





Control System Synthesis - Model Predictive Control PHD CLASS - FALL 2020



Basic idea How does MPC work? Design parameters

Important issues

Going further

Robust MPC Stochastic MPC

Stochastic MPG

Running MPC faster and explicit MPC

Adaptive and Gain-scheduled MPC

Nonlinear MPC

Data-driven MPC

1 Introduction

2 Fundamentals

3 Design techniques

- PID control
- Optimal control and LQG
- Robust control

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- Optimal control and LQG
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- Model Predictive Control
- Adaptive Control
- Data-driven Control

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MPC is a form of feedback control

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- MPC is a form of feedback control
- that solves an optimization problem to select the optimal control at each time step

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- MPC is a form of feedback control
- that solves an optimization problem to select the optimal control at each time step
- using a model to predict the future behaviour on a given horizon

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- MPC is a form of feedback control
- that solves an optimization problem to select the optimal control at each time step
- using a model to predict the future behaviour on a given horizon
- under constraints

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- MPC is a form of feedback control
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- using a model to predict the future behaviour on a given horizon
- under constraints

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sample time

- prediction horizon
- control horizon
- constraints
- weights



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sample time

- prediction horizon
- control horizon
- constraints
- weights
- \rightarrow influence the controller performances



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- sample time
- prediction horizon
- control horizon
- constraints
- weights
- \rightarrow influence the controller performances
- \rightarrow influence computational complexity



Sample time T_s

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Sample time T_s





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Sample time T_s



- **T**_s too big \Rightarrow slow response to disturbances/setpoint changes
- T_s too small \Rightarrow faster response but computational load increase

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Sample time T_s



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• T_s too big \Rightarrow slow response to disturbances/setpoint changes

MPC controller

- T_s too small \Rightarrow faster response but computational load increase
- **Recommendation:** $\frac{T_r}{20} \leq T_s \leq \frac{T_r}{10}$

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Prediction horizon N_p





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Prediction horizon N_{D}





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Prediction horizon $N_{\rm p}$

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• N_{ρ} too small \Rightarrow may be too late to react to disturbances

■ N_p too big ⇒ computational load increase (potentially unnecessary due to potential disturbances)

Past Future Predicted output Setpoint Future control actions $k \ k+1$

Prediction horizon N_p



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- N_p too small \Rightarrow may be too late to react to disturbances
- N_p too big ⇒ computational load increase (potentially unnecessary due to potential disturbances)
- Recommendation: Choose N_p to cover the significant dynamics of the system

$$N_{
m p}T_{s}pprox T_{settling}$$

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Adaptive and Gain-scheduled MPC

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Control horizon N_c

 \rightarrow usually only the first couple of control moves have a significant effect on the predicted output behavior



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Control horizon N_c

 \rightarrow usually only the first couple of control moves have a significant effect on the predicted output behavior



N_c too small \Rightarrow not enough degrees of freedom to reach the objective

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How does MPC work?

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Control horizon N_c

 \rightarrow usually only the first couple of control moves have a significant effect on the predicted output behavior





- \blacksquare N_c too small \Rightarrow not enough degrees of freedom to reach the objective
- N_c too big \Rightarrow computational load increase

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Design parameters

Control horizon N_c

 \rightarrow usually only the first couple of control moves have a significant effect on the predicted output behavior





- N_c too small \Rightarrow not enough degrees of freedom to reach the objective
- N_c too big \Rightarrow computational load increase
- **Recommendation:** 10 to 20% of N_p , minimum 2-3 steps

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MPC design

Design parameters

Constraints

- on the inputs, outputs or inputs variations
- Soft constraints
- Hard constraints



Design parameters

Constraints

- on the inputs, outputs or inputs variations
- Soft constraints: can be violated
- Hard constraints: cannot be violated



Adaptive and Gain-scheduled MPC Nonlinear MPC

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Design parameters

Constraints

- on the inputs, outputs or inputs variations
- Soft constraints
- Hard constraints
- conflicting hard constraints ⇒ may lead to an unfeasibility for the optimization problem



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MPC design

Design parameters

Constraints

- on the inputs, outputs or inputs variations
- Soft constraints
- Hard constraints
- conflicting hard constraints \Rightarrow may lead to an unfeasibility for the optimization problem
- Output constraints as soft constraints

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Design parameters

Constraints

- on the inputs, outputs or inputs variations
- Soft constraints
- Hard constraints
- conflicting hard constraints \Rightarrow may lead to an unfeasibility for the optimization problem
- Output constraints as soft constraints



Avoid having hard constraints on both inputs and inputs variations

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Weights

The MPC controller may have different objectives
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Weights

- The MPC controller may have different objectives
 - track as close as possible a reference signal

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Weights

- The MPC controller may have different objectives
 - track as close as possible a reference signal
 - smooth control moves to avoid aggressive control

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Weights

- The MPC controller may have different objectives
 - track as close as possible a reference signal
 - smooth control moves to avoid aggressive control
 - $\rightarrow\,$ Weights allow you to achieve a balanced performance between these competing goals

