Bicycle Dynamics and Control

thanks to K. J. Åström

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Some Interesting Questions

- How do you stabilize a bicycle?
  - By steering or by leaning?
- Do you normally stabilize a bicycle when you ride it?
- Why is it possible to ride with no hands?
- How is stabilization influenced by the design of the bike?
- Why does the front fork look the way it does?

The main message:
  - A bicycle is a feedback system!
  - The front fork is the key!
Tilt Dynamics

Like an inverted pendulum model

Transfer function: $\phi(s) = k \frac{s + V_0/a}{s^2 - mgl/J} \delta(s), \quad k = \frac{amlV_0}{bJ}$
The Front Fork

With a positive trail the front wheel lines up with the velocity (caster effect). The trail also creates a torque that turns the front fork into the lean. A static torque balance gives

\[ \delta = -k_1 \phi + k_2 T \]

Control variable: Handlebar torque \( T \)
Process variables: Steering angle \( \delta \), tilt angle \( \phi \)
Control variable: Handlebar torque $T$

Process variables: Steering angle $\delta$, tilt angle $\phi$

A feedback system

$$\frac{k(s + V_0/a)}{s^2 - mg\ell/J}$$
How Steer Torque Influences Steer Angle

Transfer function from $T$ to $\delta$ is

$$\frac{k_2}{1 + k_1 \frac{k(s+V_0/a)}{s^2-mgl/J}} = k_2 \frac{s^2-mgl/J}{s^2 + k_1 ks + k_1 kV_0/a - mgl/J}$$
This equation is stable if \( k_1 k V_0 / a > m g \ell / J \)

Since \( k = \frac{a m \ell V_0}{b J} \) this becomes

\[
V_0 > V_c = \sqrt{b g / k_1}
\]

where \( V_c \) is the critical velocity.

By the design of the front-fork the system is self-stabilizing for high enough speeds!

For low speeds, an extra controller (you) uses \( T \) to stabilize \( \phi \)

Leaning was not modeled above. Mainly used when turning.

Think about this next time you bike!
Warning: Response for $T = \text{impulse}$ (without lean)

The position on the road, $\gamma$, moves initially in the intended direction, but due to fork dynamics then in the wrong direction.

Some motorcycle accidents are caused by this effect, which is due to the right half plane zero.

Understanding control dynamics can save your life!
Many people have seen theoretical advantages in the fact that front-drive, rear-steered recumbent bicycles would have simpler transmissions than rear-driven recumbents and could have the center of mass nearer the front wheel than the rear. The U.S. Department of Transportation commissioned the construction of a safe motorcycle with this configuration. It turned out to be safe in an unexpected way: No one could ride it.


Same equations as above, but with $V_0 < 0$

Similar analysis as above shows that this leads to a transfer function which can be uncontrollable!
Klein’s Un-ridable Bike
Over a dozen clinics for children and adults with a wide range of disabilities, including Down syndrome, autism, mild cerebral palsy and Asperger’s syndrome. More than 2000 children aged 6-20 have been treated, see http://www.losethetrainingwheels.org
A bicycle in zero gravity is unrideable

https://www.youtube.com/watch?v=rNQdSfgJDNM