LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRTF05

Exam January 9, 2018, 8–13

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminary grades:

Betyg 3:	12 points
4:	17 points
5:	22 points

Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!

1. We have the following system

$$\dot{x} = \begin{bmatrix} 0 & -1 \\ 3 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

a. Is the system controllable? (0.5 p)

b. Is the system observable? (0.5 p)

c. Derive the transfer function of the system. Which is the order of the system? (1 p)

2. A process can be described by the following differential equation

$$\ddot{y} + \dot{y} - 12y = \dot{u} + 2u$$

a. What is the transfer function of the process? Is the process asymptotically stable? (1 p)

b. The process is controlled by a P controller. For which values of K is the system asymptotically stable? (1 p)

c. Determine the stationary error that is obtained when the reference value is a unit step change. (1 p)

3. A system is described by the following differential equation

$$2\ddot{x} + \frac{\dot{x}}{x} + x = \sqrt{(1+u)}$$

a. Which is the order of the system? Write the system on state-space form with the states $x_1 = x$ and $x_2 = \dot{x}$. (1 p)

b. Determine the stationary points of the system. (1 p)

c. Linearize the system at the point where $u = 3$. (2 p)

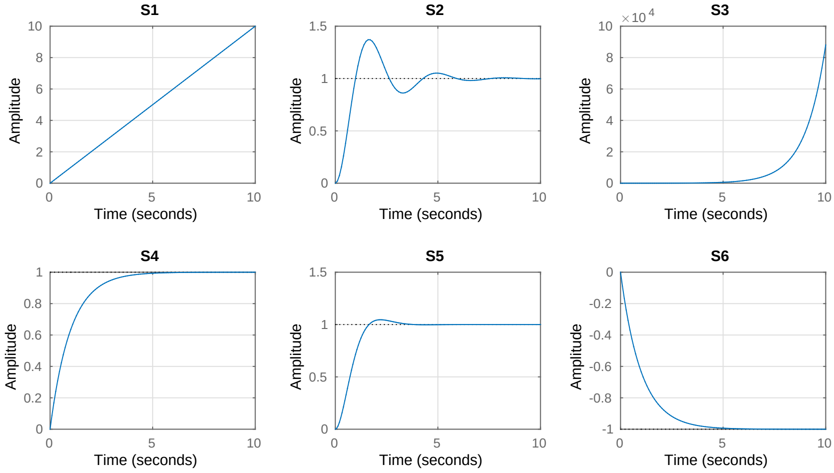
4. Figure 1 shows the step responses of six different systems and Figure 2 shows the poles of the corresponding systems. Pair the step responses with their corresponding pole diagrams. Answers must be motivated! (3 p)

5. The Nyquist plot of an unknown process is given in Figure 3.

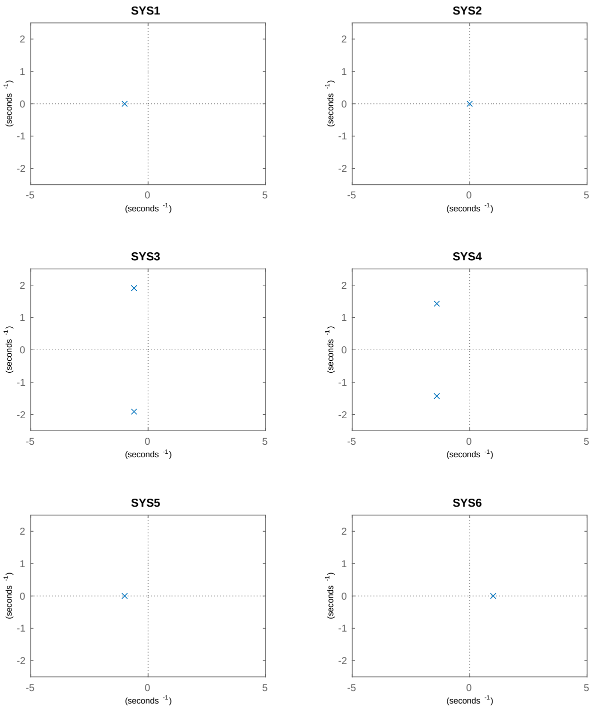
a. What is meant by gain margin and how can the gain margin be determined from a Nyquist plot? How large is the gain margin in our case? (1 p)

b. What is meant by phase margin and how can the phase margin be determined from a Nyquist plot? How large is the phase margin in our case? (1 p)

