Motion Planning - Trajectory calculation, PRM, RRT

1. Trajectory planning
   a) Lin and ptp are the two most common methods for trajectory planning, describe them briefly.
   b) The simplest way to calculate a trajectory (ptp) is a 3rd order polynomial. Why shouldn’t this be applied?
   c) Calculate the progression of a two-axis manipulator using a 5th-order polynomial. The parameters of the joints are as follows:

   \[
   \begin{align*}
   \theta(\tau = 0) &= \begin{bmatrix} 50^\circ \\ 120^\circ \end{bmatrix}, & \dot{\theta}(\tau = 0) &= \begin{bmatrix} 0^\circ/s \\ 0^\circ/s \end{bmatrix}, & \ddot{\theta}(\tau = 0) &= \begin{bmatrix} 0^\circ/s^2 \\ 0^\circ/s^2 \end{bmatrix}, \\
   \theta(\tau = 1) &= \begin{bmatrix} -30^\circ \\ 90^\circ \end{bmatrix}, & \dot{\theta}(\tau = 1) &= \begin{bmatrix} 20^\circ/s \\ 30^\circ/s \end{bmatrix}, & \ddot{\theta}(\tau = 1) &= \begin{bmatrix} 0^\circ/s^2 \\ 0^\circ/s^2 \end{bmatrix}.
   \end{align*}
   \]

   Formulate the path polynomials under the normalized time \( \tau \).
   d) Discuss a method for calculating a linear trajectory in the Cartesian space.

2. Probabilistic Random Maps (PRM)
   a) Explain what the difference between single-query and multiple-query is.
   b) In Fig. 4 are random samples marked as blue crosses. Use the PRM method to create the roadmap with \( k = 3 \).

3. Rapidly Exploring Random Trees (RRT)
   a) Describe the RRT algorithm. Create at first a schedule and explain the steps of the method.
   b) Explain the enhancements of the RRT*. Which new methods are introduced and to which optimization do they lead?
   c) Finally program the RRT and RRT* in the provided Matlab script. Test how the algorithm changes according to a variation of the parameters. Vary:
   - goal bias
   - step size
   - obstacle position and volume
   - search range
Figure 1: PRM: Start [1—1]  Goal [8—6]