LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRT 010

Exam October 27, 2015, 8–13

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminar grades:

Betyg 3: 12 points
4: 17 points
5: 22 points

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!

1. Consider a system with the transfer function,

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + w_0^2}.$$

a. Describe the system in terms of a differential equation (1 p)

b. Write the system on state-space form (1 p)

2. Figure 1 presents eight step responses. All plots have the same scales on the y-axis and the x-axis. The following transfer functions are given:

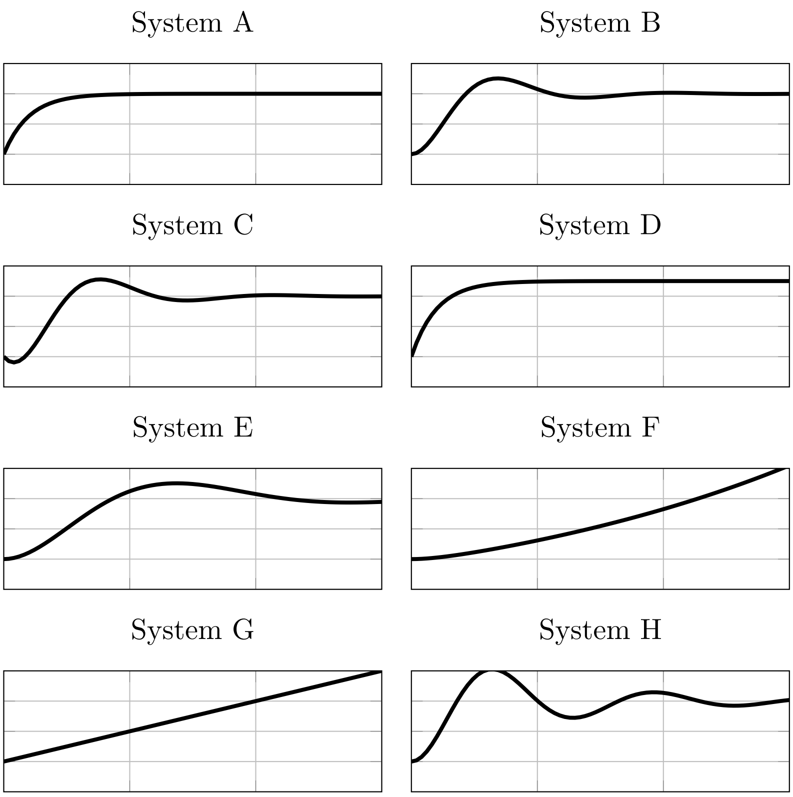
$$G_1(s) = \frac{1}{s^2 + 0.4\,s + 1} \qquad G_2(s) = \frac{1}{15\,s^2 + 14\,s - 1}$$

$$G_3(s) = \frac{1}{0.8\,s + 0.8} \qquad G_4(s) = \frac{1}{4\,s^2 + 1.6\,s + 1}$$

$$G_5(s) = \frac{1}{s + 1} \qquad G_6(s) = \frac{0.1}{s}$$

$$G_7(s) = \frac{1 - 0.5\,s}{s^2 + 0.8\,s + 1} \qquad G_8(s) = \frac{1}{s^2 + 0.8\,s + 1}$$

Pair the step responses for the systems A–H with the corresponding transfer functions $G_1(s)$-$G_8(s)$. (4 p)



Figur 1 Step responses in Problem 2.

3. Decide if the following statements are true or false. The answers must be motivated. (2 p)

a. When a process is controlled by a P controller, there will always be a stationary error after a step change in the setpoint.

b. Integral time T_i in a PI controller is the time during which the error is integrated.

c. A lead compensator gives a non-negative contribution to the phase at all frequencies.

d. Adding a pure delay to a system decreases the cross-over frequency.

