Frequency Response, Relation Between Model Descriptions

Automatic Control, Basic Course, Lecture 4

November 6, 2019

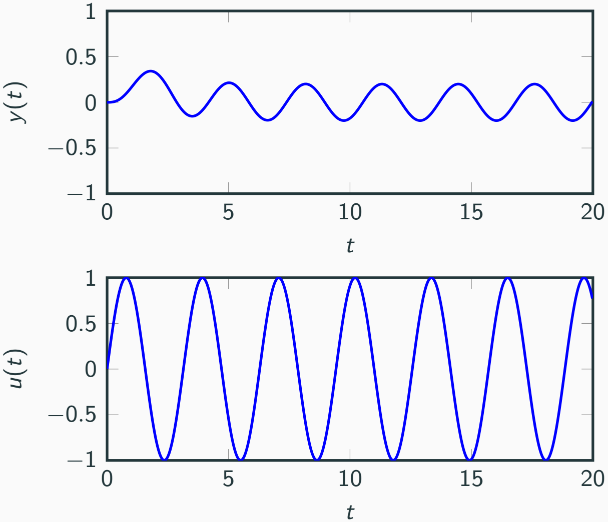
Lund University, Department of Automatic Control

Content

Frequency Response

Sinusoidal Input

Given a transfer function $G(s)$, what happens if we let the input be $u(t) = \sin(\omega t)$?



Sinusoidal Input

It can be shown that if the input is $u(t) = \sin(\omega t)$, the output[1] will be

$$y(t) = A \sin(\omega t + \varphi)$$

where

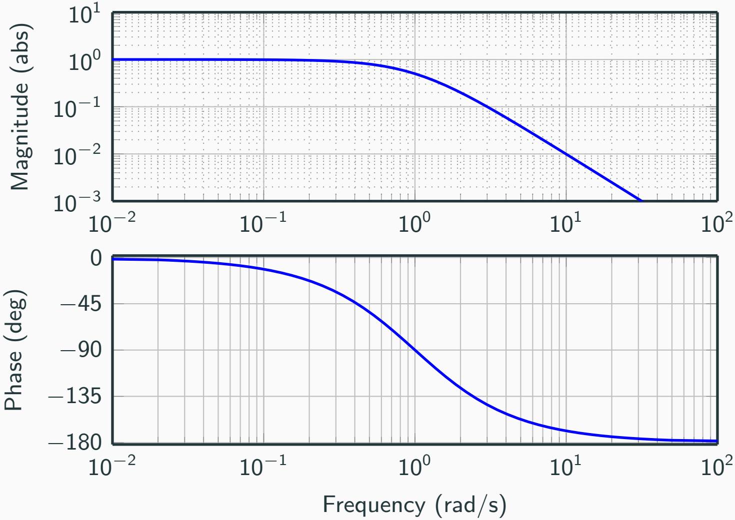
$$A = |G(i\omega)|$$
$$\varphi = \arg G(i\omega)$$

So if we determine a and φ for different frequencies ω, we have a description of the transfer function.

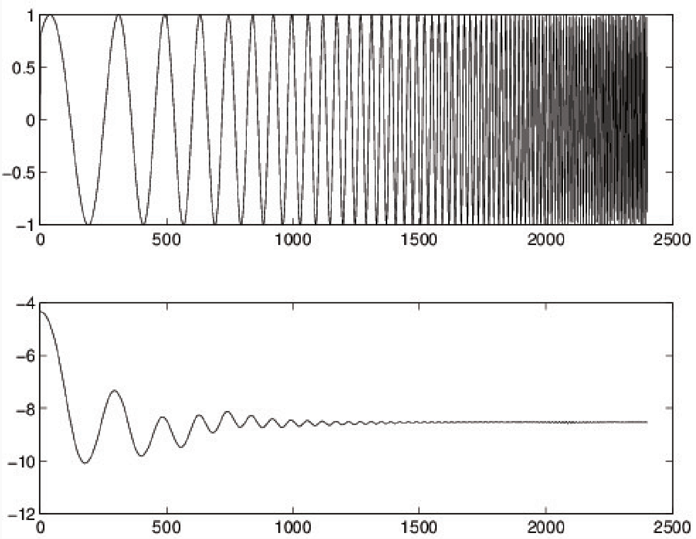
[1] after the transient has decayed

Bode Plot

Idea: Plot $|G(i\omega)|$ and $\arg G(i\omega)$ for different frequencies ω.

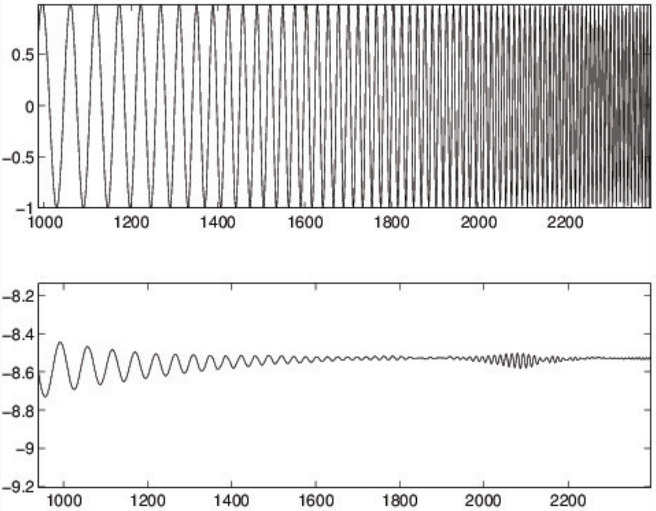


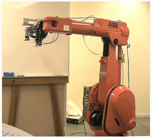
Sinusoidal Input-Output: example with frequency sweep (chirp)



Resonance frequency of industrial robot IRB2000 visible in data.

Sinusoidal Input-Output: example with frequency sweep (chirp)





Resonance frequency of industrial robot IRB2000 visible in data.

