LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRTF05

Exam January 8, 2020, 8–13

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminary grades:

Betyg 3:	12 points
4:	17 points
5:	22 points

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!

1. The following differential equation describes a system.

$$y''(t) + 3y'(t) + 4y(t) = u'(t) + 3u(t)$$

a. Determine the transfer function of the system. (1 p)

b. Determine a state-space representation of the system. (1 p)

c. Is the system stable? (0.5 p)

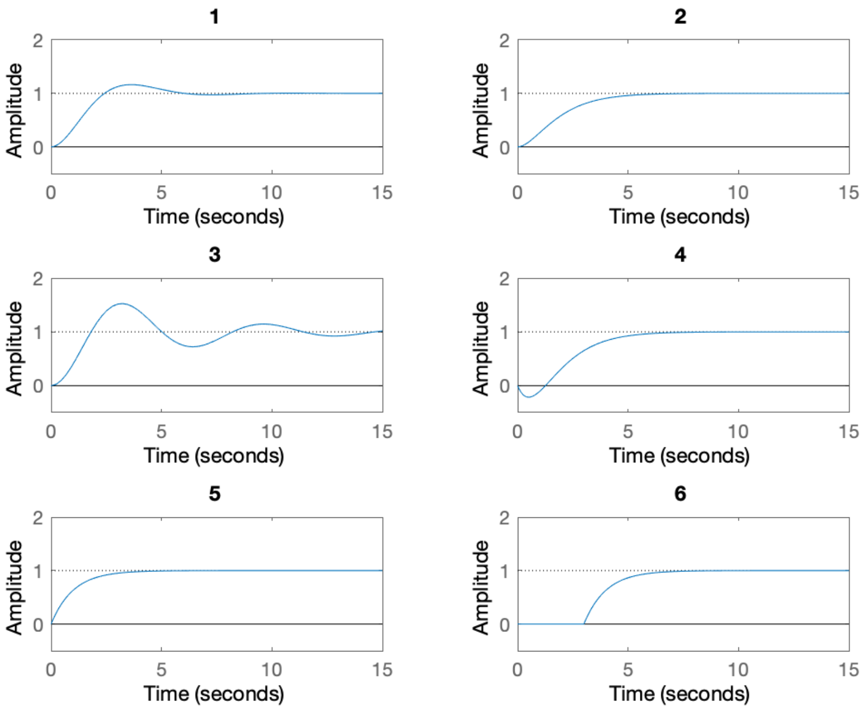
2. Compbine the step responses in Figure 1 with the right transfer function. The answers must be motivated. (3 p)

$$G_1 = \frac{e^{-3s}}{s+1} \qquad G_2 = \frac{e^{-7s}}{s+1}$$

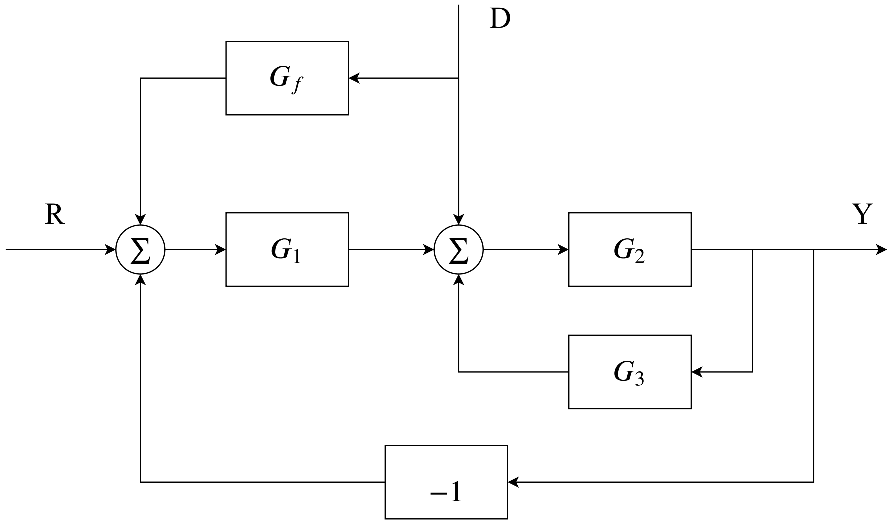
$$G_3 = \frac{1}{s^2 + 0.4s + 1} \qquad G_4 = \frac{1}{s^2 + s + 1}$$

