Lecture 15: Repetition

Bo Bernhardsson

Department of Automatic Control
Lund University, Sweden

Lecture 15 - Repetition

Course content:

- Models
- Analysis
- Control design

Models

- Differential equation

$$\dot{x} = f(x,u); \quad y = g(x,u)$$

- State space

$$\dot{x} = Ax + Bu; \quad y = Cx + Du$$

- Transfer function

$$G(s) = C(sI - A)^{-1}B + D$$

- Impulse response

$$Y_i(s) = G(s) \quad or \quad y_i(t) = h(t) = Ce^{At}B + D\delta(t)$$

Linearisation

From nonlinear diff. eq to linear diff eq.

$$\dot{x} = f(x,u); \quad y = g(x,u)$$

- Calculate stationary points

$$f(x_0,u_0) = 0; \quad y_0 = g(x_0,u_0)$$

- Linearise, $\Delta x := x - x_0$, $\Delta u := u - u_0$, $\Delta y := y - y_0$

$$\begin{aligned}
\frac{d}{dt}\Delta x &= \frac{\partial}{\partial x} f(x_0,u_0)\Delta x + \frac{\partial}{\partial u} f(x_0,u_0)\Delta u \\
\Delta y &= \frac{\partial}{\partial x} g(x_0,u_0)\Delta x + \frac{\partial}{\partial u} g(x_0,u_0)\Delta u
\end{aligned}$$

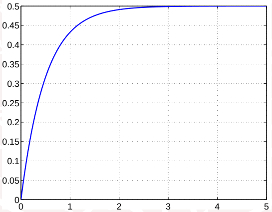
Exam Mar08 8a+8b

The transfer function $G(s)$

Input-output relation $Y(s) = G(s)U(s)$

Example: Step response of $G(s) = \frac{1}{s+2}$ is

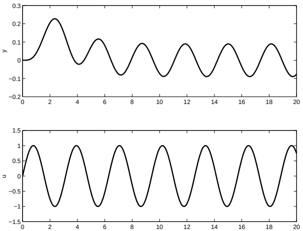
$$Y(s) = \frac{1}{s+2}\frac{1}{s} \Longrightarrow y(t) = 0.5\,\mathrm{step}(t) - 0.5\exp(-2t)$$



(Time constant=1/2. DC-gain G(0)=0.5.)

Sinusoidal inputs

$G(i\omega)$ gives the stationary response of a sinusoidal input



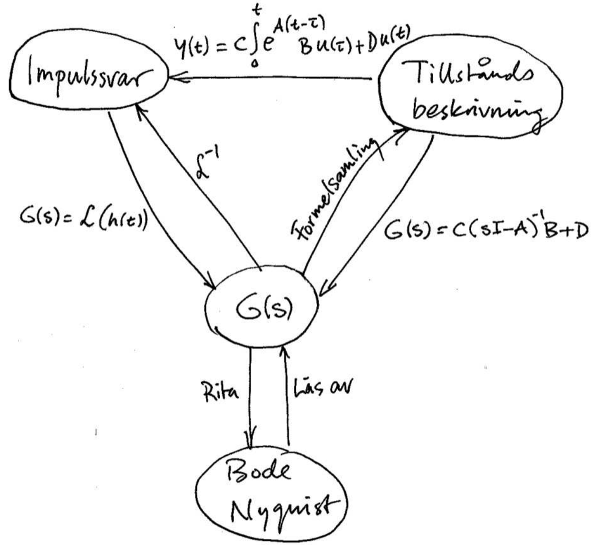
Sinusoidal input $u(t) = u_0 \sin(\omega t)$ gives (if $G(s)$ stable):

$$y(t) = u_0 \left|G(i\omega)\right| \sin(\omega t + \arg G(i\omega)) + \text{transient}$$

Amplitude gain: $|G(i\omega)|$

Phase change: $\arg G(i\omega)$

Summary of models



Some Laplace Transforms

$$\text{Step}: \; u(t) = 1, \;\; t \geq 0, \qquad U(s) = \frac{1}{s}$$

$$\text{Ramp}: \; u(t) = t, \;\; t \geq 0, \qquad U(s) = \frac{1}{s^2}$$

$$\text{Exponential}: \; u(t) = e^{-at}, \;\; t \geq 0, \qquad U(s) = \frac{1}{s+a}$$

See collection of formulas for others

Analysis

Input-output relation $Y(s) = G(s)U(s)$

If $G(s) = \frac{B(s)}{A(s)}$ then

Zeros: $B(s) = 0$

Poles: $A(s) = 0$

Final value theorem: If $sY(s)$ has all poles in left half plane

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

Static gain (DC-gain) = $G(0)$

Exam Mar08 4

Stability

Asymptotic stability: $h(t) \to 0$, e.g $G(s) = 1/(s+1)$, $h(t) = e^{-t}$

Stability: $|h(t)|$ bounded, e.g. $G(s) = 1/s$, $h(t) = 1$

If $B(s)/A(s)$ given, calculate poles from $A(s) = 0$

If (A, B, C) given, calculate eigenvalues from $\det(sI - A) = 0$

Asymptotic stability if poles in open left half plane, $\mathrm{Re}\, s < 0$

$s + a_1$ as. stable iff $a_1 > 0$

$s^2 + a_1 s + a_2$ as. stable iff $a_1 > 0, a_2 > 0$

$s^3 + a_1 s^2 + a_2 s + a_3$ as. stable iff (see collection of formulas)