Bode Plot - Products of Transfer Functions

Let

$$G(s) = G_1(s)G_2(s)G_3(s)$$

then

$$\log |G(i\omega)| = \log |G_1(i\omega)| + \log |G_2(i\omega)| + \log |G_3(i\omega)|$$
$$\arg G(i\omega) = \arg G_1(i\omega) + \arg G_2(i\omega) + \arg G_3(i\omega)$$

This means that we can construct Bode plots of transfer functions from simple "building blocks" for which we know the Bode plots.

Bode Plot of $G(s) = K$

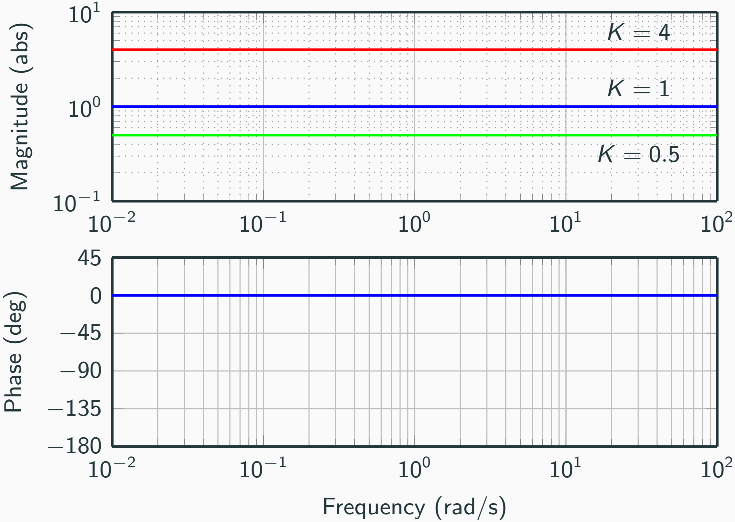
If

$$G(s) = K$$

then

$$\begin{aligned} \log|G(i\omega)| &= \log(|K|) \\ \arg G(i\omega) &= 0 \qquad (\textit{if } K > 0, \textit{ else } +180 \textit{ or } -180\,\text{deg}) \end{aligned}$$

Bode Plot of $G(s) = K$



Bode Plot of $G(s) = s^n$

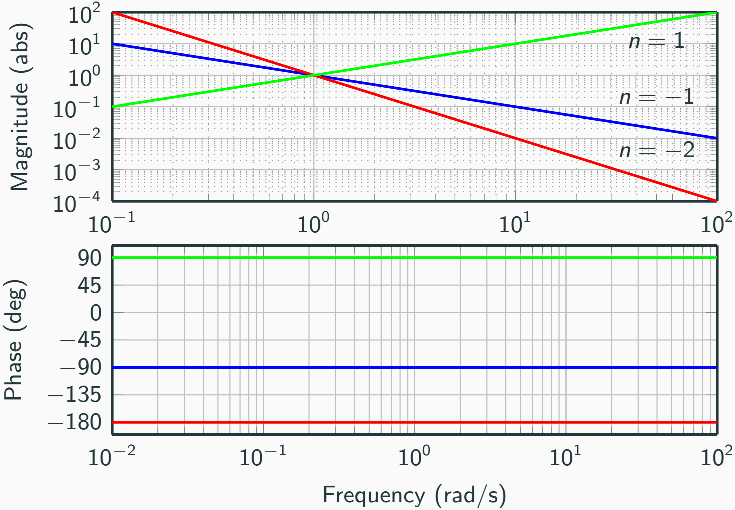
If

$$G(s) = s^n$$

then

$$\log|G(i\omega)| = n\log(\omega)$$
$$\arg G(i\omega) = n\frac{\pi}{2}$$

Bode Plot of $G(s) = s^n$



Bode Plot of $G(s) = (1 + sT)^n$

If

$$G(s) = (1 + sT)^n$$

then

$$\log|G(i\omega)| = n\log(\sqrt{1 + \omega^2 T^2})$$
$$\arg G(i\omega) = n\arg(1 + i\omega T) = n\arctan(\omega T)$$

