FRTN15 Predictive Control—Exercise 6

1. Show that the tracking error fulfills the recursive equation
$e_k(t) = [(1 - Q(q))(1 - T_c(q))]y_d(t) + [Q(q)(1 - L(q)T_c(q))]e_{k-1}(t)$
on lecture 11. What happens if $Q = 1$? If $Q \neq 1$?.

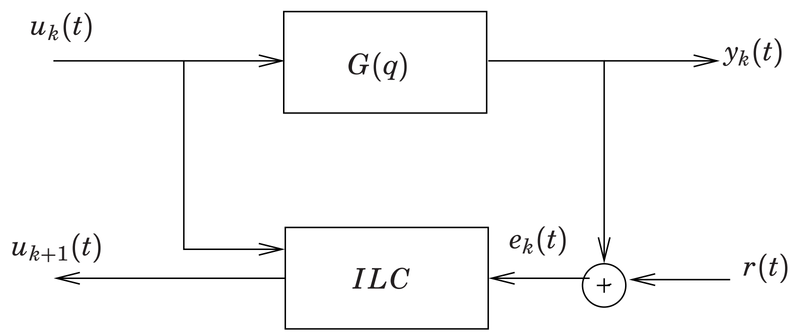
2. Consider the system

$$G(q) = \frac{0.09516}{q - 0.9048}.$$

It is controlled using ILC (see Figure 1) such that the control signal at an iteration k is given by:

$$u_{k+1}(t) = u_k(t) + L(q)e_k(t)$$

where $e_k(t) = r(t) - y_k(t)$.



Figur 1 AN ILC feedback system.

Study the convergence of the ILC iterations for $L(q) = 1$ and $L(q) = q$.
Hint: The Nyquist plots of $G(q)L(q)$ for the two chosen L are shown in Figure 2.

3.

a. Show that the system

$$\dot{x} = -x + u, \quad x(0) = x_0, \tag{1}$$

$$y = x \tag{2}$$

with transfer function

$$G_1(s) = \frac{1}{(s+1)} \tag{3}$$

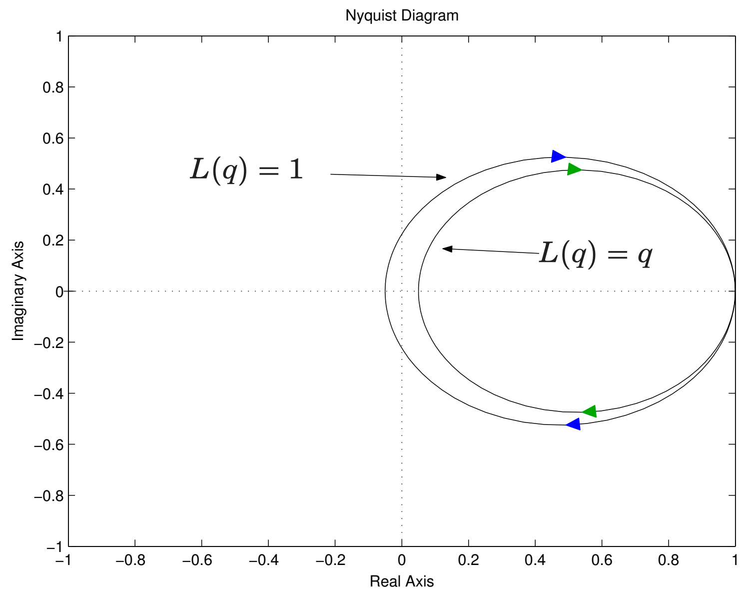
is strictly positive real (SPR) and that the storage function

$$V(x) = \frac{1}{2}x^T x$$

fulfills the passivity property

$$V(x(t)) = V(x(0)) + \int_0^t y^T(\tau)u(\tau)dt - \int_0^t x^T(\tau)x(\tau)d\tau \tag{4}$$

What is the interpretation of all the three terms on the right-hand side of Eq. (4)?



Figur 2 Nyquist plots for $G(q)L(q)$.

b. Show that the transfer function

$$G_2(s) \quad = \quad \frac{1}{(s+1)^2} \tag{5}$$

is not positive real.

4. Ex 12.2, Predictive and Adaptive Control

Consider Iterative Learning Control (ILC) given by the equations:

$$\begin{aligned} y_k(t) &= G_v(q)u_k(t) \\ e_k(t) &= r(t) - y_k(t) \\ u_k(t) &= Q(q)[u_{k-1}(t) + L(q)e_{k-1}(t)] \end{aligned}$$

where $G_c(q)$ is the closed-loop transfer function of the system and q is the forward time shift operator. Assume that $Q(q) = 1$ and that

$$G_c(q) = \frac{1}{(q-0.7)(q-0.9)}, L(q) = k(q-0.5)(q-0.7)(q-0.9)$$

where k is a positive constant. Determine a condition on k which, if fulfilled, guarantees that the error of the resulting ILC scheme converges.

5. Dead-beat ILC

Consider the system

$$y = G(q)u = \frac{q-2}{(q+0.5)(q+0.9)}u$$

Describe how to interpret the formula

$$u = \frac{(q+0.5)(q+0.9)}{q-2}y_r$$

as a non-causal filter giving a bounded signal u fullfilling $y = y_r$, where y_r is a given reference value. Simulate with y_r equal to e.g. a step function.