Today’s Goal:
◮ To know models and compensation methods for backlash
◮ Be able to analyze the effect of quantization errors

Linear and Angular Backlash

Backlash (glapp) is
◮ present in most mechanical and hydraulic systems
◮ increasing with wear
◮ bad for control performance
◮ may cause oscillations

Note: A gear box without any backlash will not work if temperature rises

Example: Parallel Kinematic Robot
Gantry-Tau robot: Need backlash-free gearboxes for high precision

The Standard Model
Assume instead
◮ \( \dot{x}_{\text{out}} = \dot{x}_{\text{in}} \) when “in contact”
◮ \( \dot{x}_{\text{out}} = 0 \) when “no contact”
◮ No model of forces or torques needed/used

Dead-zone Model
◮ Often easier to use model of the form \( x_{\text{in}}(\cdot) \rightarrow x_{\text{out}}(\cdot) \)
◮ Uses implicit assumption: \( F_{\text{out}} = F_{\text{in}}, T_{\text{out}} = T_{\text{in}} \). Can be wrong, especially when “no contact”.

Servo motor with Backlash
P-control of servo motor
◮ How does the values of \( K \) and \( D \) affect the behavior?
Effects of Backlash

Oscillations for $K = 4$ but not for $K = 0.25$ or $K = 1$. Why?
Limit cycle becomes smaller if $D$ is made smaller, but it never disappears

1 minute exercise

Study the plot for the describing function for the backlash on the previous slide.
Where does the function $-\frac{1}{N(A)}$ end for $A \to \infty$ and why?

Limit cycles?

The describing function method is only approximate.
Can one determine conditions that guarantee stability?

Note: $\dot{\theta}_m$ and $\dot{\theta}_{out}$ will not converge to zero
Idea: Consider instead $\dot{\theta}_m$ and $\dot{\theta}_{out}$

Analysis by small gain theorem

Backlash block has gain $\leq 1$ (from $\dot{\theta}_m$ to $\dot{\theta}_{out}$)
Hence closed loop is BIBO stable provided that
$G(s)$ is asymptotically stable and $|G(j\omega)| < 1$ for all $\omega$

Analysis by circle criterion

Backlash map from $\dot{\theta}_m$ to $\dot{\theta}_{out}$ is in the sector $[0, 1]$.
$-1/k_1 = \infty$ and $-1/k_2 = -1$
Hence closed loop is stable if $\text{Re } G(j\omega) > -1$ for all $\omega$.
(For our motor example this proves stability when $K < 1$)
### Backlash compensation

- Dead-zone
- Linear controller design
- Backlash inverse
- Mechanical solutions

### Linear Controller Design

Introduce phase lead to avoid the $-1/N(A)$ curve:

Instead of only a P-controller we choose $K(s) = k \frac{1+sT_2}{1+sT_1} + u \dot{\theta}_{in}$ in $\theta_{in} \theta_{out} b$

$$
\begin{cases}
\dot{\theta}_{in} = u + \hat{D} & \text{if } u(t) > u(t-) \\
\dot{\theta}_{in} = u - \hat{D} & \text{if } u(t) < u(t-) \\
\dot{\theta}_{in} = \theta_{in}(t-) & \text{otherwise}
\end{cases}
$$

- $\hat{D} = D$ then $x_{out}(t) = u(t)$ (perfect compensation)
- $\hat{D} < D$: Under-compensation (decreased backlash)
- $\hat{D} > D$: Over-compensation, often gives oscillations

### Example–Perfect compensation

Motor with backlash on input, PD-controller

### Example–Under compensation

### Example–Over compensation

Idea: Let $x_{in}$ jump $\pm 2D$ when $x_{out}$ should change sign. Works if the backlash is directly on the system input!
**Backlash–More advanced models**

Warning: More detailed models needed sometimes
Model what happens when contact is attained
Model external forces that influence the backlash
Model mass/moment of inertia of the backlash.

**Example: Parallel Kinematic Robot**

Gantry-Tau robot:
Need backlash-free gearboxes for very high precision

EU-project: SMErobot [http://www.smerobot.org](http://www.smerobot.org)

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**“Rotational to Linear motion”**

![Gear box](image)

Motor connects here

Rack-and-pinion (Swe. “kuggstång”)  

Remedy:
Use two motors in opposite directions: One motor can act as spring and brake to "reduce" backlash. Need measurements on both motor and arm-side.

**Backlash in gearbox and rails**

![Diagram](image)

Lecture 6 ⇒

\[ N(A) = \frac{4}{\pi A} \sqrt{1 - \frac{D^2}{A^2}} \]  for \( A > D \) and zero otherwise

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**Backlash compensation**

From master thesis by B. Brochier, Control of a Gantry-Tau Structure, LTH, 2006
See also master theses by J. Schiffer and L. Halt, 2009.

**Quantization**

How accurate should the converters be? (8-14 bits?)
What precision is needed in computations? (8-64 bits?)

- Quantization in A/D and D/A converters
- Quantization of parameters
- Roundoff, overflow, underflow in operations

NOTE: Compare with (different) limits for “quantizer/dead-zone relay” in Lecture 6.

**Linear Model of Quantization**

Model the quantization error as a stochastic signal \( e \) independent of \( u \) with rectangular distribution over the quantization size.

Works if quantization level is small compared to the variations in \( u \)

\[ N(A) = \frac{4}{\pi A} \sqrt{1 - \frac{D^2}{A^2}} \]  for \( A > D \) and zero otherwise

\[ Var(e) = \int_{-\infty}^{+\infty} e^2 f_e \, de = \int_{-D/2}^{D/2} e^2 \frac{1}{D} \, de = \frac{D^2}{12} \]

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**Describing Function for Deadzone Relay**

![Diagram](image)
The maximum value is $4/\pi \approx 1.27$ for $A \approx 0.71D$.
Predicts limit cycle if Nyquist curve intersects negative real axis to the left of $-\pi/4 \approx -0.79$.
Should design for gain margin $> 1/0.79 \approx 1.27$.
Note that reducing $D$ only reduces the size of the limit oscillation, the oscillation does not vanish completely.

Example – Motor with P-controller.
Roundoff at input, $D = 0.2$. Nyquist curve intersects at $-0.5K$.
Hence stable for $K < 2$ without quantization. Stable oscillation predicted for $K > 2/1.27 = 1.57$.

Quantization at A/D converter
Double integrator with 2nd order controller, $D = 0.02$
Describing function: $A_y \approx D/2 = 0.01$, period $T = 39$
Simulation: $A_y = 0.01$ and $T = 28$

Quantization Compensation
- Use improved converters, (smaller quantization errors/larger word length)
- Linear design, avoid unstable controller, ensure 1.3 gain margin
- Use the tracking idea from anti-windup to improve D/A converter
- Use analog dither, oversampling and digital low-pass filter to improve accuracy of A/D converter
Today’s Goal

- To know models and compensation methods for backlash
  
  ![Diagram of backlash](image)

- Be able to analyze the effect of quantization errors
  
  ![Diagram of quantizer](image)

No More Lecture This Week!