Exam Mar08 2a+2b+2c

Some Favorite Systems

Integrator $G(s) = \frac{1}{s}$

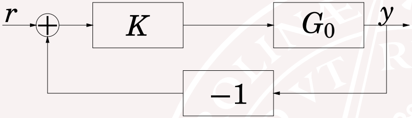
First order $G(s) = \frac{\omega_0}{s+\omega_0} = \frac{1}{1+sT}$ where $T = 1/\omega_0$

Second order $G(s) = \frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$

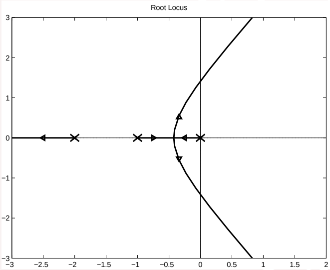
Time delay $G(s) = \exp(-sT)$

Sketch step responses, poles, Bode and Nyquist diagrams

Root locus (rotort)



$$G_0(s) = G_p(s)G_r(s) = \frac{B(s)}{A(s)}$$



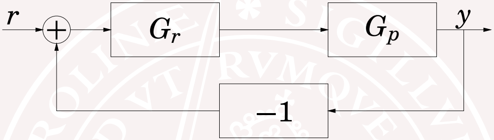
Exam Mar08, 7

Plot solutions s to

$$A(s) + KB(s) = 0$$

$K = 0$: Open loop poles
$K = 1$: Closed loop poles
$K = \infty$: Open loop zeros + rest to infinity

Block-diagrams



$G_p(s)$ open loop system

$G_r(s)$ controller

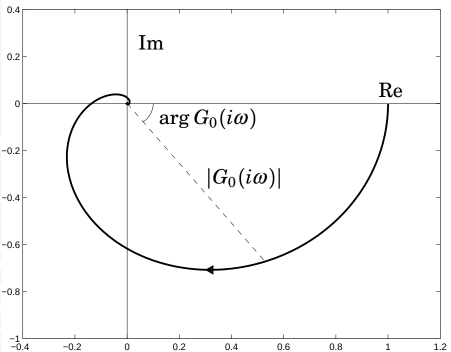
$G_0(s) = G_r(s)G_p(s)$ kretsöverföringsfunktion (Bode, Nyquist)

$G_c(s) = \dfrac{G_p G_r}{1 + G_p G_r}$ closed loop from r to y

$S(s) = \dfrac{1}{1 + G_p G_r}$, "känslighetsfunktion" (Ch7: 4 interpretations)

Exam Mar08, 3a + (8c)

Nyquist Diagram

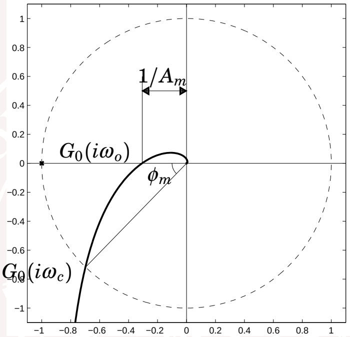


Closed loop system stable if

- $G_0(s)$ stable, and
- Nyquist curve does not encircle -1

Ex: For which K is the closed loop stable if $G_0(s) = Ke^{-s}$?

Margins in Nyquist Diagram



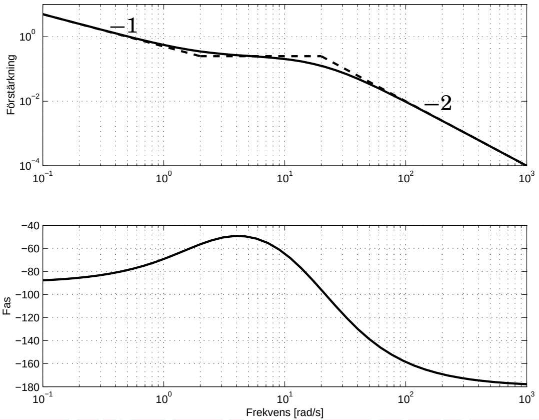
$$\text{Amplitude margin :} \quad A_m = \frac{1}{|G_0(i\omega_o)|}$$