$$G_5 = \frac{1}{s^2 + 2s + 1} \qquad G_6 = \frac{-s+1}{s^2 + 2s + 1}$$

$$G_7 = \frac{5}{s+1} \qquad G_8 = \frac{1}{s+1}$$



Figur 1 Bodediagram till uppgift 2



Figur 2 Blockdiagram till uppgift 3

3. Figur 2 shows a block diagram.

a. Determine the transfer function from R to Y. (2 p)

b. Determine feedforward G_f so that disturbance D is eliminated in Y. (1 p)

4. The following statements deal with a closed-loop system with a P, PI or PID controller. Decide whether they are true or false. The answers must be motivated. (2.5 p)

a. The stationary control error using a P controller can be reduced by reducing gain K.

b. The primary reason for using the I term in a PI controller is to get rid of the oscillations in step responses.

c. It is always best to use a PID controller since it is the most advanced controller.

d. To eliminate load disturbances there must be an integrator in the process. Integrators in the controller are of no importance.

e. If gain K in a P controller is increased, the step response becomes faster.

5. The following nonlinear system on state-space form describes a biological system where x_1 is the biomass concentration, x_2 the concentration of the substrate, and u is the dilution rate:

$$\begin{aligned}\dot{x}_1 &= x_1x_2 - ux_1\\ \dot{x}_2 &= -2x_1x_2 + (x_2 - x_1)u\\ y &= x_1\end{aligned}$$

a. Determine all stationary points (x_1^0, x_2^0, u^0) of the system. (1 p)

b. Linearize the system at the stationary point where $x_1^0 = 1$. (2 p)

6. Consider the following system on state-space form:

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

a. Design an observer

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + K(y - C\hat{x}),$$

so that the poles of the Kalman filter are placed at the distance $\omega = 4$ from the origin with relative damping $\zeta = 0.75$. (2 p)

b. When the observer is tested in practice, it turns out that the measurement noise level is higher than expected. Explain ways to modify the design of the observer to take care of this problem. (1 p)

7. A system is given on state-space form

$$\dot{x} = \begin{bmatrix} -5 & 0 \\ 1 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

a. Is the system controllable? (1 p)

b. Determine a state feedback $u = -Lx$ so that the closed-loop poles are placed at -5. (2 p)

8. Sketch the Bode plot of a system with the following properties:

- static gain K
- a zero at frequency $\omega_z = \bar{\omega}_z(\text{rad/s})$
- a pole at frequency $\omega_{p1} = 10\omega_z$
- a pole at frequency $\omega_{p2} = 10\omega_{p1}$

For the magnitude plot, it is sufficient to show the asymptotes. Note the slopes of the lines (as usual, in terms of decades per decade) in the different parts of the magnitude plot. You don't have to give any numerical values for the different varables $(K, \omega_z, \omega_{p1}, \omega_{p2})$, the important thing is to mark them correctly in the diagram, with the correct relation between them. (2 p)

9. Suppose that the transfer function $G_P(s)$ of a process is a first order system with time delay:

$$G_P(s) = \frac{2}{s+1} e^{-0.5s}$$

We want to control the process with a proportional controller $G_R(s) = K$.

a. Show that the closed-loop system is stable for $K = 1$. (1 p)

b. Suppose that we want a faster control using $K > 1$. Which is the highest cross-over frequency $\omega_c^\star$ that can be used before the system becomes unstable? (1 p)

c. The cross-over frequency that could be reached in **b.** turned out to be too low. Describe, in words, how the controller structure can be extended to provide a faster control. (1 p)