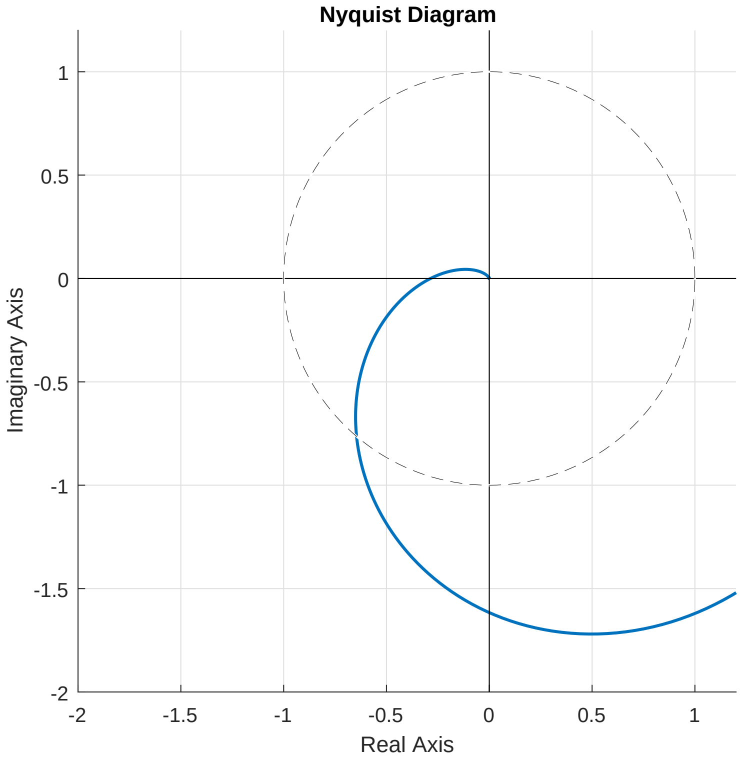
c. Suppose that the cross-over frequency is 2 rad/s. What is meant by delay margin and how large is the delay margin in our case ? (1 p)



Figur 1: Step responses in Problem 4.



Figur 2: Pole locations in Problem 4.



Figur 3: Nyquist plot for the unknown process in Problem 5.

6. Mulle Meck has found a device that he would like to use in his car. Unfortunately, he doesn't know the dynamics of the device, which makes it hard to derive a model of the device by hand.

Instead he got the idea to derive the Bode plot of the device experimentally by feeding a sinusoidal signal with unit amplitude and slowly varying frequency to the device. The amplitude and phase are then obtained from the sinusoidal outputs from the device.