4. The position z of a ball on a beam with angle ϕ relative to the horizontal plane is described by the following differential equation

$$\ddot{z}(t) = \frac{5g}{7}\sin(\phi(t))$$

where g is the acceleration of gravity. The angular velocity of the beam is controlled by a motor, where control signal u is the input voltage to the motor.The relation between the angular velocity and the input voltage is

$$\dot{\phi}(t) = 5u$$

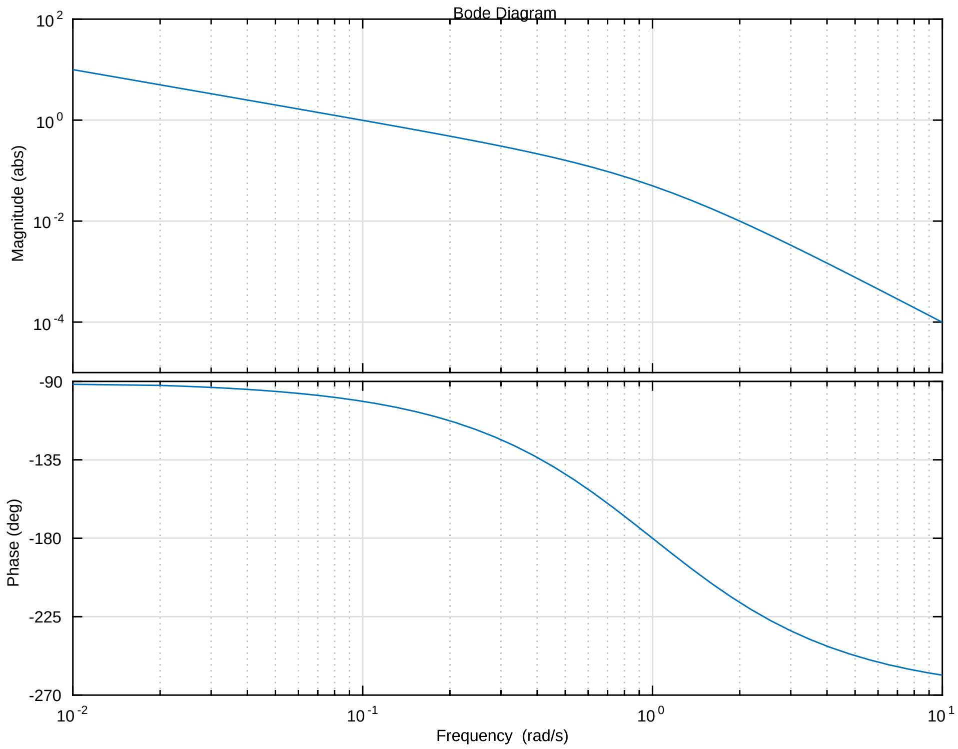
a. Write the system on state-space form, with the states $x_1 = z$, $x_2 = \dot{z}$ and $x_3 = \phi$. The measurement signal is the ball position, and the control signal is the voltage input to the motor. (1 p)

b. Linearize the system at the stationary point where $\phi^0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $z^0 = 0$, and write the state equations on matrix form. (1 p)

c. Is the system controllable? Motivate! (1 p)

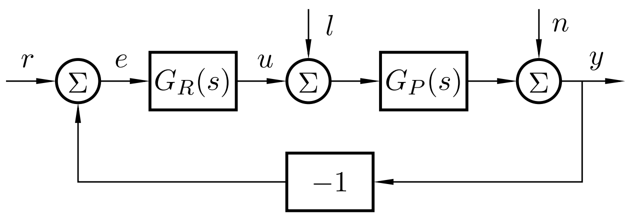
d. It is decided to control the ball on the beam using state feedback. Assume that we have access to all states of the process and design the controller $u = -Lx = -l_1x_1 - l_2x_2 - l_3x_3$ so that the closed-loop poles are placed at -5, $-5+5i$, and $-5-5i$. (2 p)

e. It turns out that even though the poles are placed to give an asymptotically stable closed-loop system, the system becomes unstable at large beam deviations. Why does the analysis not hold at large beam deviations from the horisontal plane? (1 p)



Figur 2 Bode plot of the system in Problem 5.

