Exercise for Optimal control – Week 2

Choose **one** problem to solve.

Exercise 1 (Insect control)**.** Let $w(t)$ and $r(t)$ denote, respectively, the worker and reproductive population levels in a colony of insects, e.g. wasps. At any time t, $0 \leq t \leq T$ in the season the colony can devote a fraction $u(t)$ of its effort to enlarging the worker force and the remaining fraction $u(t)$ to producing reproductives. The per capita mortality rate of workers is μ and the per capita natality rate is b when full effort is put on the worker population. Assume $\mu < b$. The two populations are governed by the equations

$$\dot{w} = (bu - \mu)w$$
$$\dot{r} = c(1 - u)w$$

with $(w(0), r(0) = (1, 0)$, where u satisfies the constraint $0 \leq u(t) \leq 1$. The objective is to maximize $r(T)$ or minimize

$$J = -r(T).$$

Exercise 2 (Time optimal control of a lunar lander)**.** Study the time optimal control of the moon lander problem (see lecture notes for the model). In addition, argue that there is an optimal feedback controller.

Exercise 3 (Minimum fuel and time control)**.** Consider the planar system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

where u satisfies the constraint $|u(t)| \leq 1$ for all $t \in [0, t_f]$. For any given initial state (ξ_1, ξ_2), find an optimal control u_* which drives the state to $(0, 0)$ while minimizing

$$J = t_f + \int_0^{t_f} |u(t)|\mathrm{d}t = \int_0^{t_f} (1 + |u(t)|\mathrm{d}t.$$

(Be careful, this exercise is difficult! In this example, we penalize both the terminal time and fuel consumption.)