Lecture 7: Anti-windup and friction compensation

- Compensation for saturations (anti-windup)
- Friction models
- Friction compensation

Material
- Lecture slides

Course Outline

Lecture 1-3 Modelling and basic phenomena
(linearization, phase plane, limit cycles)
Lecture 2-6 Analysis methods
(Lyapunov, circle criterion, describing functions)
Lecture 7-8 Common nonlinearities
(Saturation, friction, backlash, quantization)
Lecture 9-13 Design methods
(Lyapunov methods, Sliding mode & optimal control)
Lecture 14 Summary

Last lecture: Stable periodic solution

The relay gain $N(A)$ is higher for small $A$:

- Growing amplitudes
- Shrinking relay gain
- One encirclement
- Shrinking relay gain
- No encirclement
- Shrinking amplitudes
- High relay gain
- Small amplitudes

Periodic Solutions in Relay System

Today’s Goal

- To be able to design and analyze antiwindup schemes for
  - PID
  - state-space systems
  - and Kalman filters (observers)
- To understand common models of friction
- To design and analyze friction compensation schemes

Example-Windup in PID Controller

Dashed line: ordinary PID-controller
Solid line: PID-controller with anti-windup

Anti-windup for PID-Controller (‘‘Tracking’’)

Anti-windup (a) with actuator output available and (b) without
Sometimes good
Earthquakes
Friction in valves and mechanical constructions
Friction in brakes
How to model friction

Often bad
Sometimes too small

Choice of Tracking Time $T_t$

With very small $T_t$ (large gain $1/T_t$), spurious errors can saturate the output, which leads to accidental reset of the integrator. Too large $T_t$ gives too slow reaction (little effect).

The tracking time $T_t$ is the design parameter of the anti-windup.

Common choices: $T_t = T_i$ or $T_t = \sqrt{T_i T_d}$.

Antiwindup – General State-Space Controller

State-space controller:

\[
\dot{x}_c(t) = F x_c(t) + G y(t)
\]

\[
u(t) = C x_c(t) + D y(t)
\]

Windup possible if $F$ is unstable and $u$ saturates.

(a)

(b)

\[ y \quad S_A \quad u \]

\[ y \quad S_B \quad u \]

Idea:
Rewrite representation of control law from (a) to (b) such that:
(a) and (b) have same input-output relation
(b) behaves better when feedback loop is broken, if $S_B$ stable

State-space controller without and with anti-windup:

Antiwindup – General State-Space Controller

Mimic the observer-based controller:

\[
\dot{x}_e = F x_e + G y + K (u - C x_e - D y)
\]

\[
= (F - KC) x_e + (G - KD) y + Ku
\]

\[
= F_0 x_e + G_0 y + Ku
\]

Design so that $F_0 = F - KC$ has desired stable eigenvalues

Then use controller

\[
\dot{x}_e = F_0 x_e + G_0 y + Ku
\]

\[
u = \text{sat} (Cx_e + Dy)
\]

5 Minute Exercise

How would you do antiwindup for the following state-feedback controller with observer and integral action?

Process

\[ \frac{1}{s} \]

\[-L \]

Estimator

\[ \frac{1}{s} \]

\[-L \]

Observer

Saturation

Optimal control theory (later)

Multi-loop Anti-windup (Cascaded systems):

Difficult problem, several suggested solutions

Turn off integrator in outer loop when inner loop saturates

Friction

Present almost everywhere

- Often bad
  - Friction in valves and mechanical constructions

- Sometimes good
  - Friction in brakes

- Sometimes too small
  - Earthquakes

Problems

- How to model friction
- How to compensate for friction
Stick-slip Motion

\[ F_k(y - x) \]

Position Control of Servo with Friction – Hunting

3 Minute Exercise

What are the signals in the previous plots? What model of friction has been used in the simulation?

Friction

Lubrication Regimes

V \rightarrow

Sticking
Boundary lubrication
Mixed lubrication
Full fluid lubrication

Frictional Lag

Dynamics are important also outside sticking regime

Hess and Soom (1990)

Experiment with unidirectional motion \( v(t) = v_0 + a \sin(\omega t) \)

Hysteresis effect!

Classical Friction Models

For low velocity: friction increases with decreasing velocity

Stribeck (1902)

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Advanced Friction Models

See PhD-thesis by Henrik Olsson

- Karnopp model
- Armstrong’s seven parameter model
- Dahl model
- Bristle model
- Reset integrator model
- Bliman and Sorine
- LuGre model (Lund-Grenoble)

Demands on a model

To be useful for control the model should be

- sufficiently accurate,
- suitable for simulation,
- simple, few parameters to determine.
- physical interpretations, insight

Pick the simplest model that does the job! If no stiction occurs the $v = 0$-models are not needed.

Friction Compensation

- Lubrication
- Integral action (beware!)
- Dither
- Non-model based control
- Model based friction compensation
- Adaptive friction compensation

Integral Action

- The integral action compensates for any external disturbance
- Good if friction force changes slowly ($v \approx$ constant).
- To get fast action when friction changes one must use much integral action (small $T_i$)
- Gives phase lag, may cause stability problems etc

Deadzone - Modified Integral Action

Modify integral part to $I = \int^t_0 \hat{e}(t) d\tau$

where input to integrator $\hat{e} = \begin{cases} e(t) - \eta & e(t) > \eta \\ 0 & |e(t)| < \eta \\ e(t) + \eta & e(t) < -\eta \end{cases}$

Advantage: Avoid that small static error introduces limit cycle
Disadvantage: Must accept small error (will not go to zero)

Adaptive Friction Compensation

Coulomb Friction $F = a \text{sgn}(v)$

Assumption: $v$ measurable.
Friction estimator:

$\dot{z} = k_{\text{PID}} \text{sgn}(v)$
$\hat{a} = z - km|v|$
$\hat{F}_{\text{friction}} = \hat{a} \text{sgn}(v)$

Result: $e = a - \hat{a} \to 0$ as $t \to \infty$, since

$
\frac{de}{dt} = \frac{d\hat{a}}{dt} - km \frac{d|v|}{dt} = k_{\text{PID}} \text{sgn}(v) - km \frac{d|v|}{dt} = k \text{sgn}(v)(\text{PID} - m\dot{v}) = -k \text{sgn}(v)(F - \hat{F}) = -k(a - \hat{a}) = -ke
$

Remark: Careful with $\frac{d|v|}{dt}$ at $v = 0$. 

Mechanical Vibrator–Dither

Avoids sticking at $v = 0$ where there usually is high friction by adding high-frequency mechanical vibration (dither)

Cf., mechanical maze puzzle (labyrintspel)
Example–Friction Compensation

Velocity control with
a) P-controller
b) PI-controller
c) P + Coulomb estimation

Results

The Knocker
Combines Coulomb compensation and square wave dither

Tore Hägglund, Innovation Cup winner + patent 1997

Next Lecture

✦ Backlash
✦ Quantization