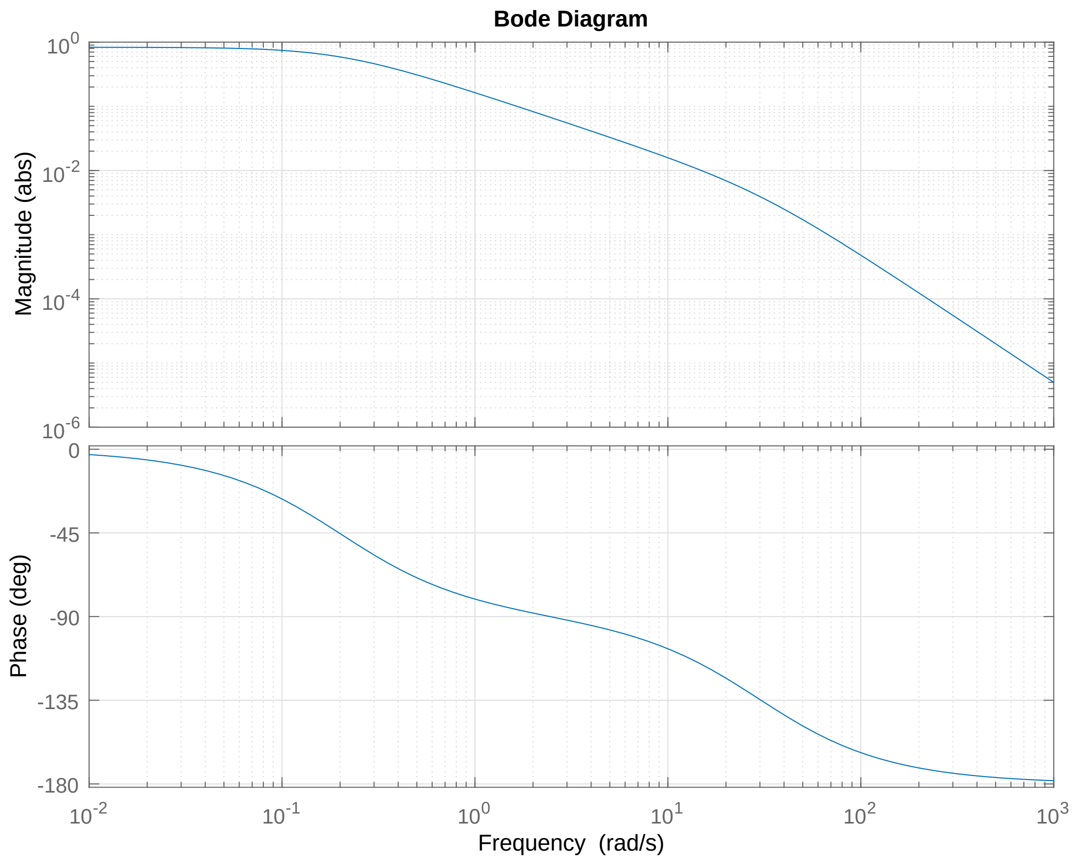
The Bode plot is shown in Figure 4. A careful look at the plot reveals that the low-frequency asymptote is $G_g(0) = \frac{5}{6}$.

a. Here Mulles control knowledge ends. Help him determining the transfer function of the device from the Bode plot. (2 p)

b. Write the dynamics on controllable state-space form. (0.5 p)

c. The properties of the device are unfortunately not acceptable, and Mulle Meck wants to change the dynamics. Help him doing this by including a state feedback so that the closed-loop poles both are placed at $s = -4$. Determine also l_r so that the static gain becomes 1.

Assume that all states are measurable. (2.5 p)



Figur 4: Bode plot of the unkown device in Problem 6.

7. The increasing consumption trend in our society has contributed to the fact that Santa's packet machine now is insufficient to meet demands before Christmas Eve. In order to keep his Christmas monopoly and not be outdone by less serious actors, Santa Claus has to solve the problem.

Investing in a further packing machine is an expensive deal when the stock market is high in magical objects. It's also not safe for the new machine to come delivered on time. Instead, Santa Claus wants to improve his current packing machine so that it is able to produce the right number of goods before Christmas.

To save Christmas, Santa Claus has hired you as a control specialist in a consulting group. The group quickly detects that the machine has a vital part that determines how fast it works. The transfer function for the vital part has been determined to

$$G(s) = \frac{1}{(s+2)^2}.$$

The vital part is now to be controlled to speed up the machine.

