Exercise for Optimal control – Week 5

Choose **one** problem to solve.

Exercise 1. Use tent method to derive the KKT condition (google it if you don't know) for the nonlinear optimization problem:

$$\min f(x)$$

subject to

$$\begin{aligned} g_i(x) &\leq 0, \quad i = 1, \cdots, m \\ h_j(x) &= 0, \quad j = 1, \cdots, l \end{aligned}$$

where f, g_i, h_j are continuously differentiable real-valued functions on $\mathbb{R}^n$.

Exercise 2. Find a variation of inputs u_ϵ near u_* that generate the deviation vectors of the form $\sum_{i=1}^q k_i v_i(t_f)$ for $k_i \geq 0$, where $v_i(t_f)$ is generated by

$$u_{i,\epsilon}(t) = \begin{cases} w_i, & t \in (\tau_i - \epsilon, \tau_i] \\ u_*(t), & \text{otherwise} \end{cases}$$

See lecture note. *Hint*: consider the combined needle variation

$$u_\varepsilon(t) = \left\{ \begin{array}{ll} w_i, & t \in (\tau_i - k_i\varepsilon, \tau_i] \text{ for some } i \in \{1, \cdots, q\} \\ u_*(t), & \text{otherwise} \end{array} \right. .$$

Then find $x_\epsilon(t_f)$ and $\frac{\partial x_\epsilon(t_f)}{\partial t}|_{\epsilon=0+}$. Start with $q = 2$.

Exercise 3. Consider driving a cart (a unicycle, or Dubins car) on the plane

$$\begin{aligned} \dot{x} &= v \cos\theta \\ \dot{y} &= v \sin\theta \\ \dot{\theta} &= \omega \end{aligned}$$

where (x, y) represents the position of the cart and θ the heading angle, the driving speed is a constant $v > 0$, see Figure 1. There is only one control: the turning rate ω, which is bounded by

$$|\omega| \leq \frac{v}{R}$$

for some positive constant R. Study the time optimal control problem of driving the cart from initial position at

$$(x(0), y(0), \theta(0))^\top = (0, 0, 0)^\top$$

to

$$(x(t_f), y(t_f), \theta(t_f))^\top = (x_f, y_f, \theta_f)^\top \in \mathbb{R}^3$$

What are the possible types of trajectories joining the initial and terminal states? *Note 1: there may exist singular arcs! Check lecture note 4. Note 2: there may exist several solutions to the maximum principle.*

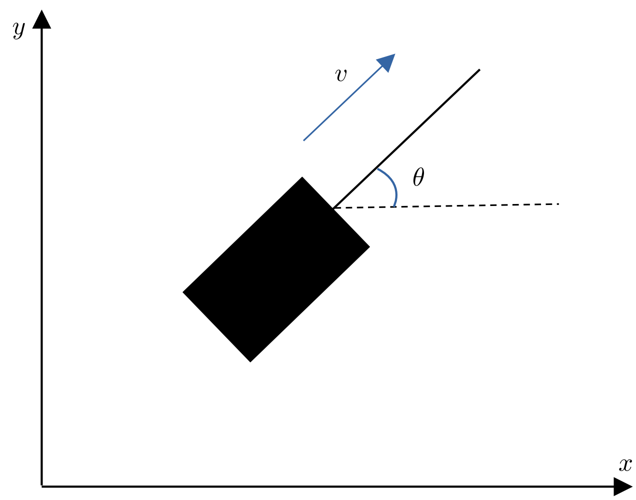

Figure 1: A unicycle.