LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

Automatic Control, Basic Course FRTF05

Exam October 23, 2017, 8–13

Points and grades

All solutions must be well motivated. The whole exam gives 25 points. The number of points are presented after each problem. Preliminary grades:

Betyg 3: 12 points
4: 17 points
5: 22 points

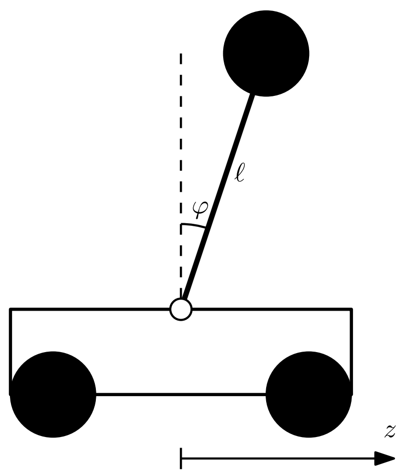
Aids

Mathematical collections of formulae (e.g. TEFYMA), collections of formulae in automatic control, and calculators that are not programmed in advance.

Results

The results are presented through LADOK. Time and place for exam presentation will be announced on the course web page.

Good luck!



Figur 1 Inverted pendulum.

1. A system is given by the following transfer function

$$G(s) = \frac{s+2}{s^2+4s+5}$$

a. Is the system asymptotically stable? (0.5 p)

b. Write the system on controllable state-space form. (0.5 p)

c. Write a differential equation describing the system. (1 p)

d. Introduce the states $x_1 = y$, $x_2 = \dot{y} - u$ and write the system on state-space form. (1 p)

2. An inverted pendulum on a cart according to Figure 1 is described by the differential equation

$$\ell\ddot{\varphi} - g\sin\varphi = -u\cos\varphi, \tag{1}$$

where φ is the angle of the pendulum defined in the figure, $\ell > 0$ is the length of the pendulum, $g > 0$ is the gravity acceleration, and $u = \ddot{z}$ is the acceleration of the cart which is the control signal (input signal).

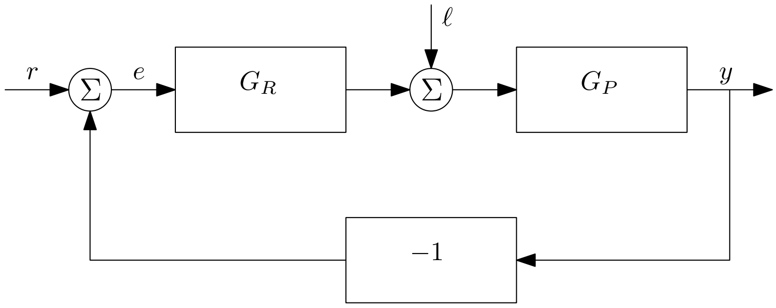
a. Introduce the angle and the angular velocity as states: $x_1 = \varphi$ och $x_2 = \dot{\varphi}$ and write the nonlinear system (1) on state-space form. The angle φ is chosen as measurement signal (output signal). (1 p)

b. Verify that $(x_1, x_2, u) = (0, 0, 0)$ is a stationary point and linearize the system at this point. (2 p)

3. After many complaints about cold student houses during the winter season, the responsible have decided to recruit you as a consultant to investigate the problem. The indoor temperature can be modelled by the differential equation

$$\dot{y}(t) = -\alpha y(t) + \beta u(t),$$

where $y(t)$ is the temperature deviation (given in °C) from the reference temperature 20 °C (i.e. $y(t) = T(t) - 20$ °C, where $T(t)$ is the indoor temperature). The input signal $u(t)$ consists of two parts: $u(t) = u_1(t) + \gamma u_2(t)$, where $u_1(t)$ is the heat from the radiators and $u_2(t)$ is the outdoor temperature given as deviation from the mean value 15 °C.



Figur 2 The simple feedback loop.

a. Determine the transfer function $G_P(s)$ from input $u(t)$ to temperature $y(t)$. (0.5 p)

b. You are informed that the temperature control is made using a simple feedback according to Figure 2, where $\ell(t) = \gamma u_2(t)$ is the influence from the outdoor temperature and the controller output is $u_1(t)$. You discover that the controller $G_R(s)$ that is used is a simple P controller: $G_R(s) = K$. Determine the two transfer functions $G_{ry}(s)$ and $G_{\ell y}(s)$ from reference $r(t)$ to process output $y(t)$ and from load disturbance $\ell(t)$ to process output $y(t)$, respectively. (1 p)

c. When the outdoor temperature is decreased by 5 °C from the mean 15 °C, a load disturbance occurs that can be described as

$$\ell(t) = \begin{cases} 0, & t < 0 \\ -5\gamma, & t \geq 0 \end{cases}. \tag{2}$$