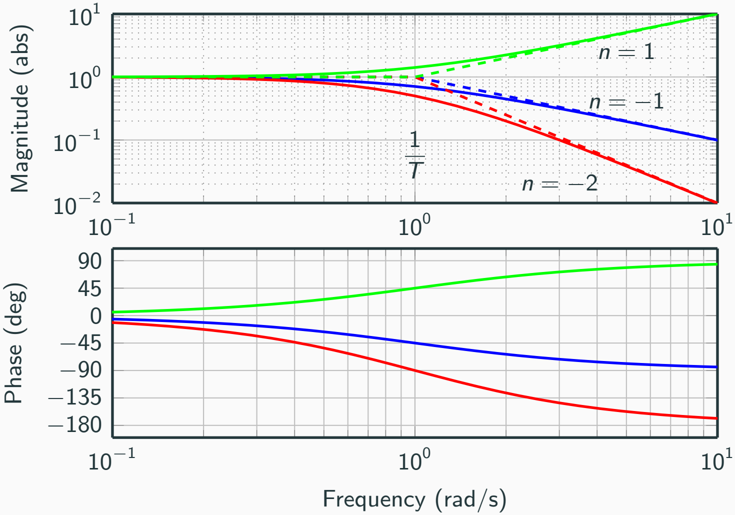
For small ω

$$\log|G(i\omega)| \to 0$$
$$\arg G(i\omega) \to 0$$

For large ω

$$\log|G(i\omega)| \to n\log(\omega T)$$
$$\arg G(i\omega) \to n\frac{\pi}{2}$$

Bode Plot of $G(s) = (1 + sT)^n$



Bode Plot of $G(s) = (1 + 2\zeta s/\omega_0 + (s/\omega_0)^2)^n$

$$G(s) = (1 + 2\zeta s/\omega_0 + (s/\omega_0)^2)^n$$

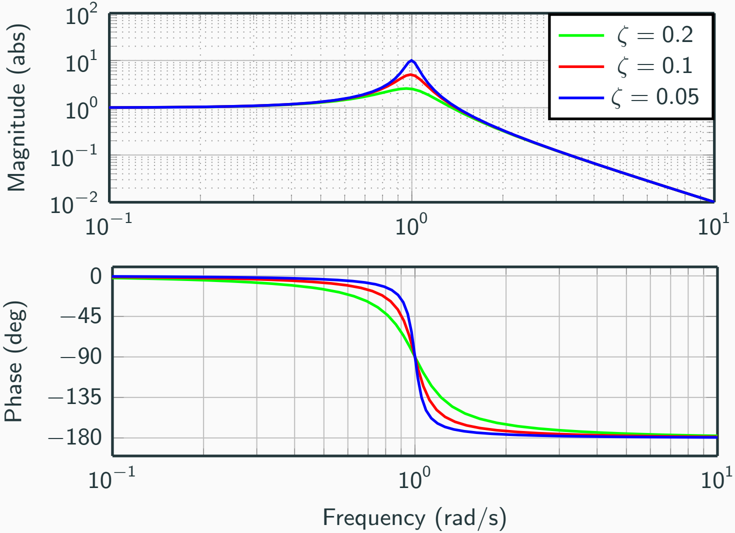
For small ω

$$\log|G(i\omega)| \to 0$$
$$\arg(i\omega) \to 0$$

For large ω

$$\log|G(i\omega)| \to 2n \log\left(\frac{\omega}{\omega_0}\right)$$
$$\arg G(i\omega) \to n\pi$$

Bode Plot of $G(s) = (1 + 2\zeta s/\omega_0 + (s/\omega_0)^2)^n$



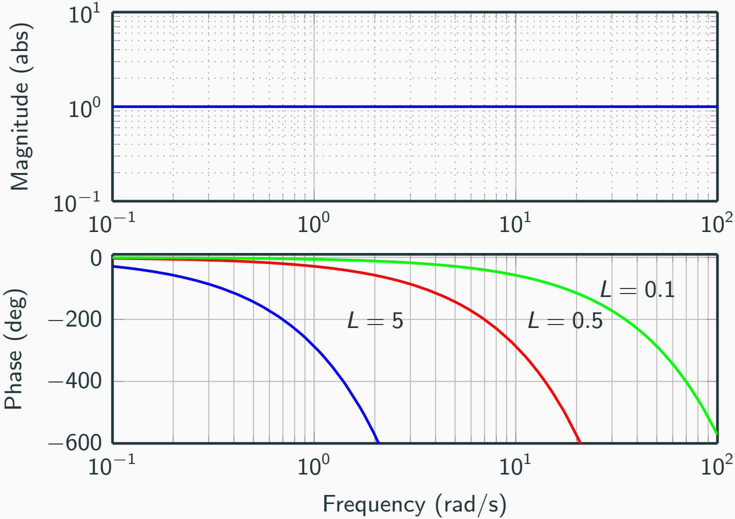
Bode Plot of $G(s) = e^{-sL}$

$$G(s) = e^{-sL}$$

Describes a pure time delay with delay L, i.e, $y(t) = u(t - L)$

$$\log |G(i\omega)| = 0$$
$$\arg G(i\omega) = -\omega L$$

Bode Plot of $G(s) = e^{-sL}$

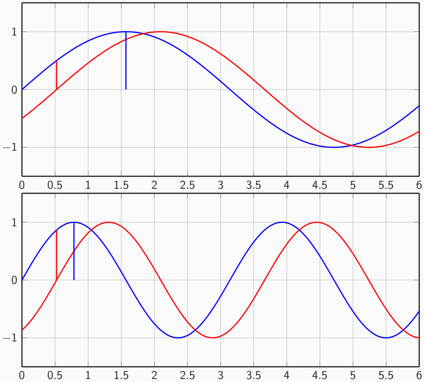


Bode Plot of $G(s) = e^{-sL}$

Same delay may appear as **different phase lag** for different frequencies!

Example

Delay ≈ 0.52 sec between input and output.



(Upper): Period time $= 2\pi \approx$ 6.28 sec. Delay represents phase lag of $\frac{0.52}{6.28} \cdot 360 \approx 30\,\text{deg}$

(Lower): Period time $= \pi \approx$ 3.14 sec. Delay represents phase lag of $\frac{0.5}{3.14} \cdot 360 \approx 60\,\text{deg}$.

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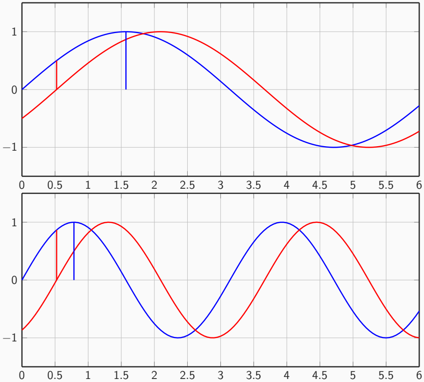


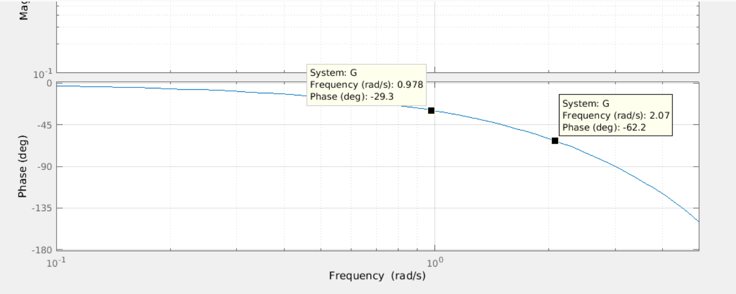
(Upper): Period time $= 2\pi \approx 6.28$ sec. Delay represents phase lag of $\frac{0.52}{6.28} \cdot 360 \approx 30\,\text{deg}$

(Lower): Period time $= \pi \approx 3.14$ sec. Delay represents phase lag of $\frac{0.5}{3.14} \cdot 360 \approx 60\,\text{deg}$.

