line $\theta_2 = \theta_1 - a/b$. The dot indicates the convergence point.