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Weights

- The MPC controller may have different objectives
 - track as close as possible a reference signal
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 - $\rightarrow\,$ Weights allow you to achieve a balanced performance between these competing goals
- relative weighting between the different objectives

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Weights

- The MPC controller may have different objectives
 - track as close as possible a reference signal
 - smooth control moves to avoid aggressive control
 - → Weights allow you to achieve a balanced performance between these competing goals
- relative weighting between the different objectives
- different weights for different signals for MIMO systems

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Weights

- The MPC controller may have different objectives
 - track as close as possible a reference signal
 - smooth control moves to avoid aggressive control
 - → Weights allow you to achieve a balanced performance between these competing goals
- relative weighting between the different objectives
- different weights for different signals for MIMO systems
- \rightarrow affect the controller performances

Linear system

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Linear system

Linear constraints



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Linear system

- Linear constraints
- Quadratic cost function



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- Linear constraints
- Quadratic cost function
- \rightarrow LTI MPC



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Linear system

- Linear constraints
- Quadratic cost function
- \rightarrow LTI MPC
 - convex optimization problem

Summary

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Linear system

MPC design

- Linear constraints
- Quadratic cost function
- \rightarrow LTI MPC
 - convex optimization problem
 - single global optimum of the cost function

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MPC design Summary

MPC design

How does MPC work?

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- Adaptive and Gain-scheduled MPC

- Linear system
- Linear constraints
- Quadratic cost function
- LTI MPC \rightarrow
 - convex optimization problem
 - single global optimum of the cost function
 - well-studied problem, many numerical methods and software to solve it

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Linear system

MPC design

- Linear constraints
- Quadratic cost function

\rightarrow LTI MPC

Summary

- convex optimization problem
- single global optimum of the cost function
- well-studied problem, many numerical methods and software to solve it

Compared to LQR:

- MPC solves the optimization problem in a smaller time window ⇒ suboptimal solution
- MPC is an online technique and can handle deviations of the system from the model

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Linear system

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Summary

- Linear constraints
- Quadratic cost function
- \rightarrow LTI MPC
 - convex optimization problem
 - single global optimum of the cost function
 - well-studied problem, many numerical methods and software to solve it

The state is often used in the cost function \rightarrow state estimator may be needed

Important issues

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Feasibility \rightarrow soft state or output constraints introducing a slack variable ε

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Feasibility → soft state or output constraints introducing a slack variable *ε* Stability

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Feasibility \rightarrow soft state or output constraints introducing a slack variable ε

- Stability → either use the minimizer of the cost function as a Lyapunov function or require the norm of the state *x* to shrink
 - Ferminal constraint $x(k + N_p) = 0$ or set (involves the LQ gain) or terminal cost
 - Infinite Output Prediction Horizon $N_p = \infty$
 - Contraction Constraint $||x(t+1)|| \le \alpha ||x(t)||$ with $\alpha < 1$

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- Feasibility \rightarrow soft state or output constraints introducing a slack variable ε
- Stability → either use the minimizer of the cost function as a Lyapunov function or require the norm of the state x to shrink
- Computational cost

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- Feasibility \rightarrow soft state or output constraints introducing a slack variable ε
- **Stability** → either use the minimizer of the cost function as a Lyapunov function or require the norm of the state *x* to shrink
- Computational cost \rightarrow design parameters and stability constraints (in the LTI MPC case)

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Basic MPC:

- the system and the prediction model are the same
- no unmeasured noise/disturbance

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Basic MPC:

- the system and the prediction model are the same
- no unmeasured noise/disturbance

Fundamental question: what about robustness to model uncertainty and noise?

Bemporad, A., and Morari, M. (1999). *Robust model predictive control: A survey.*, in *Robustness in identification and control*, Springer.

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Basic MPC:

- the system and the prediction model are the same
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Fundamental question: what about robustness to model uncertainty and noise?

Bemporad, A., and Morari, M. (1999). *Robust model predictive control: A survey.*, in *Robustness in identification and control*, Springer. **Robustness**:

- specific uncertainty range: $P \in P$, $w \in W$
- robust stability, robust performance and robust constraints fulfillment

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Nominal vs robust MPC



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Nominal vs robust MPC

Robust performances:min $\max_{\substack{U \\ w \in \mathcal{P}}} J(U, x, P, w)$

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Nominal vs robust MPC

Robust performances: $\begin{matrix} \max \\ U \end{matrix} \\ P \in \mathcal{P} \\ w \in \mathcal{W} \end{matrix}$

Robust constraints fulfillment: Constraint Tightening MPC (nominal prediction model + changing the constraints to achieve robustness).

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Robust constraints fulfillment: Constraint Tightening MPC (nominal 2 prediction model + changing the constraints to achieve robustness).

 $P \in \mathcal{P}$

 $w \in \mathcal{W}$

Open-loop or a closed-loop prediction scheme? 3

Robust performances:min max J(U, x, P, w)U

 \rightarrow include a feedback term into the prediction model

$$u = Fx + v$$

Nominal vs robust MPC

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Nominal vs robust MPC

Robust constraints fulfillment: Constraint Tightening MPC (nominal prediction model + changing the constraints to achieve robustness).