Bode Plot of $G(s) = e^{-sL}$

Check phase in Bode diagram for $e^{-0.52s}$ for

- $\sin(t) \Rightarrow \omega = 1.0$ rad/s
- $\sin(2t) \Rightarrow \omega = 2.0$ rad/s




```
>> s=tf('s')
>> G=exp(-0.52*s);
>> bode(G,0.1 ,5) % Bode plot in frequency-range [0.1 .. 5] rad/s
```

Bode Plot of Composite Transfer Function

Example

Draw the Bode plot of the transfer function

$$G(s) = \frac{100(s+2)}{s(s+20)^2}$$

First step, write it as product of simple transfer functions:

$$G(s) = \frac{100(s+2)}{s(s+20)^2} = 0.5 \cdot s^{-1} \cdot (1+0.5s) \cdot (1+0.05s)^{-2}$$

Then determine the corner frequencies (break points):

$$G(s) = \frac{100(s+2)}{s(s+20)^2} = 0.5 \cdot s^{-1} \cdot \overbrace{(1+0.5s)}^{w_{c_1}=2} \cdot \overbrace{(1+0.05s)^{-2}}^{w_{c_2}=20}$$

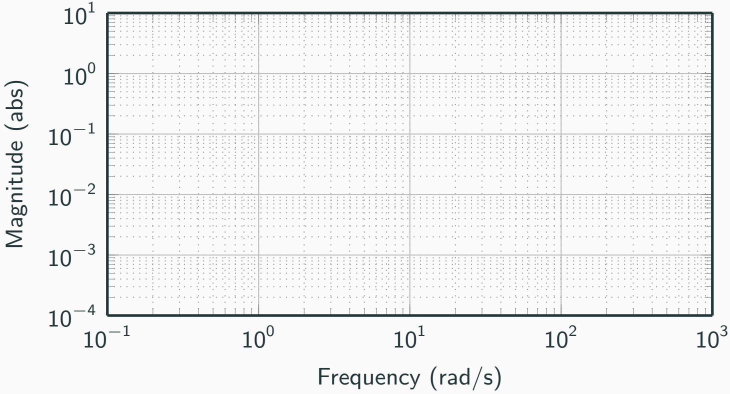
Sort from LOW to HIGH frequencies:

Start with LOW frequencies

(make sure the other TFs asymptotically reduce to **1**).

Bode Plot of Composite Transfer Function

$$G(s) = \frac{100(s+2)}{s(s+20)^2} = 0.5 \cdot s^{-1} \cdot \overbrace{(1+0.5s)}^{w_{c_1}=2} \cdot \overbrace{(1+0.05s)^{-2}}^{w_{c_2}=20}$$



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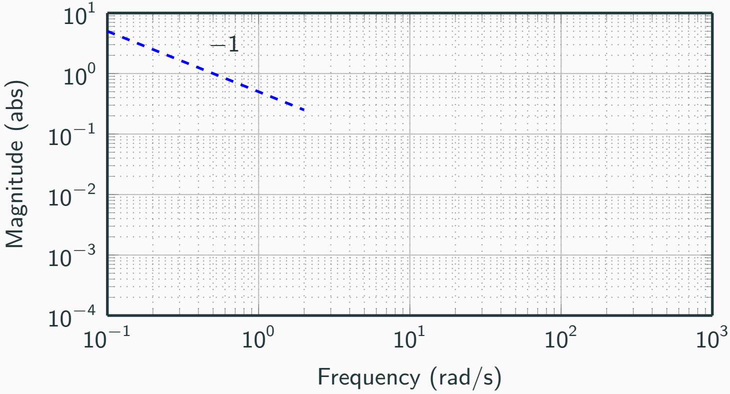

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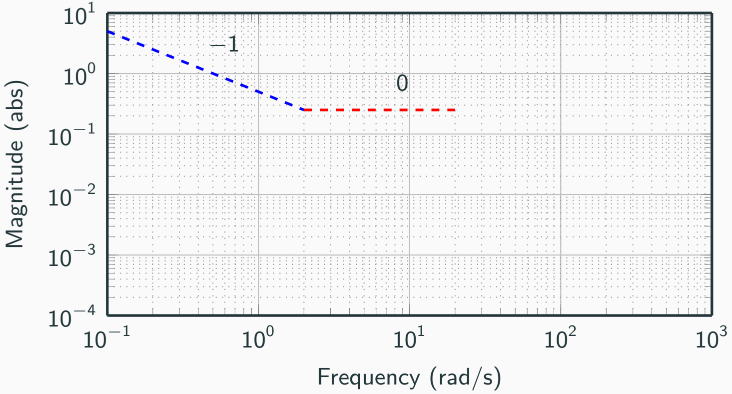

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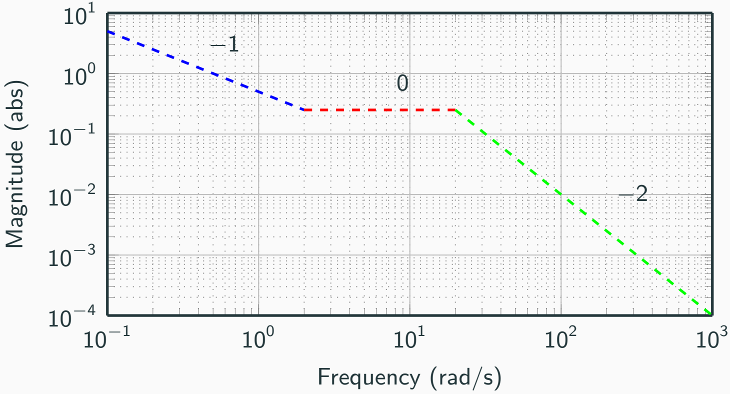

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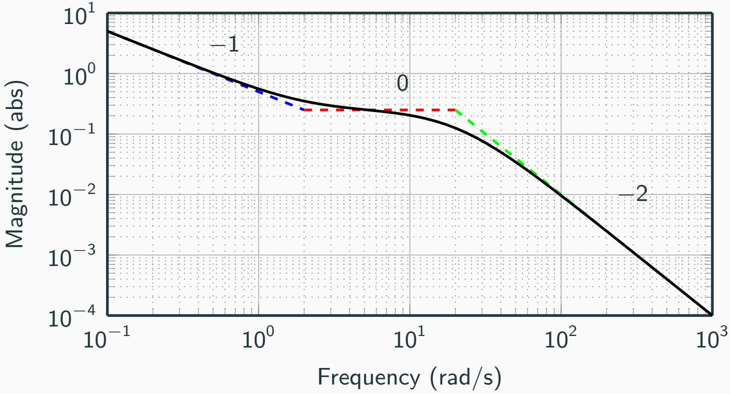