Determine the stationary change in the indoor temperature, i.e.. $y(t)$ when $t \to \infty$ (we assume that $r(t) = 0$), that results from this disturbance. Assume that $\alpha = 8$, $\beta = 4$, $K = 2$ and $\gamma = 2$. (1 p)

d. You suggest that the P controller is replaced by the PI controller

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)\,\mathrm{d}\tau\right),$$

where $K > 0$ and $T_i > 0$. The other parameter values are the same as before. Determine the transfer function $G_{\ell y}(s)$ from $\ell(t)$ to $y(t)$ and determine the stationary value of the process output $y(t)$ when $r(t) = 0$ and the load disturbance is given by (2). (1.5 p)

4. An electric motor used to control a robot arm can be described by

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -4 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x,$$

where x_1 is the motor angle, and also process output y, x_2 is the angular velocity of the motor, and control signal u is the current to the motor.

a. To have to robot arm moving as we want, we need a good controller. We choose to let the control signal be a linear combination of the states and the reference signal, i.e.

$$u = \ell_r r - Lx, \tag{3}$$

where ℓ_r and $L = [\,\ell_1 \quad \ell_2\,]$ are design parameters that we have to determine. We want the characteristic polynomial of the closed-loop system, i.e. the system that is controlled by our controller and has r as input, to be

$$p_1(s) = s^2 + 2\zeta\omega s + \omega^2, \tag{4}$$

for some given values of ω and ζ. Determine parameters ℓ_1 and ℓ_2 (expressed using ω and ζ) so that the closed-loop system gets the characteristic polynomial (4). (2 p)

b. Determine ℓ_r so that the static gain between r and y becomes 1. (1 p)

c. To use our controller, we must know the values of the state variables x_1 and x_2, but the only signals available are u och y. Using a Kalman filter

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + Bu + K(y - \hat{y})\\ \hat{y} &= C\hat{x},\end{aligned}$$

where $K = [\,k_1 \quad k_2\,]^T$ are design parameters, we get estimates $\hat{x}_1$ and $\hat{x}_2$ of the state variables x_1 och x_2. We assume that the polynomial (4) has the values $\zeta = 0.25$ and $\omega = 2$. To obtain a state estimation that is faster than the dynamics of the closed-loop system, we want the characteristic polynomial of the Kalman filter to have poles with the double distance from the origin. Therefore we choose $\omega = 4$ and

$$p_2(s) = s^2 + 2s + 16. \tag{5}$$

Determine the values of k_1 and k_2 so that the estimation error $\widetilde{x} = x - \hat{x}$ declines according to the characteristic polynomial (5). (2 p)

d. Vad kan det finnas för problem med att, för en verklig process, välja det karakteristiska polynomet (5) för skattningsfelet så att felet avtar väldigt snabbt, d.v.s. vilken avvägning måste göras i valet av snabbheten för Kalmanfiltret? (1 p)

5. A process that is disturbed by a load disturbance v according to Figure 3 is to be controlled. The goal is to design controllers R_1 and R_2 so that the process output y follows the reference r well.

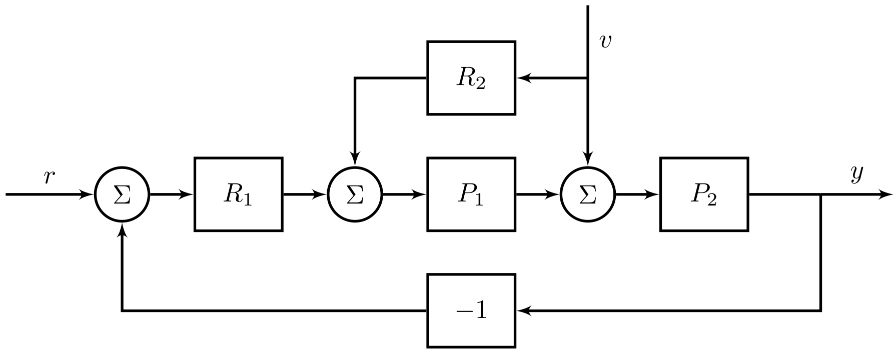
a. Determine the transfer functions to y from r and v, respectively. (1 p)

b. How should R_1 and R_2 be determined in order to eliminate the effects of the load disturbance on the process output? Are these choices always useful in practice? (1 p)

The system is given by $P_1 = \frac{1}{s+4}$ and $P_2 = \frac{1}{s}$, and it is desired to control it using proportional controllers $R_1 = K_1$ and $R_2 = K_2$.

c. Determine K_1 so that the closed-loop system gets a double pole in -2. (1 p)

d. Determine K_2 so that constant load disturbances are eliminated. (1 p)



Figur 3 Block diagram for Problem 5

6. We want to control a process given by

$$G_P(s) = \frac{3}{(s+4.4)^2}$$

and decide to use a PI controller

$$G_R(s) = K\left(1 + \frac{1}{sT_i}\right).$$

The closed-loop system is shown in Figure 4. If the integral time is chosen to be $T_i = 1/37$, the loop transfer function becomes

$$G_0(s) = G_P(s)G_R(s) = \frac{3K(s+37)}{s(s+4.4)^2}.$$

The Bode plot for $G_0(s)$ when $K = 1$ is presented in Figure 5.