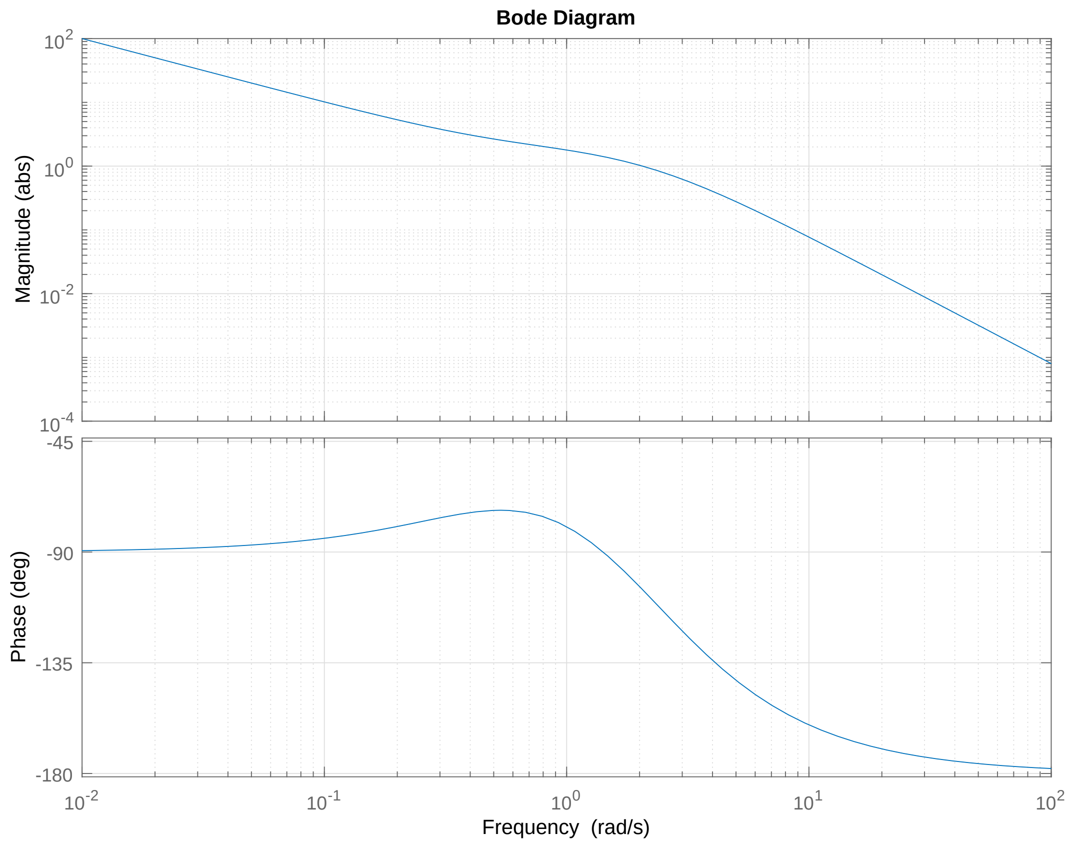
a. As a first attempt, your group decides to use a PID controller designed using pole placement. The poles should be placed on the form $(s+a)(s^2+2\zeta\omega s+\omega^2)$

so that all poles get the distance 4 to the origin and the two complex conjugate poles get a relative damping of 0.8.

Being the specialist in the team, it's now up to you to determine parameters K, T_i, and T_d so that the closed-loop system gets the desired pole placements. (2 p)

b. After some negotiations, you convince your colleagues that it will be hard to meet the specifications using just a PID controller. Instead, you suggest that a PI controller that takes care of the stationary error should be used combined with a lead-lag compensator to speed up the system.

Let $K = 8$, $T_i = 2$ in the PI controller and introduce a suitable compensator so that the system becomes 5 times faster with a phase margin of 50°. For your help, the Bode plot of the loop transfer function is given in Figure 5. (3 p)



Figur 5: Bode plot of the loop transfer function $G_0(s)$ in Problem 7 b.