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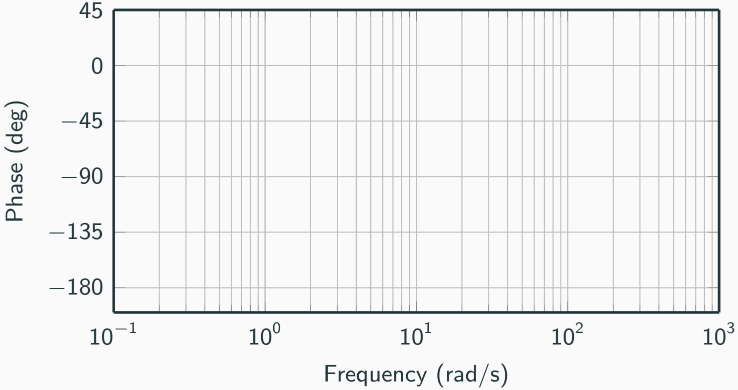

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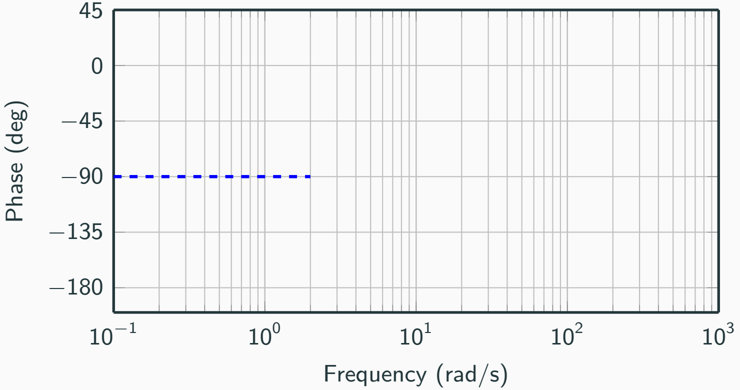

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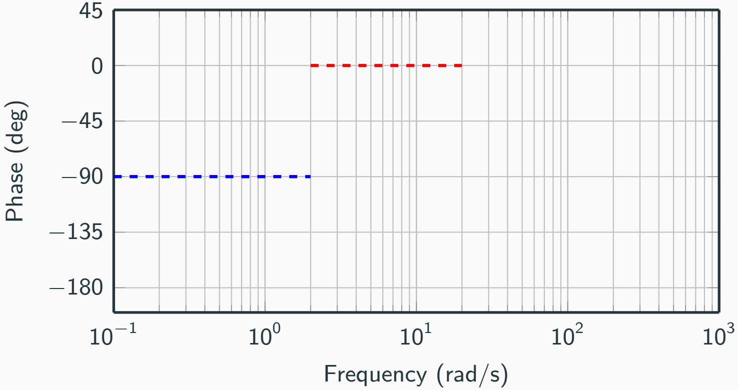

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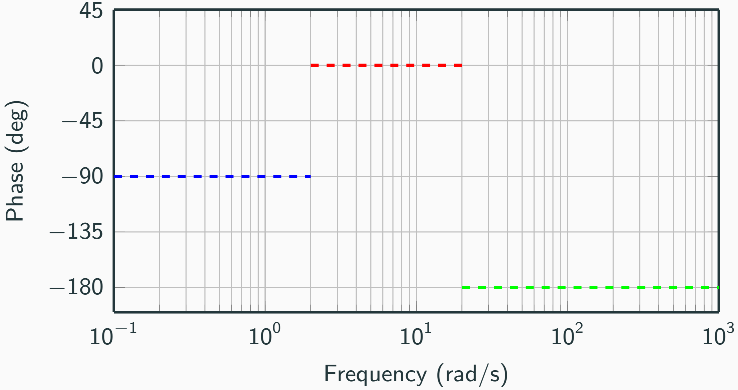

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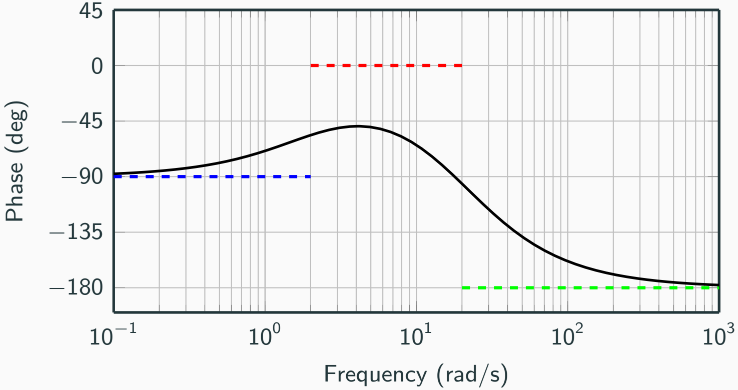
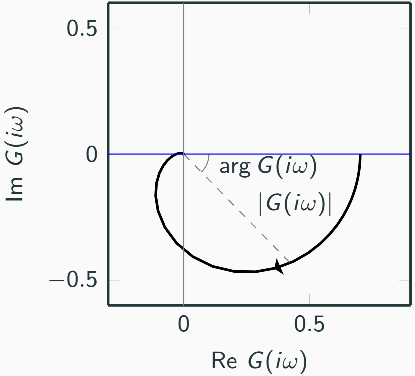


Nyquist Plot

By removing the frequency information, we can plot the transfer function in one plot instead of two.



