# Home Assignment 4: The Hodgkin and Huxley model

2018

*Preparation: Go through the exercises in Chapter 8 of the exercise manual.*

In this assignment you will explore Hodgkin and Huxley's (HH) model of the action potential. In step 1–6 you will fill in the missing parts of the code-template given to you on the course homepage, `hw4CodeShell.m`. In step 7–9 you will use your code in order to simulate the behavior of the membrane potential and the gating variables at different input currents. Furthermore, you will determine the threshold potential as well as the refractory period of the specific neuron simulated.

In the HH-model, the dynamics governing the membrane potential $V$ is given by

$$C_m \frac{dV}{dt} = -I_{\mathrm{Na}} - I_{\mathrm{K}} - I_L + I_{\mathrm{ext}}$$

where $C_m$ is the capacitance of the membrane, $I_{\mathrm{ext}}$ is the external (input) current and $I_\bullet$ are the respective ion currents. The ion currents are given by

$$\begin{aligned}
I_{\mathrm{Na}} &= g_{\mathrm{Na}} \cdot m^3 h \,(V - E_{\mathrm{Na}}) \\
I_{\mathrm{K}} &= g_{\mathrm{K}} \cdot n^4 \,(V - E_{\mathrm{K}}) \\
I_{\mathrm{L}} &= g_{\mathrm{L}} \cdot (V - E_{\mathrm{L}})
\end{aligned}$$

where conductances $g_\bullet$ and reversal potentials $E_\bullet$ are fixed parameters while $m$, $h$ and $n$ are functions of time governed by the following differential equations

$$\begin{aligned}
\frac{dm}{dt} &= \alpha_m(V)(1-m) - \beta_m(V)m \\
\frac{dh}{dt} &= \alpha_h(V)(1-h) - \beta_h(V)h \\
\frac{dn}{dt} &= \alpha_n(V)(1-n) - \beta_n(V)n
\end{aligned}$$

with rate functions, units [1/ms],

$$\begin{aligned}
\alpha_m(V) &= 0.1\tfrac{V+45}{1-e^{-(V+45)/10}} & \beta_m(V) &= 4e^{-(V+70)/18} \\
\alpha_h(V) &= 0.07e^{-(V+70)/20} & \beta_h(V) &= \tfrac{1}{1+e^{-(V+40)/10}} \\
\alpha_n(V) &= 0.01\tfrac{V+60}{1-e^{-(V+60)/10}} & \beta_n(V) &= 0.125e^{-(V+70)/80}.
\end{aligned}$$

Open hw4CodeShell.m.

1. In *Section 1- Model parameters*, declare the following values

$$\begin{aligned} &C_m = 1\ [\mu\text{F/cm}^2] && \\ &E_{\text{Na}} = 45\ [\text{mV}] && g_{\text{Na}} = 120\ [\text{mS/cm}^2] \\ &E_{\text{K}} = -82\ [\text{mV}] && g_{\text{K}} = 36\ [\text{mS/cm}^2] \\ &E_{\text{L}} = -60\ [\text{mV}] && g_{\text{L}} = 0.3\ [\text{mS/cm}^2]. \end{aligned}$$

   Notice that the reversal potentials are given in [mV] and thus that the potential $V$ will be given in the same unit. Time is given in [ms] due to the rate functions given above.

2. In *Section 2 - Channel gating kinetics, rate functions*, declare the rate functions of the gating variables' dynamics as function handles. (Hint: see Exercise 8.4.)

3. In *Section 3 - Membrane currents*, declare the functions of the ion currents.

4. In *Section 4 - External current*, state that the external current is constant zero. We will use this setup for our initial simulations.

5. In *Section 5 - Declaring the differential equations*, declare the differential equations of $V$, $m$, $h$ and $n$ by a function handle on matrix form denoted `dAlldt` with variable `X = [V m h n]`.

6. Run sections 1–5. You have now declared the parameters and the functions of the model and are ready to simulate its behavior. In *Section 6 - Steady state*, you should determine appropriate initial values of the variables $V$, $m$, $h$ and $n$ to use in the following section. As can be seen in the code, the initial values, `initial_values`, are all set to zero. Run the section. The solution of the differential equations is given by `ode45` and a figure will be created which includes three subplots. The top subplot shows the membrane potential $V$ during the simulation period, the one in the middle displays the dynamics of the gating variables while the bottom subplot gives the external current. Determine the steady state values of the variables $V$, $m$, $h$ and $n$ from the plot with one significant decimal and use them as the initial values in the following section.

7. Now you have implemented the HH-model. In *Section 7 - Find the threshold potential and specify the refractory period*, use the external current to determine the threshold potential of the neuron. For instance, gradually increase the external current by some factor $k > 0$ until the neuron fires.

8. During an action potential, the gating variables are varying in a specific way. Connect this behavior with the corresponding physiological behavior.

9. What is the refractory period of the neuron? Again, use the external current in order to investigate this.