Open-loop or a closed-loop prediction scheme ?

 \rightarrow include a feedback term into the prediction model

$$u = Fx + v$$

4 Robust stability enforcement

- indirectly: performance objective and uncertainty description
- directly: type of robust contraction constraint

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A. Mesbah, *Stochastic Model Predictive Control: An Overview and Perspectives for Future Research*, in IEEE Control Systems Magazine, 2016.

Heirung, T. A. N., Paulson, J. A., O'Leary, J., Mesbah, A. *Stochastic model predictive control—how does it work?*, Computers Chemical Engineering, 2018.

Probabilistic descriptions of uncertainties (can be challenging)

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A. Mesbah, *Stochastic Model Predictive Control: An Overview and Perspectives for Future Research*, in IEEE Control Systems Magazine, 2016.

Heirung, T. A. N., Paulson, J. A., O'Leary, J., Mesbah, A. *Stochastic model predictive control—how does it work?*, Computers Chemical Engineering, 2018.

- Probabilistic descriptions of uncertainties (can be challenging)
- Noise and disturbances are stochastic variables, making the prediction model stochastic

MPC design

Basic idea How does MPC work? Design parameters

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- Probabilistic descriptions of uncertainties (can be challenging)
- Noise and disturbances are stochastic variables, making the prediction model stochastic
- Chance constraints: constraints satisfied with at least a priori specified probability level
- rooted in stochastic programming and chance-constrained optimization

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\blacksquare online optimization problem at each time step \rightarrow computationally complex
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online optimization problem at each time step → computationally complex
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online optimization problem at each time step → computationally complex usually formulated as a QP problem

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- increasing number of states/constraints, prediction and control horizons → increase complexity
- okay for systems with slow dynamics (process industry)



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- limited memory
- → design parameters
- → model order reduction
- → lower complexity
- \rightarrow maximum number of optimization iterations
 - \rightarrow suboptimal solution but still satisfies constraints

How to run MPC faster?

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Explicit MPC precomputes the optimal solution in different state-regions



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Explicit MPC precomputes the optimal solution in different state-regions
 as linear functions that are piecewise affine and continuous in x



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Explicit MPC precomputes the optimal solution in different state-regions
 as linear functions that are piecewise affine and continuous in x
 Online evaluation the state region and of u



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- Explicit MPC precomputes the optimal solution in different state-regions
 as linear functions that are piecewise affine and continuous in x
- Online evaluation the state region and of u
- Many regions \rightarrow time consuming and memory needed



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Adaptive and Gain-scheduled MPC

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Nonlinear systems + linear constraints + quadratic cost function

ightarrow linearization to benefit from the nice properties of linear MPC

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Nonlinear systems + linear constraints + quadratic cost function

→ linearization to benefit from the nice properties of linear MPC
 Adaptive MPC: linearized model computed on the fly, updated at each time step



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Nonlinear systems + linear constraints + quadratic cost function

- \rightarrow linearization to benefit from the nice properties of linear MPC
- Adaptive MPC: linearized model computed on the fly, updated at each time step
- Gain-scheduled MPC: switch between different MPC controllers for different operating points of interest



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 \blacksquare use of a nonlinear plant model \rightarrow more accurate prediction



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Nonlinear systems + nonlinear constraints + nonlinear cost function

- use of a nonlinear plant model \rightarrow more accurate prediction
- \blacksquare non convex problem, multiple local optima \rightarrow more challenging to solve

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Nonlinear systems + nonlinear constraints + nonlinear cost function

- use of a nonlinear plant model \rightarrow more accurate prediction
- \blacksquare non convex problem, multiple local optima \rightarrow more challenging to solve
- \rightarrow the efficiency depends on the used nonlinear solver

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Nonlinear systems + nonlinear constraints + nonlinear cost function

- use of a nonlinear plant model \rightarrow more accurate prediction
- \blacksquare non convex problem, multiple local optima \rightarrow more challenging to solve
- \rightarrow the efficiency depends on the used nonlinear solver
- economic MPC is part of nonlinear MPC

Going further Dealing with non-linearities

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Good MPC closed-loop performance implies an accurate model

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Good MPC closed-loop performance implies an accurate modelWhen no accurate model is available:

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- Good MPC closed-loop performance implies an accurate modelWhen no accurate model is available:
 - robust MPC (reduced performance due to worst-case conservative assumptions)

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- When no accurate model is available:
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- Good MPC closed-loop performance implies an accurate model
- When no accurate model is available:
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Huang, B., Kadali, R. *Dynamic modeling, predictive control and performance monitoring: a data-driven subspace approach*, Springer, 2008.



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- MPC and learning:

Hewing, L., Wabersich, K. P., Menner, M., Zeilinger, M. N. *Learning-based model predictive control: Toward safe learning in control*, Annual Review of Control, Robotics, and Autonomous Systems, 2020.

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 - Learning for closed-loop performances