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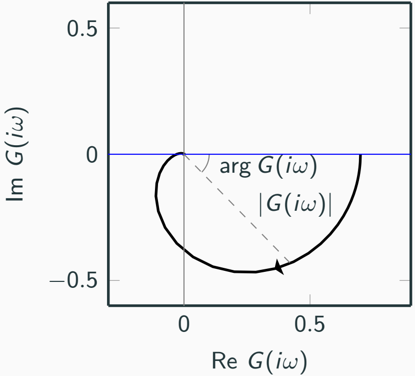
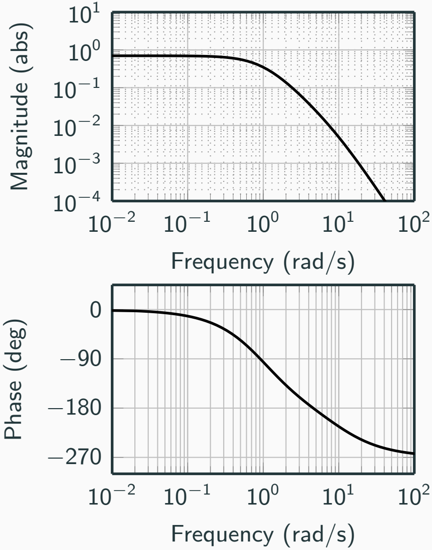


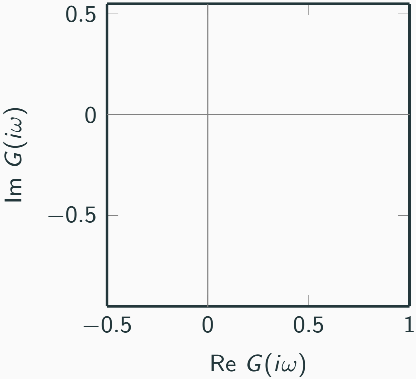
Split the transfer function into real and imaginary part:

$$G(s) = \frac{1}{1+s} \quad G(i\omega) = \frac{1}{1+i\omega} = \frac{1}{1+\omega^2} - i\frac{\omega}{1+\omega^2}$$

Is this the transfer function in the plot above?

From Bode Plot to Nyquist Plot





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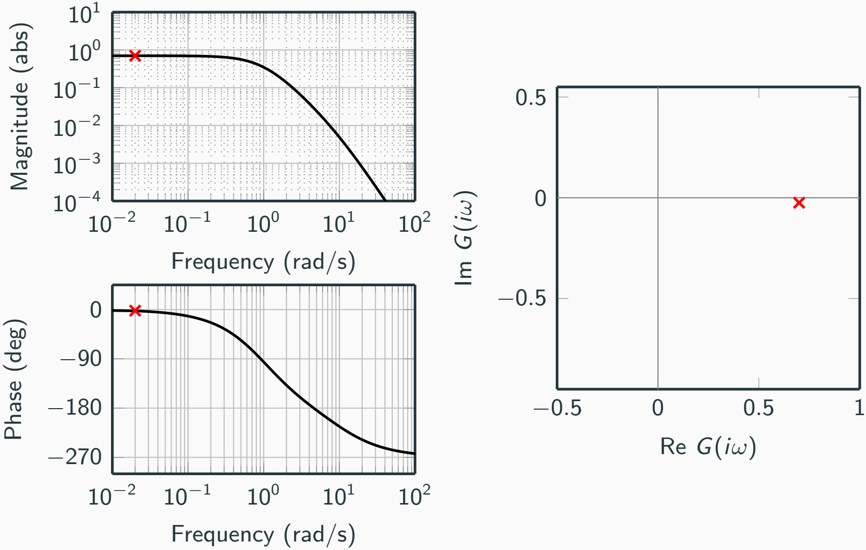

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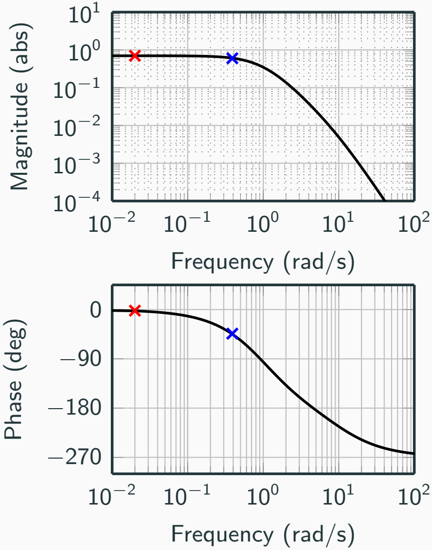

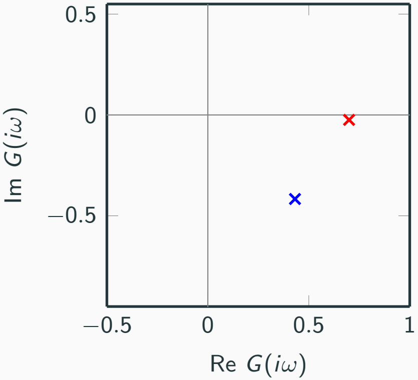

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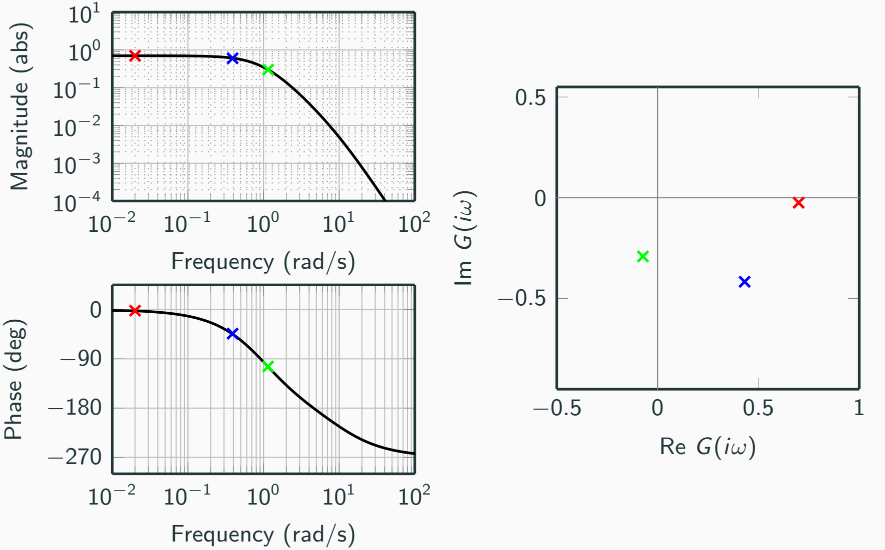