$$\text{Phase margin :} \quad \phi_m = \pi + \arg G_0(i\omega_c)$$

$$\text{Delay margin :} \quad L_m = \frac{\phi_m}{\omega_c}$$

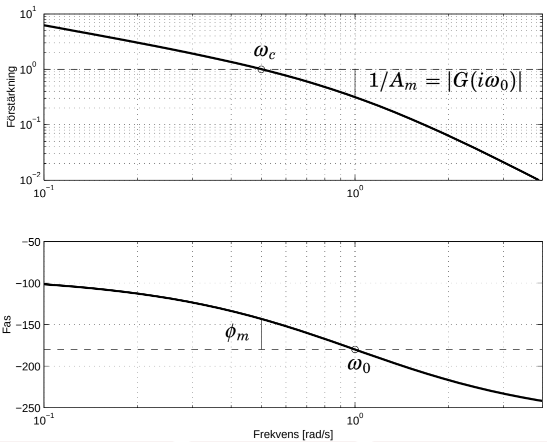
Exam Mar08, 1

Bode Diagram

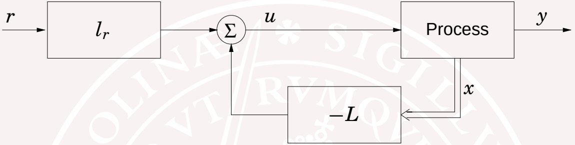


$$G(s) = \frac{100(s+2)}{s(s+20)^2)} = 0.5s^{-1} \cdot (1+0.5s) \cdot (1+0.05s)^{-2}$$

Margins in Bode Diagram



Design: State Feedback



$$\dot{x} = Ax + Bu, \qquad u = -Lx + l_r r$$

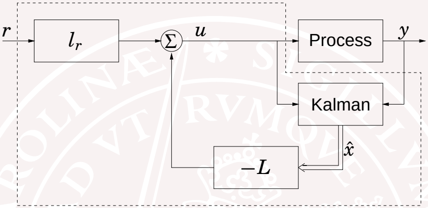
$$\text{closed loop poles:} \quad \det(sI - A + BL) = 0$$

$$C(-A + BL)^{-1} B l_r = 1$$

If (A, B) **controllable** then closed loop poles can be moved arbitrarily by proper choice of L

$$\text{Controllable} \iff \text{rank} \begin{pmatrix} B & AB & \dots & A^{n-1}B \end{pmatrix} = n$$

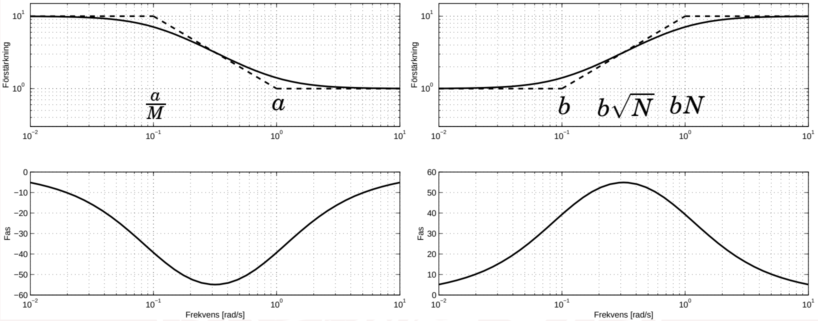
Kalman Filter



$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x})$$
$$u = -L\hat{x} + l_r r$$
$$\text{closed loop poles:}\quad \det(sI - A + BL)\det(sI - A + KC) = 0$$
$$C(-A + BL)^{-1}Bl_r = 1$$
$$(A, C)\textbf{observable} \iff \text{rank}\begin{pmatrix} C \\ \vdots \\ CA^{n-1} \end{pmatrix} = n$$

Exam Mar08, 5

Compensation in Frequency Domain

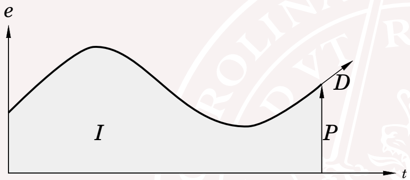


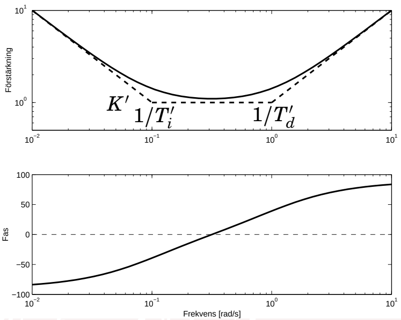
Fasretarderande länk - Fasavancerande länk

$$G_R(s) = \frac{s+a}{s+a/M}, \qquad G_R(s) = KN\frac{s+b}{s+bN}$$

Exam Mar08, 6

Design: PID control



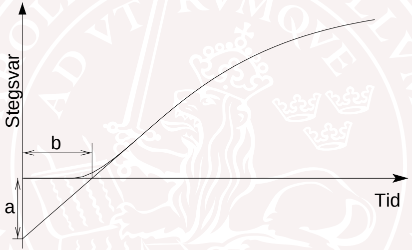


Series form $G_R'(s) = K'(1 + \frac{1}{sT_i'})(1 + sT_d')$

Parallel form $G_R(s) = K(1 + \frac{1}{sT_i} + sT_d)$

- Improved P-part: $K(br - y)$
- Improved I-part: Antiwindup
- Improved D-part: $\frac{1+sT_d}{1+sT_f} y$

Design: PID tuning



Ziegler Nichols methods (step response or self-oscillation)

See collection of formulas

Other Control Structures

- Cascade
- Feedforward
- Otto-Smith