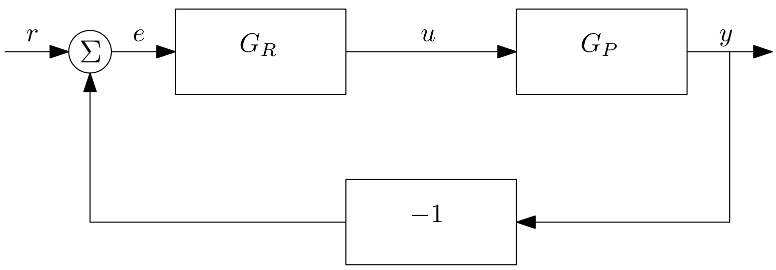
a. Use the Bode plot to determine the largest value of K that can be used in order to keep the closed-loop system stable. (1 p)

b. Assume that $K = 1$ and that a deadtime is added to the loop transfer function $G_0(s)$. Use the Bode plot to determine how long this deadtime may be without causing instability. (1 p)

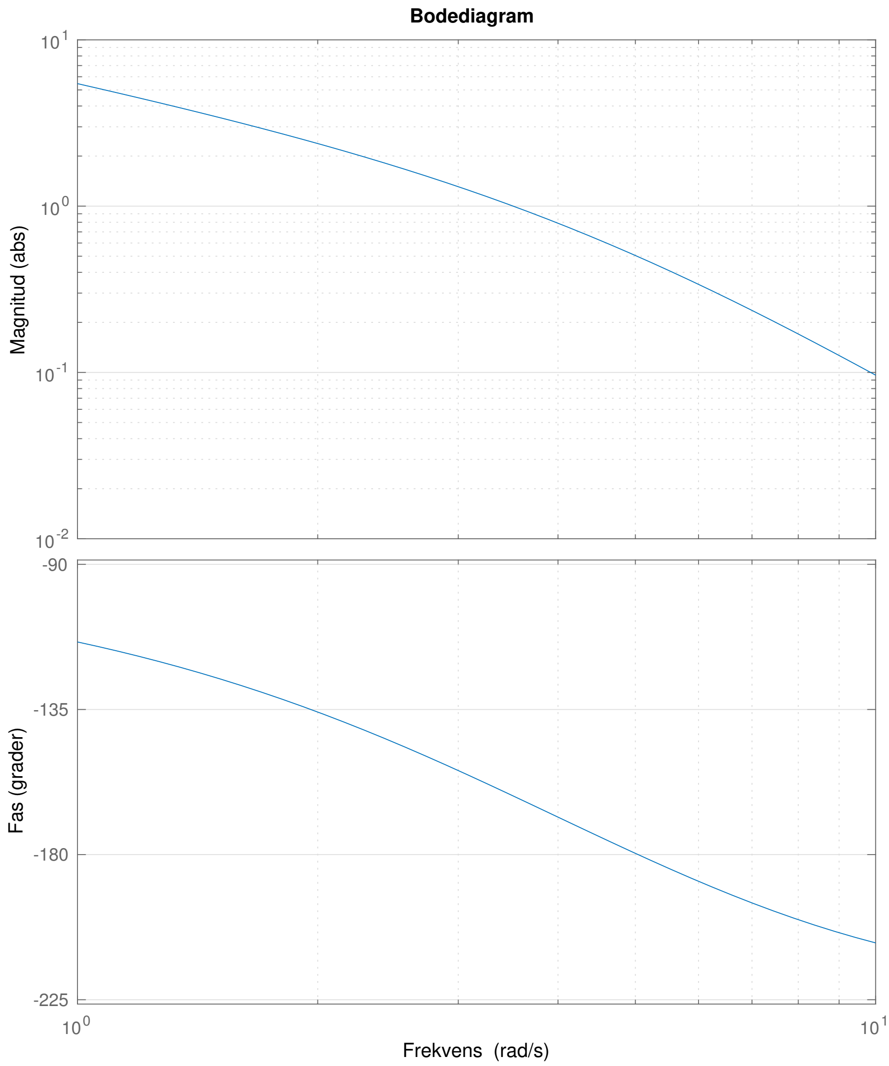
c. We now want to modify the controller so that the closed-loop system becomes faster, still retaining a desired robustness. Therefore, we replace the PI controller with the controller

$$G_R(s) = K\left(1 + \frac{1}{sT_i}\right)G_K(s),$$

where $G_K(s)$ is a transfer function that is to be determined. We assume once again that $K = 1$, $T_i = 1/37$, and that no deadtime is present. To make the closed-loop system faster, the cross-over frequency of the loop transfer function is to be increased, and to obtain the desired robustness a certain phase margin is specified. Determine $G_K(s)$ so that the cross-over frequency becomes 5 rad/s and the phase margin 30°. (3 p)



Figur 4 The feedback loop in Problem 6.



Figur 5 The Bode plot of the loop transfer function in Problem 6.