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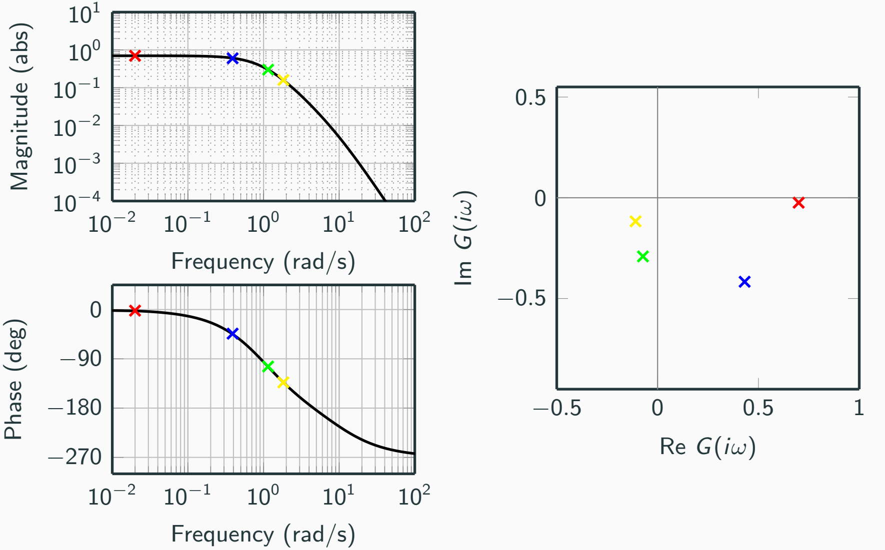

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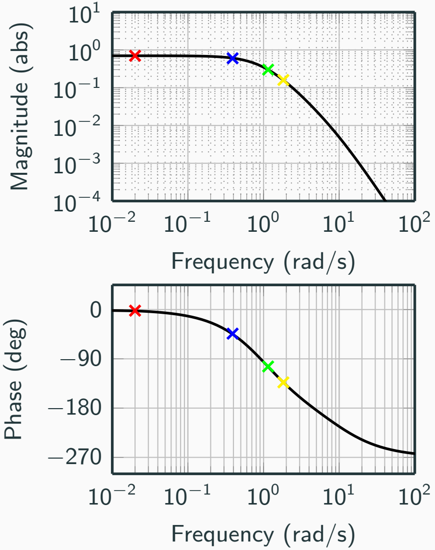
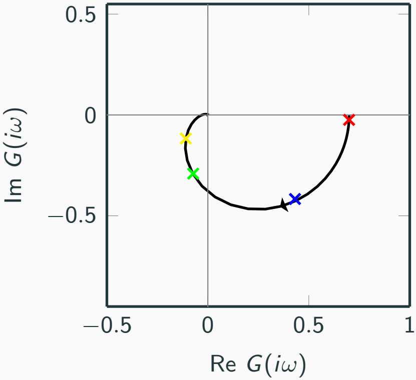
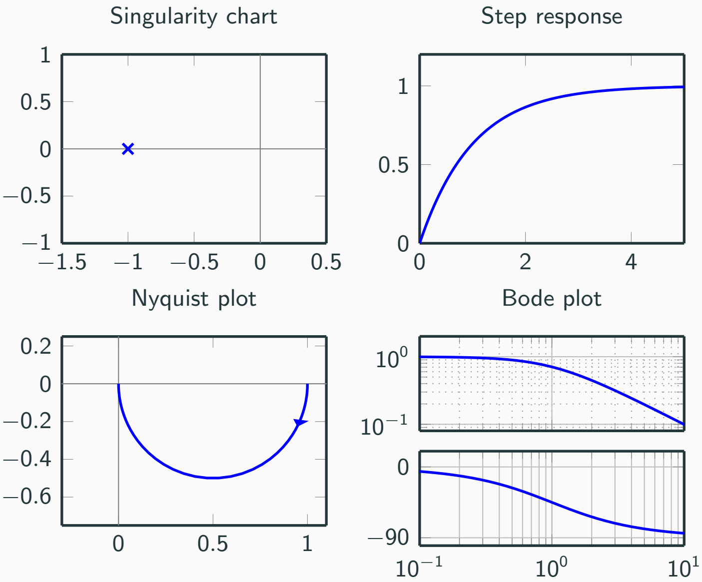


