Nonlinear Control (FRTN05)

Computer Exercise 3

Last updated: November 29, 2019

The following exercises should be solved with help of computer programs and simulation tools. Remember that computers can help to give insight, but you still need to understand the underlying theory.

1. Consider a tank where liquid flows in trough a controlled pump, and exits trough a hole in the bottom (those who read the basic control course at LTH will recognize this as the "water tanks" from lab one and two). A simple first order model describes the liquid level:

$$\dot{h} = ku - a\sqrt{h} \quad \text{for} \quad h \geq 0 \tag{1}$$

where k and a are constants. Use $a = 1/3000$ and $k = 1/1500$ for simulations. Around the operating point h_0 the linearized dynamics are given by

$$\frac{d\Delta h}{dt} = ku - \frac{1}{2}\frac{1}{\sqrt{h_0}}a\Delta h$$

The inflow is controlled by a PI controller, which in the Laplace domain is given by:

$$G_c(s) = K\frac{1 + sT_i}{T_i s} \tag{2}$$

The controller is designed by pole-placement. The denominator of the closed loop close to the operating point should be $s^2 + 2\zeta\omega + \omega^2$. Recall that ω corresponds to the speed of the system and ζ to the damping. It is easy to show that the desired closed loop is achieved by choosing

$$\begin{aligned} K &= \frac{1}{k}\left(2\zeta\omega - 1/T\right) \\ T_i &= \frac{Kk}{\omega^2} \end{aligned} \tag{3}$$

where $T = \frac{2\sqrt{h_0}}{a}$. Let $h_0 = 3$ and use $\omega = 1/3000$ and $\zeta = 0.9$ for simulation, corresponding to an open loop with reasonable speed and damping. Verify that the system responds as desired when changing from normal operation at $h = 3\,\text{m}$ to $3.8\,\text{m}$, and back. Notice which range the control operates within under the changes.

2. The pump has limitations: First of all, it can not suck liquid out of the tank. The lowest flow it can provide, is zero flow. Also, it has a maximum capacity, in this case, use $u_{max} = 1$. Insert the described saturation in the simulation. Rerun the previous experience. Do you see any issues? Can you explain them?

3. Investigate what happens when the reference changes from normal operation ($h = 3\,\text{m}$) to a higher value ($h = 5\,\text{m}$) and then back to $h = 3\,\text{m}$ after, say $5 \cdot 10^4$ seconds. Run the simulation for $2 \cdot 10^5$ seconds.

4. Assuming that you can measure the saturated flow, design an integrator anti-windup scheme so that the observed problems do not occur.
5. Assuming that you are aware of the saturation, but that you can NOT measure the saturated flow, design an integrator anti-windup scheme to solve the problem.

Hints and Answers

1. The control signal reaches values that are outside the restrictions specified in subtask 2.
2. The initial input u is bigger than u_{max}, leading to saturation. During the saturation the integral term increases much more than is needed to reach the set point. This leads to integrator wind up and the control signal will be too large when y reaches the set point, resulting in a large overshoot. After a while, the integrator term will decrease, and the set point will be reached.
3. The saturation in combination with the integral part of the controller, results in windup. Thus, when changing the reference back to $h = 3$ m, the controller still acts to keep a large height to compensate for the historical error. This behavior is undesired. After a while, the integrated error approaches 0, and the controller then acts to decrease the height.
4. See lecture slides on anti-windup.
5. See lecture slides on anti-windup.