5. The Bode plot of a system is shown in Firgure 2. It is desired to make the system 10 times faster without reducing the phase margin more than 50%. Furthermore, there must not be any stationary error after step changes of the setpoint. Design a controller that fulfils all these requirements. (3 p)



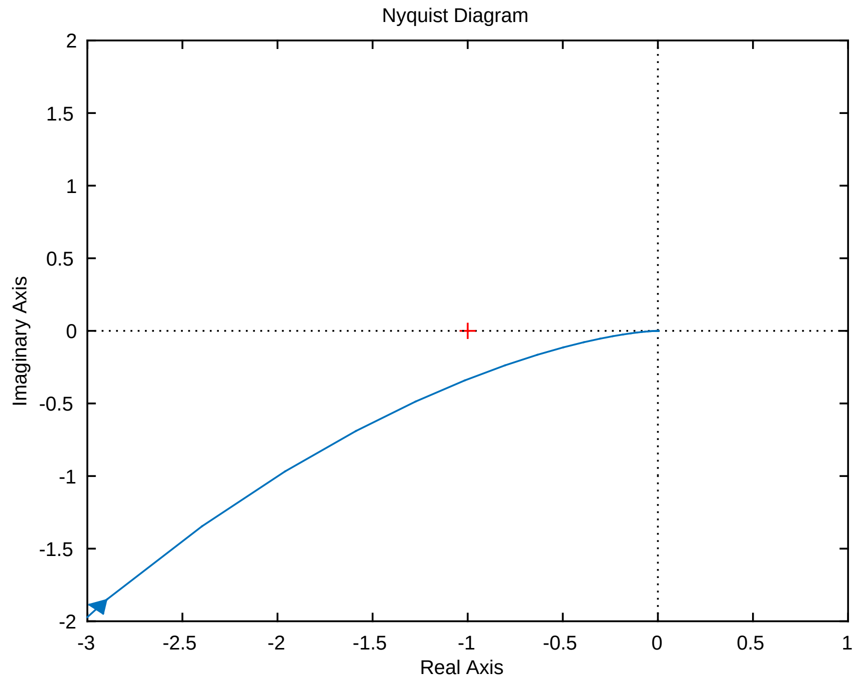
Figur 3 The system in Problem 6.

6. Consider the system in Figure 3. Remember the definition of the sensitivity function

$$S(s) = \frac{1}{1 + G_R(s)G_P(s)}.$$

a. Assume that $r(t) = 0$. Show that the sensitivity function can be interpreted as a measure of the impact of feedback on the influence of disturbances L and N on measurement signal Y. (2 p)

b. In Figure 4 the Nyquist plot of $G_o = G_p G_R$ is shown. Draw a diagram that shows for which frequences, i.e. where in the Nyquist plot, disturbances are amplified by introducing feedback. Be careful with the marks on the axis. (2 p)



Figur 4 Nyquistplot för Problem 6

7. A servo motor with transfer function

$$G_p(s) = \frac{1}{s(0.025\, s + 1)}$$

is to be controlled.

a. Design a P controller (with transfer function $G_c(s) = K$) so that the closed-loop system gets a double pole. (2 p)

b. The servo motor and the controller will be placed on different geographical places. We would like to control the process over Internet, and are going to invest in an Internet connection between the two places. In the table below, four alternative connections are presented together with the delays (Process → Controller → Process) they will introduce in this case, as well as their prices.

Type of connection	Typical delay	Price
GPRS	500 ms	$
3G	150 ms	$$
LTE	50 ms	$$$
Fiber	10 ms	$$$$

Motivate, based on the process, the controller, and the table above, which Internet connection to choose. We would, of course, not like to pay more than necessary. (2 p)