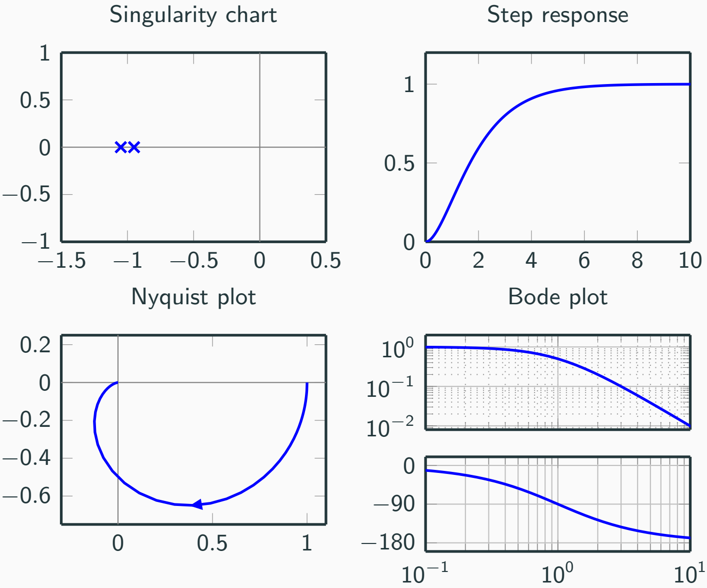


Relation between Model Descriptions

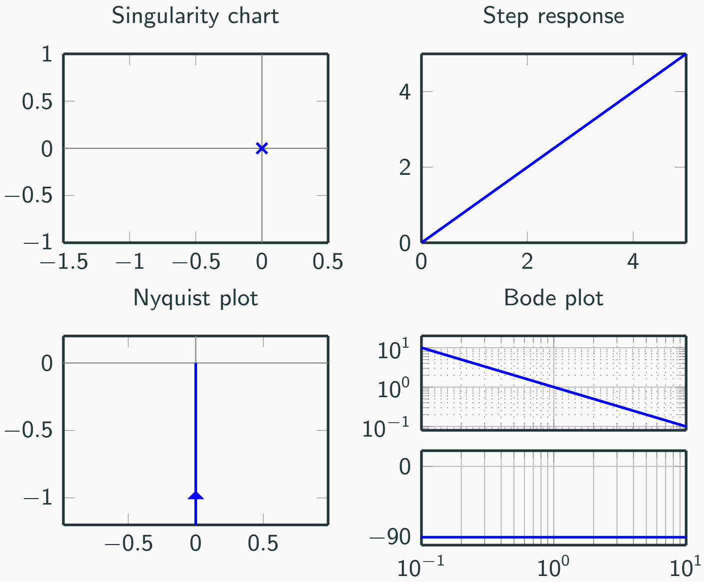
Single-capacitive Processes $\frac{K}{sT+1}$



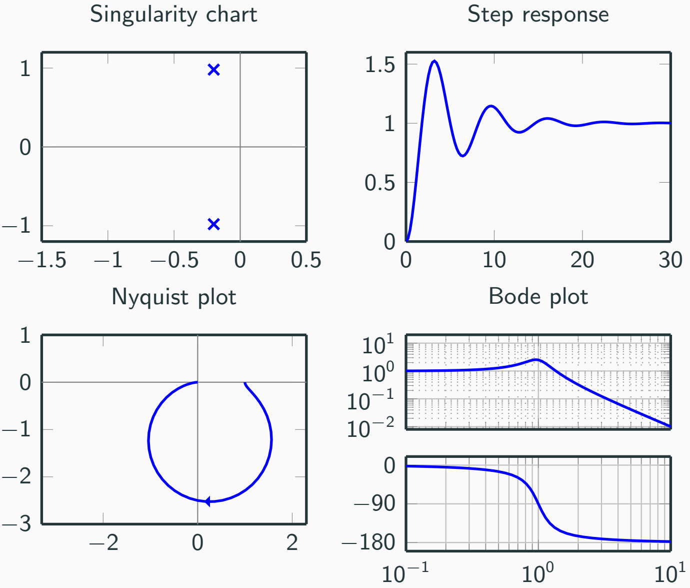
Multi-capacitive Processes $\frac{K}{(sT_1+1)(sT_2+1)}$



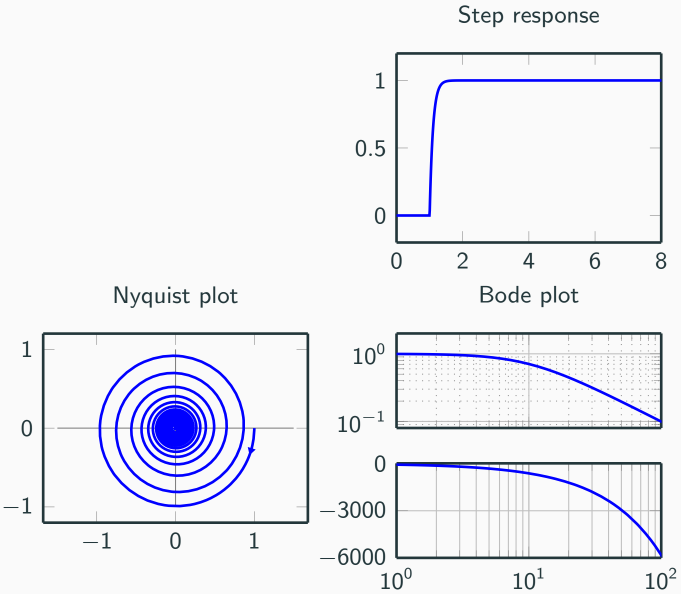
Integrating Processes $\frac{1}{s}$



Oscillative Processes $\frac{K\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$, $0 < \zeta < 1$

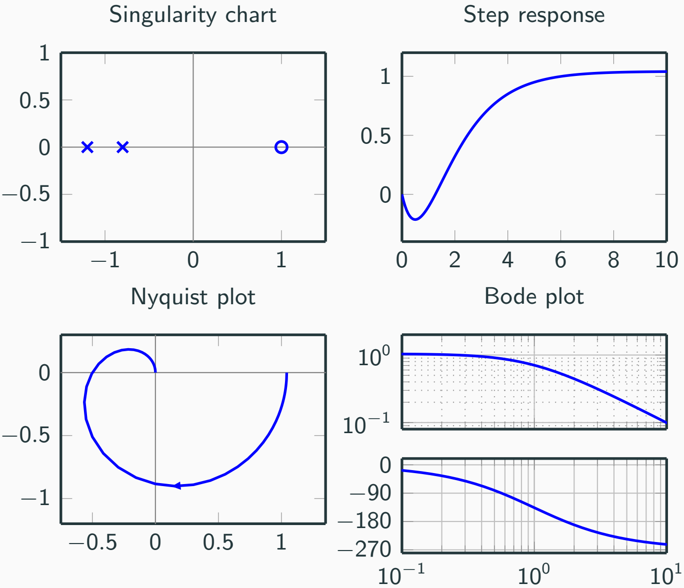


Delay Processes $\frac{K}{sT+1}e^{-sL}$



Process with Inverse Responses

$$\frac{-sa+1}{(sT_1+1)(sT_2+1)}$$



Content

This lecture

1. Frequency Response

2. Relation between Model Descriptions

Next lecture

- Feedback - The Steam Engine
- Stability